Theoretical Aspects Regarding the Optimal Taxation of Effort With More Conditions

Prof. Dumitru MARIN PhD (dumitru.marin@csie.ase.ro)
Bucharest University of Economic Studies, Bucharest

Prof. Ion PARTACHI PhD (ionpartachi@yahoo.fr)
Academy of Economic Studies of Moldova

Assoc. prof. Madalina ANGHEL PhD (madalinaanghel08@yahoo.com)
Artifex University, Bucharest

ABSTRACT

In this article, the authors propose to conduct a pertinent analysis of the use of mathematical models in relation to optimal effort taxation. The effort taxation must be a balance between allocative efficiency and distribution. It is a question of determining the optimal tax level for three conditions. The theory is that the shift to a fixed amount, a single share, is not feasible for wage compensation, being important to note that this condition is one that must be considered. The mathematical model proposed by the authors regarding optimal taxation of multi-condition effort is relevant and suggestive for the analysis. It is well known that, in order to increase tax revenue, they must inevitably be based on the unobservable variant, which is the potential gain or, most importantly, on the individual productivity of labor. Solving a problem of this sensibility, such as the optimal taxation of multi-state effort, needs to be analyzed and conditioned by the establishment of a mathematical-econometric model that highlights both the aspects of a minimum expectation threshold or a situation that depends on the economic outcome. The authors consider the existing variants and establish variants based on a mathematical model that is analyzed to best answer the problem of the adverse selection ie the balance between allocation efficiency and distribution. We analyze the optimization problem from Lagrange's function and associated multiplier, highlighting the mathematical functions that can be used. Further, the authors consider the case of adverse selection based on asymmetric information. For the objective function and budget constraint, the authors consider that they need to add incentive restrictions, especially when they are taxed on income. By analyzing this hypothesis in depth, the authors propose and demonstrate a mathematical function that best suits these adjustment needs and solve the adverse selection. The way in which the authors suggest and demonstrate the implementation condition that is equivalent to the assertion that the plurality of admissible solutions is unclear gives a solution perspective applicable in most cases. The authors propose, by synthesizing, conditions (sentences) that demonstrate that the model used is one that must be considered and can be successfully used in optimal taxation of the multi-state

effort. In the explanatory approach, it is also appreciated that the combination of the Kuhn-Tucker multipliers of the two constructions can determine the Lagrange function. This feature built by all Kuhn-Tucker multipliers is strictly positive and the level of effort is optimal given by a series of equalities that imply a final aspect of its use.
Keywords: *optimal taxation, optimal contract, adverse selection, participation restriction, incentive restriction*
JEL Classification: *C62, H21*

INTRODUCTION

This article aims to solve and perform a scientific analysis of how to balance balance between allocation efficiency and allocation. Income, as inputs or effort, as an output must be analyzed in the two-state version of the most commonly known fixed amount and most often used in practice. However, fixed-rate taxation is limited by the lowest income, as is the case in Romania's economy, when talking about the minimum wage on the economy. From this point of view, the authors find that a single tax rate applied is not feasible under these circumstances. In the article, the authors inventory a number of analyzes made by other economists, modellers, concerned with establishing a mathematical function with regard to optimal taxation of effort that is characterized by several states. The authors identify and propose a mathematical model to be used at least in two situations. The first is an optimal solution, a context in which the authors identify in their demonstration a series of sentences to be considered by the one who tries this optimization. We will identify from several sentences, and the authors only deal with sentence number one, when symmetric information implies an optimal tax imposed by the government with some characteristics such as budget constraints, marginal utilities, usage levels, Marginal cost of labor, marginal productivity, optimal taxes. All this is the principle from which the authors go. Of course, they demonstrate from Lagrange's function that the multiplier associated with the participation restriction is in fact a mathematical function that any analyst or user of certain information needs to consider in order to impose management at micro- or macroeconomic level. The authors also deal with the asymmetric information selection based on the fact that the objective function and the budgetary restriction still have to be based on some restrictions in case of income tax. It is demonstrated on the basis of mathematical elements that the computational relationships reached by the authors are those that must mean the way of analysis in order to simplify the optimal optimization of the effort. Appreciating the association of the Kuhn-Tucker multiplier, we analyze the three constraints on which the Lagrange function based on the theorem, „all Kuhn-Tucker multipliers are strictly positive", and the optimal effort is given by mathematical equality. Finally, the authors consider that the established mathematical relationship is the one and only the one that gives the desired results.

LITERATURE REVIEW

Albanesi and Sleet (2006) are concerned about optimal, dynamic taxation. Aizenman and Frenkel (1985) analyze the correlation between optimal remuneration indexation, markets and monetary policies. Anghelache, Anghelache, Anghel, Niţă and

Sacală (2016) describe a model for analyzing financial investments and budget execution. Anghelache and Anghel (2016a), Biji, Biji, Anghelache and Lilea (2002), Anghelache (2008) describe the concepts and methods of economic statistics. Anghelache and Anghel (2016b) are concerned with the application of econometric tools in economic and financial analyzes. Benninga and Czaczkes (1997), Anghelache and Anghel (2014), Arsene and Dumitru (2007) are concerned with the usefulness of modeling in financial-monetary-banking analysis, Musiela and Rutkowski (1997) focus on martingale methods. Bollerslev, Chou and Kroner (1994, 1992), Weiss (1986) discusses the applicability of ARCH models, Fiorentini and Maravall (1996) analyzes their unobserved components. Lee and Hansen (1994) study the asymptotic theory for a GARCH specific estimator. Elliott and Kopp (1998) develops mathematical tools dedicated to financial markets. Tam and Reinsel (1997) analyze some features of the ARIMA models. Birchenhall, Bladen-Hovell, Chui, Osborn and Smith (1989) develops a seasonal consumption pattern. Burmeister, Flood and Turnovsky (1981) analyze the macroeconomic stability according to the equilibrium of the money market and the human resources market. Capinski and Kopp (1999), Stirzaker (1999) analyzes some aspects of mathematical probabilities. Chen (1996) analyzes the correlation between interest rate dynamics, derivative prices and risk management. Bisin and Gottardi (2006), Guerrieri, Shimer and Wright (2010) consider the application of adverse selection in the context of competitive equilibrium. Karatzas and Shreve (1998), Blyth and Robertson (1998), Capinski and Zastawniak (2003) are reference works in the field of financial mathematics. Cullen and Gordon (2006) assess the impact of tax reforms on entrepreneurial activities. Elton and Gruber (1995) is concerned with portfolios theory and investment analysis, Muthuraman and Kumar (2006) focus on portfolios optimization. Harvey (1993) describes the modeling and analysis of time series. Hansen (1985) studies the correlation between invisible work and the economic cycle. Jacobs and Bovenberg (2011) analyze the correlation between optimal taxation of human capital and the gain function. Wilmott (2001), Pliska (1997) describe fundamental aspects of financial mathematics. Marinescu, Ramniceanu and Marin (2008) analyze the Pareto-type optimality for contracts and the measurement of staff satisfaction. Blanchard and Kahn (1980) study the solving of linear models with rational expectations. Chetty (2012) analyzes some aspects of elasticity. Dumitru, Stancu and Marinescu (2004) presents the theory of general equilibrium, including practical approaches to major concepts. Farhi and Werning (2013) assess some aspects of insurance and taxation, Grochulski and Piskorski (2010) focus on tax enforcement cases. Judd (1987) analyzes welfare costs associated with factor taxation in the context of a perfect forecasting model. Marston (1984) develops on real wages and indexation rules in an open economy. Piketty and Saez (2012) theorizes optimal capital taxation. Rose (1977) is concerned about some features in using ARIMA models.

RESEARCH METHODOLOGY AND DATA

A central theme of Adverse Selection is that of the balance between allocative efficiency and distribution. This conflict was highlighted by Mirrlees (1971), winner of the Nobel Prize in Economics.

The issue of optimal taxation of income or input was first analyzed by Mirrlees (1988) in the two-state version of fixed, non-taxable amount of taxation. Fixed charge is limited by the lowest income (single rate is not feasible).

It has been shown that in order to increase tax revenue, it must be based inevitably on the unobservable variable (as potential gains) or on the productivity of the individual.

Maskin and Riky (1989) analyzed how tax levels would be affected if instead of observing „income," the government would have noticed the „input" individually, that is, the equivalent workload, the number of hours worked.

In the paper we determine the optimum level of taxation for three states, both in symmetric information and in asymmetric information. Local and global (upward and downward) inciting locality restrictions are analyzed. Applies when the change in fixed amount (single rate) is not feasible.

Fees must be based on the possible earnings or productivity of the individual.

1. The mathematical model

Assume that an output q (an income) is obtained with an input (effort) e, according to the productivity function $q=\theta F(e)$.

The parameter θ is one of productivity and can take three values (three tax installments) denoted θ_L, θ_M, θ_G, cu $\theta_L<\theta_M<\theta_G$ (low, medium and high productivity).

We assume that the proportions in which individuals are L, M or G (subjective or objective probabilities) are Π_L, Π_M, Π_G, cu $\Pi_L + \Pi_M + \Pi_G = 1$ and all strictly positive.

Economic agents (individuals) have the same utility function as:

$U(q'-t-\psi(e))$,

where:

$U'(\bullet)>0$, $U''(\bullet)<0$, and the variables have the following meanings:

q = output-ul;

t = net tax that the individual has to pay or receive from the government (subsidy);

$\psi(e)$ = the effort function, increasing and convex.

Then the government's budget cut is:

$\Pi_L t_L + \Pi_M t_M + \Pi_G t_G \geq u(p)$,

where:

$t_L \geq 0$, $t_M \geq 0$, $t_G \geq 0$, and u a minimum threshold expected by the government.

In the absence of adverse selection (symmetric information), if the government seeks to maximize social utility (the sum of individual utilities weighted with probabilities of realization) then the following problem should be solved:

P(1) Max $t_L, t_M, t_G, l_L, l_M, l_G$ $\{\Pi_L \cdot U[\theta_L l_L - t_L - \psi(l_L)] + \Pi_M \cdot U[\theta_M l_M - t_M - \psi(l_M)] + \Pi_G \cdot U[\theta_G l_G - t_G - \psi(l_G)]\}$

s.r. $\Pi_L t_L + \Pi_M t_M + \Pi_G t_G \geq u$

It is obvious that if the Government, as MAJOR or DECIDENT, can observe i, then it can clearly specify what effort an individual puts forward, when output q is observable and quantifiable. Under these circumstances, the government can control the individual's input (effort).

Solving P1 optimization problem

The sentence 1. In the case of symmetric information, the optimal tax imposed by the government has the following characteristics:

i) Budgetary (participation) restriction is saturated;

ii) Marginal utilities are equal between the three types of individuals;

iii) Utility levels are equal for the three types of individuals;

iv) The marginal cost of effort equals marginal productivity for each type;

v) Optimal taxes are also influenced by the proportions of low-, medium- or high-productivity individuals (in the total population).

Demonstration

The function of LAGRANGE, with the λ multiplier associated with the participation restriction, is written:

$$L(t_L, t_M, t_G, l_L, l_M, l_G ; \lambda) = \Pi_L \, \boldsymbol{U} \, \theta_L l_L - t_L - \psi(l_L)] + \Pi_M \, \boldsymbol{U} \, \theta_M l_M - t_M - \psi(l_M)] +$$
$$+ \Pi_G \, \boldsymbol{U} \, \theta_G l_G - t_G - \psi(l_G)] + \lambda \, [\Pi_L t_L + \Pi_M t_M + \Pi_G t_G - \underline{u}]$$

By canceling the partial derivatives in relation to taxes, supposedly non-zero, we obtain:

$$\frac{\partial L}{\partial t_L} = -\Pi_L U'[\theta_L l_L - t_L - \psi(l_L)] + \lambda \, \Pi_L = 0 \qquad (2^o)$$

$$\frac{\partial L}{\partial t_M} = -\Pi_M U'[\theta_M l_M - t_M - \psi(l_M)] + \lambda \, \Pi_M = 0 \qquad (3^o)$$

$$\frac{\partial L}{\partial t_G} = -\Pi_G U'[\theta_G l_G - t_G - \psi(l_G)] + \lambda \, \Pi_G = 0 \qquad (4^o)$$

It is noted that Lagrange λ is strictly positive and eliminating it results in equality:

$U'[\theta_L l_L - t_L - \psi(l_L)] = U'[\theta_M l_M - t_M - \psi(l_M)] = U'[\theta_G l_G - t_G - \psi(l_G)]$($5^o$), ie (ii).

In addition, the budget restriction is saturated ($\lambda > 0$) so (i) is true.

If the utility function is strictly concave, that is individuals with risk aversion, then the sequence of equality (5^o) becomes:

$[\theta_L l_L - t_L - \psi(l_L)] = [\theta_M l_M - t_M - \psi(l_M)] = [\theta_G l_G - t_G - \psi(l_G)]$ (5^o), which implies identical levels of utility for the three types of individuals, ie (iii).

We cancel the partial derivatives in relation to effort levels l_L, l_M and l_G and we obtain:

$$\frac{\partial L}{\partial t_L} = \Pi_L [\theta_L - \psi'(l_L)] U'[\theta_L l_L - t_L - \psi(l_L)] = 0 \qquad (7^o)$$

$$\frac{\partial L}{\partial t_M} = \Pi_M [\theta_M - \psi'(l_M)] \, U'[\theta_M l_M - t_M - \psi(l_M)] = 0 \qquad (8^o)$$

$$\frac{\partial L}{\partial t_G} = \Pi_G [\theta_G - \psi'(l_G)] \, U'[\theta_G l_G - t_G - \psi(l_G)] = 0 \qquad (9^o)$$

Relationships (7^o), (8^o) and (9^o) lead to the equations:

$\Psi'(l_L) = \theta_L$ (10^o)

$\Psi'(l_M) = \theta_M$ (11^o)

$\Psi'(l_G) = \theta_G$ (12^o), from which conclusion (iv) of sentence 1.

The solution of the system formed by the equations (10°), (11°) and (12°) leads to the optimal values of the input (level of effort), namely:

$$\tilde{e}_L = (\psi')^{-1}(\theta_L),\ \tilde{e}_M = (\psi')^{-1}(\theta_M) \text{ and } \tilde{e}_G = (\psi')^{-1}(\theta_G) \qquad (13°)$$

We combine the equations (6°) and the tight budget constraint and taking into account the optimal effort levels (13°) we obtain the system:

$$\begin{aligned} &\Pi_L t_L + \Pi_M t_M + \Pi_G t_G = \underline{u} \\ &-t_L \quad +t_M \qquad = \theta_M\tilde{e}_M - \psi(\tilde{e}_M) - (\theta_L\tilde{e}_L - \psi(\tilde{e}_L)) = A \\ &\quad -t_M \ + \quad t_G = \theta_G\tilde{e}_G - \psi(\tilde{e}_G) \ - (\theta_M\tilde{e}_M - \psi(\tilde{e}_M) = B \end{aligned}$$

The system can be resolved by Cramer's rule, namely:

$$\Delta = \begin{vmatrix} \Pi_L & \Pi_M & \Pi_G \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = \Pi_L + \Pi_M + \Pi_G = 1$$

$$\Delta_{t_L} = \begin{vmatrix} 0 & \Pi_M & \Pi_G \\ A & 1 & 0 \\ B & -1 & 1 \end{vmatrix} = -A(1 - \Pi_L) - \Pi_G B = A\Pi_L - B\Pi_G - A$$

$$\Delta_{t_M} = \begin{vmatrix} \Pi_L & 0 & \Pi_G \\ -1 & A & 0 \\ 0 & B & 1 \end{vmatrix} = A\Pi_L + \Pi_G(-B) = A\Pi_L - B\Pi_G$$

$$\Delta_{t_G} = \begin{vmatrix} \Pi_L & \Pi_M & 0 \\ -1 & 1 & A \\ 0 & -1 & B \end{vmatrix} = \Pi_L\,(A+B) - \Pi_M$$

Then the optimal taxes are:

$$\tilde{t}_L = A\Pi_L - B\Pi_G - A,\ \tilde{t}_M = A\Pi_L - B\Pi_G,\ \tilde{t}_G = \Pi_L(A+B) - \Pi_M$$

3. Case of ADVERSE SELECTION (asymmetric information)

We also need to add incentive restrictions (when income is taxed) to the objective function and budget constraint.

Restrictions are upward (local and global)

$$\theta_G l_G - t_G - \psi(l_G) \geq \theta_M l_M - t_M - \psi\left(\frac{\theta_M l_M}{\theta_G}\right) \qquad (1°)$$

According to this restriction, the G (best) productivity type θ_G chooses the contract (q_G, l_G) to the detriment of the contract (q_M, t_M).

It would produce q_M and would pay t_M with the effort $\left(\frac{\theta_M l_M}{\theta_G}\right)$ given the productivity θ_G.

$$\theta_M l_M - t_M - \psi(l_M) \geq \theta_L l_L - t_L - \psi\left(\frac{\theta_L l_L}{\theta_M}\right) \qquad (2°)$$

$$\theta_G l_G - t_G - \psi(l_G) \geq \theta_L l_L - t_L - \psi\left(\frac{\theta_L l_L}{\theta_G}\right) \qquad (3°)$$

with an interpretation similar to the one above.

Restrictions (1^o) and (2^o) are local upward restrictions and (3^o) is a global ascending restriction.

The following restrictions are descending (local and global) and ensure that worse-placed agents (with lower productivity) prefer contracts for them than other contracts.

$$\theta_L l_L - t_L - \psi(l_L) \geq \theta_M l_M - t_M - \psi\left(\frac{\theta_M l_M}{\theta_L}\right) \qquad (4^o)$$

$$\theta_M l_M - t_M - \psi(l_M) \geq \theta_G l_G - t_G - \psi\left(\frac{\theta_G l_G}{\theta_M}\right) \qquad (5^o)$$

$$\theta_L l_L - t_L - \psi(l_L) \geq \theta_G l_G - t_G - \psi\left(\frac{\theta_G l_G}{\theta_L}\right) \qquad (6^o)$$

Restrictions (4^o) and (5^o) are downstream local restrictions and (6^o) is a downward global restriction.

Suppose that $\frac{\theta_G}{\theta_M} = \frac{\theta_M}{\theta_L} = \theta > 1$ and note $f(e) = \psi(e) - f(\theta_e)$.

Proposition 2. The function f(·)is negative and decreasing.

Demonstration. Obviously $e \leq \theta_e$ and how $\psi'(\cdot) > 0$, it follows that $\psi(e) \leq \psi(\theta_e)$, where from $f(e) \leq 0$.

By deriving the function f(·)we obtain:

$f'(e) = \psi'(e) - \theta\psi'(\theta_e) < \psi'(e) - \psi'(\theta_e) < 0$ because $\psi'(\cdot) > 0$.

Proposition 3. The model implementation feasibility is $\theta_G l_G \geq \theta_M l_M \geq \theta_L l_L$

Demonstration. The implementation condition is equivalent to the assertion that the set of admissible solutions is unclear.

We collect member with inequalities (1^o) and (5^o) and we obtain:

$-\Psi(l_G) - \Psi(l_M) \geq -\Psi(\theta l_G) - \Psi\left(\frac{l_M}{\theta}\right)$ or

$\Psi(l_G) - \Psi(\theta l_G) + \Psi(l_M) - \Psi\left(\frac{l_M}{\theta}\right) \leq 0$

The above inequality becomes:

$f(l_G) - f\left(\frac{l_M}{\theta}\right) \leq 0$ and from (P_2) results

$l_G \geq \frac{l_M}{\theta}$, ie $l_G \cdot \theta \geq l_M$ or $l_G \cdot \frac{\theta_G}{\theta_M} \geq l_M$, ie $l_G\theta_G \geq l_M\theta_M$.

Analogously, summing (2^o) and (4^o) we obtain:

$-\Psi(l_M) - \Psi(l_L) \geq -\psi\left(\frac{\theta_L l_L}{\theta_M}\right) - \psi\left(\frac{\theta_M l_M}{\theta_L}\right)$

or by regrouping the terms we obtain:

$-\Psi(l_M) - \Psi(\theta\, l_M) + \psi(\theta \cdot \frac{l_L}{\theta}) + \psi(\frac{l_L}{\theta}) \leq 0$, ie $f(l_M) - f(\frac{l_L}{\theta}) \leq 0$

Again, taking into account sentence 1, we obtain: $l_M \geq \frac{l_L}{\theta}$ or $l_M\,\theta \geq l_L$, where from $l_M\theta_M \geq l_L\,\theta_L$ and the sentence is demonstrated.

Proposition 4. If the local upstream restrictions are checked, then the global ascending restriction is checked.

Demonstration. We add the restrictions (1^o) and (2^o) and we obtain:

$\theta_G l_G - t_G - \psi(l_G) - \Psi(l_M) \geq \theta_L l_L - t_L - \psi\left(\frac{\theta_M l_M}{\theta_G}\right) - \psi\left(\frac{\theta_L l_L}{\theta_M}\right)$ or

$\theta_G l_G - t_G - \psi(l_G) \geq \theta_L l_L - t_L - \psi\left(\frac{\theta_M l_M}{\theta_G}\right) - \psi\left(\frac{\theta_L l_L}{\theta_G}\right) + \psi\left(\frac{\theta_L l_L}{\theta_G}\right) + \Psi(l_M) - \psi\left(\frac{\theta_M l_M}{\theta_G}\right) - \psi\left(\frac{\theta_L l_L}{\theta_M}\right) =$

$= \geq \theta_L l_L - t_L - \psi\left(\frac{\theta_L l_L}{\theta_G}\right) + \psi(\theta \cdot \frac{l_M}{\theta}) - \psi\left(\frac{\theta_M l_M}{\theta_G}\right) + \psi(\frac{l_L}{\theta^2}) - \psi(\frac{l_L}{\theta^2}) = \theta_L l_L - t_L - \psi\left(\frac{\theta_L l_L}{\theta_G}\right) - f(\frac{l_M}{\theta}) - f(\frac{l_L}{\theta^2}) \geq \theta_L l_L - t_L - \psi\left(\frac{\theta_L l_L}{\theta_G}\right)$, because the function f(·) is negative for any strictly positive argument.

Proposition 5. If the downstream local restrictions are checked, then the downstream global restriction is checked.

Demonstration. We collect the relations (4^o) and (5^o) and we obtain:

$\theta_L l_L - t_L - \Psi(l_L) - \Psi(l_M) \geq \theta_G l_G - t_G - \psi\left(\frac{\theta_M l_M}{\theta_L}\right) - \psi\left(\frac{\theta_G l_G}{\theta_M}\right)$ or

$\theta_L l_L - t_L - \Psi(l_L) \geq \theta_G l_G - t_G - \psi\left(\frac{\theta_G l_G}{\theta_L}\right) + \psi\left(\frac{\theta_G l_G}{\theta_L}\right) + \Psi(l_M) - \psi\left(\frac{\theta_M l_M}{\theta_L}\right) - \psi\left(\frac{\theta_G l_G}{\theta_M}\right) =$

$= \theta_G l_G - t_G - \psi\left(\frac{\theta_G l_G}{\theta_L}\right) + \psi(\theta^2 l_G) + \Psi(l_M) - \psi(\theta l_M) - \psi(\theta l_G) = \theta_G l_G - t_G - \psi\left(\frac{\theta_G l_G}{\theta_L}\right) - f(\theta l_G) + f(l_M)$

$\geq \theta_G l_G - t_G - \psi\left(\frac{\theta_G l_G}{\theta_L}\right)$

Since $\theta l_G = \frac{\theta_G}{\theta_M} l_G \geq l_M$, according to the properties of the function f, ie $-f(\theta l_G) + f(l_M) \geq 0$

By a convenient notation, namely:

$U_G = \theta_G l_G - t_G - \psi(l_G)$

$U_M = \theta_M l_M - t_M - \psi(l_M)$

$U_L = \theta_L l_L - t_L - \psi(l_L)$, the incentive restrictions become:

$$U_G \geq U_M - f(\frac{l_M}{\theta}) \quad (7^o)$$

$$U_M \geq U_L - f(\frac{l_L}{\theta}) \quad (8^o)$$

$$U_L \geq U_M + f(l_M) \quad (9^o)$$

$$U_M \geq U_G + f(l_G) \quad (10^o)$$

We remove the variables t_L, t_M, t_G from the above transformations and the problem becomes successively:

First, the objective function will take the following form:

Max $U_G, U_M, U_L, l_G, l_M, l_L$ $[\Pi_L U_{(u_L)} + \Pi_M U_{(u_M)} + \Pi_G U_{(u_G)}]$

(maximizing expected utility)

Government Restriction (Budget Restriction) takes the following form:

$\Pi_L [\theta_L l_L - \Psi(l_L)] + \Pi_M [\theta_M l_M - \Psi(l_M)] + \Pi_G [\theta_G l_G - \Psi(l_G)] - [\Pi_L U_L + \Pi_M U_M + \Pi_G U_G] \geq \underline{u}$, where $\underline{u}$ has the meaning of an expected minimum level of government revenue.

We will ignore the last two incitement restrictions (local descending restrictions) and finally we will show that the solution thus obtained also checks these restrictions.

If worse placed agents accept the contract, the better placed the contract accepts.

The initial optimization problem (P1) simplifies (fewer restrictions) and becomes:

Max $U_{G,}$ $U_{M,}$ $U_{L,}$ l_G, l_M, l_L $[\Pi_L U_{(u_L)} + \Pi_M U_{(u_M)} + \Pi_G U_{(u_G)}]$

s.r.

(P2) $\Pi_L[\theta_L l_L - \psi(l_L)] + \Pi_M[\theta_M l_M - \psi(l_M)] + \Pi_G[\theta_G l_G - \psi(l_G)] - [\Pi_L U_L + \Pi_M U_M + \Pi_G U_G] \geq \underline{u}$

$U_G \geq U_M - f(\frac{l_M}{\theta})$

$U_M \geq U_L - f(\frac{l_L}{\theta})$

$U_L \geq 0$, $U_M \geq 0$, $U_G \geq 0$,

$l_L \geq 0$, $l_M \geq 0$, $l_G \geq 0$

We associate the KUHN-TUCKER multipliers λ, μ și ρ of the three constraints and construct Lagrange's function as follows:

$L(u_L, u_M, u_G, l_L, l_M, l_G ; \lambda, \mu, \delta) = \Pi_L U_{(u_L)} + \Pi_M U_{(u_M)} + \Pi_G U_{(u_G)} + \lambda[\Pi_L(\theta_L l_L - \Psi(l_L) - U_L] + \Pi_M[(\theta_M l_M - \Psi(l_M) - U_M] + \Pi_G[(\theta_G l_G - \Psi(l_G) - U_G] + \mu[U_G - U_M + f(\frac{l_M}{\theta})] + \delta [U_M - U_L + f(\frac{l_L}{\theta})]$

The KUHN-TUCKER conditions of first order (necessary and sufficient according to the properties of utility functions) and f are written:

$$\frac{\partial L}{\partial u_L} = \Pi_L u'(u_L) + \lambda \Pi_L - \delta = 0 \qquad (11^o)$$

$$\frac{\partial L}{\partial u_M} = \Pi_M u'(u_M) - \lambda \Pi_M - \mu + \delta = 0 \qquad (12^o)$$

$$\frac{\partial L}{\partial u_G} = \Pi_G u'(u_G) - \lambda \Pi_G + \mu - \delta = 0 \qquad (13^o)$$

$$\frac{\partial L}{\partial u_L} = \lambda\Pi_L[\theta_L - \Psi'(l_L)] + \delta \frac{1}{\theta} f'(\frac{l_L}{\theta}) = 0 \qquad (14^o)$$

$$\frac{\partial L}{\partial u_M} = \lambda\Pi_M[\theta_M - \Psi'(l_M)] + \mu \frac{1}{\theta} f'(\frac{l_M}{\theta}) = 0 \qquad (15^o)$$

$$\frac{\partial L}{\partial u_G} = \lambda\Pi_G[\theta_G - \Psi'_G] = 0 \qquad (16^o)$$

The final result is included in the following theorem:

Theorem 1. All_KUHN- multipliers are strictly positive and the level of optimal effort is given by the equality:

$\theta_G = \Psi'(l_G)$

$u_G = u_M - f(\frac{l_M}{\theta})$

$u_M = u_L - f(\frac{l_L}{\theta})$

Conclusion

Determining the optimal levels $\tilde{e}_{G,} \tilde{e}_{M,}$ $\tilde{e}_L$ and the variables $\tilde{u}_{G,} \tilde{u}_{M,}$ $\tilde{u}_L$ allows to write optimal taxes according to the formula:

$$\tilde{t}_G = \theta_G \, \tilde{e}_G - \tilde{u}_G - \Psi(\tilde{e}_G)$$
$$\tilde{t}_M = \theta_M \, \tilde{e}_M - \tilde{u}_M - \Psi(\tilde{e}_M)$$
$$\tilde{t}_L = \theta_L \, \tilde{e}_L - \tilde{u}_L - \Psi(\tilde{e}_L)$$

In this paper (study), the authors have departed from the practical necessity of optimal taxation of effort. In current activity, optimal effort taxation involves several states. From the study, analysis and demonstration, it is clear that there is a well-articulated mathematical model presented in the article's content which best describes deterministically the way in which a balance between allocative efficiency and distribution is ensured. The article did not intend to do a practical study as, first of all, the theoretical relationship shown in the article must first be demonstrated and then used, applied in concrete cases. For a country's economy, such a mathematical model is useful because it refers to a number of issues that are important in terms of how best-effort taxation can be applied. It is easy to apply the expected mathematical model to a concerted situation based on the data that is encountered in an economy.

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