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About Directed d-Convex Simple Graphs

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Abstract

In this article we introduce a pseudo-metric on directed graphs, which forms there a family of convex sets. The graphs without d-convex sets, except empty set, sets of one vertex and set of all vertexes, are called d-convex simple. We give an iterative method of description of the set of all directed d-convex simple graphs. Then we research the structure of directed d-convex simple graphs and do this by using some new operations and new graphs. After that we show that the set of directed d-convex simple graphs contains all known undirected d-convex simple graphs.

1 An Iterative Method of Description for Directed d-Convex Simple Graphs.

We are going to study below directed graphs, without loops or multiple arcs. A directed graph $\vec{G} = (X,\ \vec{U})$ is called to be *strongly connected* graph, if for each two vertexes $x,\ y \in X$ there is at least one path (directed chain), from vertex x to vertex y and at least one path from vertex y to vertex x. Let $D = (x = z_1,\ z_2,\ \ldots,\ z_p = y)$ be a path from x to y. In this case we will say, also, that path D joins the vertexes x and y in the indicated ordering, which are called the *extremities* of D. The number p is called the *length* of the path

$$D = (x = z_1,\ z_2,\ \ldots,\ z_p = y)$$

and we will write $l(D) = p$.

Let $\mathcal{D}(x,\ y)$ be the family of all paths form $\vec{G}$ that joins the vertexes x and y. The length of the shortest path from $\mathcal{D}(x,\ y)$ will be called

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the *distance* between vertexes x and y and will be denote by $d(x,\ y)$. So, we have

$$d(x,\ y) = \min_{D \in \mathcal{D}(x,\ y)} \{l(D)\}.$$

In the case when between two vertexes $x,\ y \in X$ a path that joins them does not exist, it is considered that $d(x,\ y) = \infty$.

It is easy to see that the notion of distance, introduced by this way, does not respect the commutative propriety, i. e.

$$d(x,\ y) \neq d(y,\ x).$$

This distance is a function $d : X \times X \to \mathbb{N}$, that respects the properties:

1. $d(x,\ y) \geq 0$, for each two vertexes $x,\ y \in X$, and $d(x,\ y) = 0$ if and only if $x = y$;

2. $d(x,\ y) \leq d(x,\ z) + d(z,\ y)$, for each three vertexes $x,\ y,\ z \in X$.

We have, that the defined above distance function $d : X \times X \to \mathbb{N}$ is a *pseudo-metric* in directed graph $\vec{G}$.

The set $\langle \overrightarrow{x,\ y} \rangle = \{z \in X \mid d(x,\ z) + d(z,\ y) = d(x,\ y)\}$ is called *directed d-segment* from x to y. Obviously, the notion of directed segment $\langle \overrightarrow{x,\ y} \rangle$ has sense only if there is at least one path that joins x with y. From these considerations, we are going to study below only strongly connected directed graphs.

Definition 1.1 *The set $A \subset X$ is called to be **d-convex** set in the graph $\vec{G} = (X,\ \vec{U})$ if for each $x,\ y \in A$, considered in the indicated order, there is the relation $\langle \overrightarrow{x,\ y} \rangle \subset A$.*

We observe that each set A, $|A| = 0$ or $|A| = 1$, is d-convex.

It is easy to see that in the arbitrarily directed graph the sets $\langle \overrightarrow{x,\ y} \rangle$ and $\langle \overrightarrow{y,\ x} \rangle$ are not indispensably equal. Let A be an arbitrary set of vertexes from graph $\vec{G} = (X,\ \vec{U})$. According to definition 1.1, if A is a d-convex set, then both directed d-segments $\langle \overrightarrow{x,\ y} \rangle$ and $\langle \overrightarrow{y,\ x} \rangle$ belong to A. This implies that the union $\langle \overrightarrow{x,\ y} \rangle \cup \langle \overrightarrow{y,\ x} \rangle$, which contains all vertexes of at least one circuit that passes through vertexes x and y, also belongs to A. More, the set $\langle \overrightarrow{x,\ y} \rangle \cup \langle \overrightarrow{y,\ x} \rangle$ contains the vertexes of all circuits of minimal length that pass trough x and y.

Lemma 1.1 *If A and B are two d-convex sets of a directed graph $\vec{G} = (X, \vec{U})$, then the intersection $A \cap B$ is, also, a d-convex set in $\vec{G}$.*

Proof: Let $\vec{G} = (X, \vec{U})$ be a directed graph, and A and B be two d-convex sets of it. If $|A \cap B| \leq 1$, then according to definition 1.1 the assertion of lemma is true. Let us suppose that $|A \cap B| \geq 2$, and let x, y be any two vertexes from $A \cap B$. Because A and B are d-convex simple sets, we have:

$$\langle \overrightarrow{x,\, y} \rangle \subseteq A,$$

$$\langle \overrightarrow{x,\, y} \rangle \subseteq B,$$

that implies relation $\langle \overrightarrow{x,\, y} \rangle \subseteq A \cap B$. So we have that $A \cap B$ is d-convex simple set in $\vec{G}$. □

By analogy with classical model of d-convex hull notion [8], in case of directed graphs we have:

Definition 1.2 *Intersection of all d-convex sets of a directed graph $\vec{G} = (X, \vec{U})$, that contains a subset of vertexes $B \subseteq X$, is called* ***d-convex hull*** *of the set B and denoted by* ***d-conv(B)****.*

Obviously, if $B \subseteq X$ is already a d-convex set in $\vec{G}$, then $d-conv(B) = B$.

For an arbitrary subset of vertexes S of the graph $\vec{G}$, we will define the next operation

$$P(S) = \bigcup_{\forall x,\, y\, \in S} \langle \overrightarrow{x,\, y} \rangle.$$

Then d-convex hull of the set B can be iteratively described like follows:

$$B_0 = B,$$

$$B_1 = \bigcup_{\forall x,\, y\, \in B_0} \langle \overrightarrow{x,\, y} \rangle = P(B_0),$$

$$B_2 = \bigcup_{\forall x,\, y\, \in B_2} \langle \overrightarrow{x,\, y} \rangle = P(B_1) = P(P(B_0)) = P^2(B_0) \text{ and } P(B_1) \neq B_1,$$

$$\ldots,$$

$$B_{q-1} = \bigcup_{\forall x,\, y \in B_{q-2}} \langle \overrightarrow{x,\, y} \rangle = P(B_{q-2}) = P^{q-1}(B_0) \ and \ P(B_{q-2}) \neq B_{q-2},$$

$$B_q = \bigcup_{\forall x,\, y \in B_{q-1}} \langle \overrightarrow{x,\, y} \rangle = B_{q-1} = d - conv(B).$$

By this way, the construction of d-convex hull $d-conv(B)$ is reduced to construction of a sequence of subsets:

$$B = B_0 \subset B_1 \subset B_2 \subset \ldots \subset B_{q-1} = B_q,$$

where B_i, $1 \leq i \leq q$, is determined by using the P operation, described above. In case of infinite graphs, the iteratively constructed sequence could be also infinite. Then, d-convex hull of the set B is computed using relation:

$$d - conv(B) = \bigcup_{i=0}^{\infty} B_i.$$

By definition 1.2, it is easy to see that next relations are true:

1. $d - conv(\emptyset) = \emptyset$;
2. $d - conv(\{x\}) = \{x\}$;
3. $d - conv(X) = X$;
4. $A \subseteq d - conv(A)$;
5. $d - conv(d - conv(A)) = d - conv(A)$.

By the relations $1 - 5$, we can say that the notion of d-convexity for the directed graphs do not get out of the general axiomatic theory of the convexity [8].

In undirected graphs any subset of vertexes $A \neq X$, which induce a complete subgraph and the set of all vertexes X, is always d-convex. We can say the same thing about directed graphs. The sets that are always d- convex in any directed graph are the empty set, the sets with one vertex, the sets that induce a complete subgraph and the set of all vertexes X. This is because the introduced by this way notion of convexity in directed graphs is the extension of the notion of convexity in the undirected graphs, and this thing will be shown below. Further the mentioned sets will be called *trivial* d-convex sets in directed graphs.

Definition 1.3 *Directed and strongly connected graph* $\vec{G} = (X,\ \vec{U})$ *is called* ***d-convex simple*** *if it does not contain d-convex sets* $A \subset X$*, such that* $1 < |A| < |X|$.

From the definition it results that in a directed d-convex simple graph, between any two vertexes there can be only one of two possible arcs. Indeed, if for some two vertexes $x,\ y$ in a d-convex simple graph both arcs $(x,\ y)$ and $(y,\ x)$ exist, then the set $A = \{x,\ y\}$ will be d-convex, that contradicts to the definitions assertion of d-convex simple graph. This remark means that all directed d-convex simple graphs are *antisymmetric.*

Theorem 1.1 *Next assertions are equivalent:*

1. $\vec{G} = (X, \vec{U})$ *is directed d-convex simple graph;*
2. $d - conv(\{x,\ y\}) = X$ *for any two distinct vertexes* $x,\ y \in X$*;*
3. $d - conv(\{x,\ y\}) = X$ *for any two adjacent vertexes* $x,\ y \in X$.

Proof: $1 \to 2$. Let $\vec{G} = (X, \vec{U})$ be a directed d-convex simple graph. By definition 1.3 it does not contain d-convex sets with more than one vertex or less than the number of vertexes of X. It follows that it does not contain d-convex sets of cardinal two. This fact implies $d - conv(\{x,\ y\}) = X$, for any two distinct vertexes $x,\ y \in X$.

$2 \to 3$. Relation $d - conv(\{x,\ y\}) = X$ is true for any two distinct vertexes $x,\ y \in X$, so it is true for adjacent vertexes, too. It results that the assertion 3 is true.

$3 \to 1$. Let us suppose that for any two adjacent vertexes $x,\ y \in X$, the relation $d - conv(\{x,\ y\}) = X$ is true, but the directed graph $\vec{G}$ is not d-convex simple. This means that in $\vec{G}$ exists a d-convex set $A \subset X$ and $1 < |A| < |X|$. From $|A| > 1$ it results that in A there exist two vertexes $p,\ q$, such that $\langle\overrightarrow{p,\ q}\rangle\ \subset A$. This implies the existence of two adjacent vertexes $x,\ y$ in A. Because A is the d-convex simple set, there is the relation $d - conv(x,\ y) \subseteq A$. On the other hand, from condition 3 we have $d - conv(x,\ y) = X$. It follows that $A = X$, that contradicts the supposition $1 < |A| < |X|$. □

Now we consider the next class of directed graphs $\mathfrak{D}$, that is defined recursively as follows:

I. In the class $\mathfrak{D}$ there are all graphs $\vec{G}_0 = (X_{\vec{G}_0},\ \vec{U}_{\vec{G}_0})$, where:
$X_{\vec{G}_0} = \{x_1,\ x_2,\ \ldots,\ x_n\},\ n \geq 3$,
$\vec{U}_{\vec{G}_0} = \{(x_n,\ x_1)\} \cup \{(x_i,\ x_{i+1})|\ i = 1,\ 2,\ \ldots,\ n-1\}$,
i. e. $\vec{G}_0$ is an elementary circuit with n vertexes;

II. From the graph $\vec{G}_{i-1}$, $(i \geq 1)$ we construct the graph $\vec{G}_i = (X_{\vec{G}_i},\ \vec{U}_{\vec{G}_i})$, where:
$X_{\vec{G}_i} = X_{\vec{G}_{i-1}} \cup \{y_1,\ y_2,\ \ldots,\ y_m\},\ m \geq 1$,
$\vec{U}_{\vec{G}_i} = \vec{U}_{\vec{G}_{i-1}} \cup \{(a,\ b)\ |\ a,\ b \in X_{\vec{G}_i}$, not both are in $X_{\vec{G}_{i-1}}$, and such that conditions a), b) are satisfied $\}$;

   a) For each vertex y_i, there exist two distinct vertexes $p,\ q \in X_{\vec{G}_i}$, such that $y_i \in d - conv(\{p,\ q\})$;
   b) For any two adjacent vertexes $a,\ b \in X_{\vec{G}_i}$, there exist two distinct vertexes $p,\ q \in X_{\vec{G}_{i-1}}$, such that the following relations are satisfied:
      1. $p,\ q \in d - conv(\{a,\ b\})$;
      2. $d_{\vec{G}_{i-1}}(p,\ q) = d_{\vec{G}_i}(p,\ q)$;
      3. $d_{\vec{G}_{i-1}}(q,\ p) = d_{\vec{G}_i}(q,\ p)$;

III. In $\mathfrak{D}$ there are no other graphs, except the graphs iteratively described in I. and II.

The class of directed graphs $\mathfrak{D}$ is a union of graphs families, recursively obtained, according to the procedures described above:

$$\mathfrak{D} = \mathfrak{D}_0 \cup \mathfrak{D}_1 \cup \ldots \cup \mathfrak{D}_i \ldots,$$

where $\mathfrak{D}_0$ represents all graphs $\vec{G}_0 = (X_{\vec{G}_0},\ \vec{U}_{\vec{G}_0})$, described at step I of construction of class $\mathfrak{D}$, and $\mathfrak{D}_i$, $i \geq 1$, represent the family of all graphs that are obtained from $\mathfrak{D}_{i-1}$, applying the operation II.

Theorem 1.2 *All graphs of the class $\mathfrak{D}$ are d-convex simple graphs.*

Proof: We will prove that $\mathfrak{D}_0,\ \mathfrak{D}_1,\ \ldots,\ \mathfrak{D}_i,\ \ldots$ are families of directed d-convex simple graphs, using mathematical induction method by the index $i = 0,\ 1,\ 2,\ \ldots$ of the family of graphs.

It is easy to see, that if $i = 0$, then $\vec{G}_0 = (X_{\vec{G}_0},\ \vec{U}_{\vec{G}_0})$ from $\mathfrak{D}_0$ is d-convex simple, because it is an elementary circuit with $n \geq 3$ vertexes.

Let us prove that $\mathfrak{D}_i$, $i \geq 1$, is a family of directed d-convex simple graphs, in conditions that all graphs from $\mathfrak{D}_0,\ \mathfrak{D}_1,\ \ldots,\ \mathfrak{D}_{i-1}$ have already this property. We choose a graph $\vec{G}_i \in \mathfrak{D}_i$ with $n \geq 3$ vertexes. Let $\vec{G}_i = (X_{\vec{G}_i},\ \vec{U}_{\vec{G}_i})$ be obtained from $\vec{G}_{i-1} \in \mathfrak{D}_{i-1}$ as the result of application of the operation II of construction of the class $\mathfrak{D}$. According to theorem 1.1 for this, it is necessary to prove that for any two adjacent vertexes $a,\ b \in X_{\vec{G}_i}$ there is the relation $d - conv_{\vec{G}_i}(\{a,\ b\}) = X_{\vec{G}_i}$. Let us suppose that between a and b there exists the arc $(a,\ b) \in \vec{U}_{\vec{G}_i}$. According to the condition II b) of the class $\mathfrak{D}$ description, there exist two distinct vertexes $p,\ q \in X_{\vec{G}_{i-1}}$, such that $p,\ q \in d - conv_{\vec{G}_i}(\{a,\ b\})$. This implies the relation

$$d - conv_{\vec{G}_i}(\{p,\ q\}) \subset d - conv_{\vec{G}_i}(\{a,\ b\}).$$

The fact that, by induction, $\vec{G}_{i-1}$ is d-convex simple, and in process of construction of the graph $\vec{G}_i$, new arcs were not added to vertexes from $\vec{G}_{i-1}$, and the distance between p and q has kept unchanged in $\vec{G}_i$ (see condition II b)), results in:

$$X_{\vec{G}_{i-1}} \subset d - conv_{\vec{G}_i}(\{p,\ q\}) \subset d - conv_{\vec{G}_i}(\{a,\ b\}).$$

From condition II a) we obtain:

$$X_{\vec{G}_{i-1}} \cup \{y_1,\ y_2,\ \ldots,\ y_m\} = X_{\vec{G}_i} \subset d - conv_{\vec{G}_i}(\{a,\ b\}).$$

Reverse inclusion is obvious and so we have $X_{\vec{G}_i} = d - conv_{\vec{G}_i}(\{a,\ b\})$. This means that directed graph $\vec{G}_i$ is d-convex simple. So $\mathfrak{D}_i$ is a family of directed d-convex simple graphs. □

Theorem 1.3 *A directed strongly connected graph $\vec{G} = (X,\ \vec{U})$, $|X| \geq 3$ is d-convex simple if and only if $\vec{G} \in \mathfrak{D}$.*

Proof:

Necessity: Let $\vec{G} = (X,\ \vec{U})$ be a directed d-convex simple graph. Thus $|X| \geq 3$ results that in $\vec{G}$ there exist two distinct vertexes u and v. Because $\vec{G}$ is a strongly connected graph then there exists at least one circuit that passes trough these vertexes. So, we obtained that in $\vec{G}$ there exists at least one elementary circuit. We consider thus elementary circuit of minimal length. We also observe that if $C = [z_0, z_1, \ldots, z_p, z_{p+1} = z_0]$ is a circuit of minimal length in graph $\vec{G}$, then the subgraph generated by the set of vertexes $\{z_0, z_1, \ldots, z_p\}$ is isomorphic with C. It follows that in $\vec{G}$ do not exist arcs that join any two vertexes $z_i,\ z_j$, where $|i-j| > 1$. We will denote by $\vec{G}_0 = (X_{\vec{G}_0},\ \vec{U}_{\vec{G}_0})$ the subgraph generated by the set of vertexes $\{z_0,\ z_1,\ \ldots,\ z_p\}$, and by $\vec{G}_1$ denote the graph $\vec{G}$ itself. It is easy to verify that $\vec{G}_1$ is obtained from $\vec{G}_0$ by adding the sets $X \backslash X_{\vec{G}_0}$ of vertexes and $\vec{U} \backslash \vec{U}_{\vec{G}_0}$ of arcs, according to condition II of description of class $\mathfrak{D}$. It follows that $\vec{G} \in \mathfrak{D}$.

Sufficiency: It results from theorem 1.2. □

2 Operations Over Directed d-Convex Simple Graphs.

Let $\vec{G}_1 = (X_1,\ \vec{U}_1)$ and $\vec{G}_2 = (X_2,\ \vec{U}_2)$ be two directed graphs, where we choose by one pair of nonadjacent vertexes: $x_1,\ x_2$ in $\vec{G}_1$ and $y_1,\ y_2$ in $\vec{G}_2$. By analogy with [2, 4, 5] we will denote by $M_{x_2=y_2}^{x_1=y_1}(\vec{G}_1,\ \vec{G}_2)$ the graph obtained from $\vec{G}_1$ and $\vec{G}_2$ as the result of joining the vertexes x_1 with y_1 and x_2 with y_2. For the graph $\vec{G} = M_{x_2=y_2}^{x_1=y_1}(\vec{G}_1,\ \vec{G}_2)$ the following relations are true:

$$|X_{\vec{G}}| = |X_1| + |X_2| - 2,$$

$$\vec{U}_{\vec{G}} = \vec{U}_1 \cup \vec{U}_2.$$

In order to simplify notations we will write $\vec{G} = M(\vec{G}_1,\ \vec{G}_2)$ if the pairs of vertexes that take part in generation of new graph are known.

Let $\vec{G} = (X,\ \vec{U})$ be a directed graph and x be any vertex from X. We denote by

$$\Gamma^+(x) = \{y \in X \mid (x,\ y) \in \vec{U}\} \text{ and}$$

$$\Gamma^-(x) = \{y \in X \mid (y,\ x) \in \vec{U}\}$$

the set of *successors* of the vertex x and the set of *predecessors* of it respectively. The vertex which does not have predecessors is called *source* vertex, and the vertex which does not have successors - *destination* vertex.

We denote by $\mathcal{P}(p,\ q;\ r)$, $r > 0$, the directed graph where a source vertex p, and a destination vertex q are fixed and which satisfies the conditions:

1. in $\mathcal{P}$ there exist paths that join vertex p with vertex q;
2. any vertex and any arc from $\mathcal{P}$ belong to at least one path that joins p with q;
3. all paths that join vertexes p and q are of the same length $r > 0$;
4. other vertexes or arcs in $\mathcal{P}$ do not exist.

Theorem 2.1 *For any two graphs* $\mathcal{P}_1(p_1,\ q_1;\ r_1)$ *and* $\mathcal{P}_2(p_2,\ q_2;\ r_2)$, *the graph*

$$\vec{G} = M^{p_1=q_2}_{q_1=p_2}(\mathcal{P}_1,\ \mathcal{P}_2)$$

is d-convex simple.

Proof: In the graph $\vec{G}$ all elementary circuits are of the same length, equal to $r_1 + r_2$, and they pass through vertexes $p_1 = q_2,\ q_1 = p_2$ and any vertex of graph $\vec{G}$ belongs to at least one circuit of this type. It follows that $\vec{G}$ is strongly connected graph and for any two vertexes $x,\ y$ from $\vec{G}$ we have:

$$p_1 = q_2,\ q_1 = p_2 \in \langle \overrightarrow{x,\ y} \rangle \cup \langle \overrightarrow{y,\ x} \rangle,$$

but $d - conv(\{p_1 = q_2,\ q_1 = p_2\}) = X_{\vec{G}}$. From this it results that $\vec{G}$ is d-convex simple graph.

Theorem is proved. □

Let $\mathcal{P}_1(p_1,\ q_1;\ r_1),\ \mathcal{P}_2(p_2,\ q_2;\ r_2),\ \ldots,\ \mathcal{P}_s(p_s,\ q_s;\ r_s)$ be s-directed graphs. We denote by:

$$M_{q_s=p_1}^{q_1=p_2,\ q_2=p_3,\ \ldots,\ q_{s-1}=p_s}(\mathcal{P}_1,\ \mathcal{P}_2,\ \ldots,\ \mathcal{P}_s)$$

the graph obtained from $\mathcal{P}_1,\ \mathcal{P}_2,\ \ldots,\ \mathcal{P}_s$ as the result of joining the vertexes q_1 with p_2, q_2 with p_3, $\ldots$, q_{s-1} with p_s and q_s with p_1. For the graph $\vec{G} = M_{q_s=p_1}^{q_1=p_2,\ q_2=p_3,\ \ldots,\ q_{s-1}=p_s}(\mathcal{P}_1,\ \mathcal{P}_2,\ \ldots,\ \mathcal{P}_s)$ the following relations are true:

$$|X_{\vec{G}}| = |X_{\mathcal{P}_1}| + |X_{\mathcal{P}_2}| + \ldots + |X_{\mathcal{P}_s}| - s,$$

$$\vec{U}_{\vec{G}} = \vec{U}_{\mathcal{P}_1} \cup \vec{U}_{\mathcal{P}_2} \cup \ldots \cup \vec{U}_{\mathcal{P}_s}.$$

Corollary 2.1 *For any $s \geq 2$ directed graphs*

$$\mathcal{P}_1(p_1,\ q_1;\ r_1),\ \mathcal{P}_2(p_2,\ q_2;\ r_2),\ \ldots,\ \mathcal{P}_s(p_s,\ q_s;\ r_s),$$

the graph $\vec{G} = M_{q_s=p_1}^{q_1=p_2,\ q_2=p_3,\ \ldots,\ q_{s-1}=p_s}(\mathcal{P}_1,\ \mathcal{P}_2,\ \ldots,\ \mathcal{P}_s)$ is d-convex simple.

Proof: From graphs $\mathcal{P}_2(p_2,\ q_2;\ r_2), \mathcal{P}_3(p_3,\ q_3;\ r_3), \ldots, \mathcal{P}_s(p_s,\ q_s;\ r_s)$ we build a new graph $\tilde{\mathcal{P}}(p_2,\ q_s;\ r_2 + r_3 + \ldots + r_s)$ by joining the vertexes q_2 with p_3, $\ldots$, q_{s-1} with p_s. Then graph $\vec{G} = M_{q_s=p_1}^{q_1=p_2,\ q_2=p_3,\ \ldots,\ q_{s-1}=p_s}(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_s)$ coincides with graph $M_{q_s=p_1}^{q_1=p_2}(\mathcal{P}_1, \tilde{\mathcal{P}})$. From theorem 2.1 the last graph is d-convex simple. This means the graph $\vec{G}$ is d-convex simple, too. □

Theorem 2.2 *If $\vec{H} = (X_{\vec{H}},\, \vec{U}_{\vec{H}})$ is a directed d-convex simple graph, where there exists a pair of vertexes $x_1,\ x_2$, such that $d_{\vec{H}}(x_1,\, x_2) = r > 1$, then for any graph $\mathcal{P}(p,\, q;\, r)$ the graph $\vec{G} = M^{x_1=p}_{x_2=q}(\vec{H},\, \mathcal{P})$ is d-convex simple.*

Proof: By theorem 1.2, in order to prove the assertion of this theorem it is sufficient to show, that the graph $\vec{G}$ can be obtained from the graph $\vec{H}$ with respect of condition II of description of the class $\mathfrak{D}$. Let us consider $\vec{G}_{i-1} = \vec{H}$ and $\vec{G}_i = \vec{G}$. From construction of graph $\vec{G} = M^{x_1=p}_{x_2=q}(\vec{H},\, \mathcal{P})$ and condition $r > 1$, it results that the vertexes of graph $\vec{H}$ were not joined with new arcs. More than that:

a) for any $y_j \in X_{\mathcal{P}}$ there exist vertexes $x_1 = p,\ x_2 = q \in X_{\vec{H}}$, such that: $y_j \in d - conv(\{p,\, q\}) = d - conv(\{x_1,\, x_2\})$;

b) for any two adjacent vertexes $a,\, b \in X_{\vec{G}}$, there exist two distinct vertexes $x_1,\ x_2 \in X_{\vec{H}}$, such that the following relations are true:

0. $x_1 = p,\ x_2 = q$;
1. $x_1,\ x_2 \in d - conv(\{a,\, b\})$;
2. $d_{\vec{H}}(x_1,\, x_2) = d_{\vec{G}}(x_1,\, x_2) = d_{\vec{G}}(p,\, q) = r$;
3. $d_{\vec{G}}(q,\, p) = d_{\vec{H}}(q,\, p) = d_{\vec{H}}(x_2,\, x_1)$.

It follows that $\vec{G} = M^{x_1=p}_{x_2=q}(\vec{H},\, \mathcal{P}) \in \mathfrak{D}$. According to theorem 1.2, this means the $\vec{G}$ is d-convex simple graph.

The theorem is proved. □

Theorem 2.3 *If $\vec{G}_1$ and $\vec{G}_2$ are two d-convex simple graphs, where there exist the nonadjacent vertexes $x_1,\ x_2 \in \vec{G}_1$ and $y_1,\ y_2 \in \vec{G}_2$, which satisfy conditions $d_{\vec{G}_1}(x_1,\, x_2) = d_{\vec{G}_2}(y_1,\, y_2)$ and $d_{\vec{G}_1}(x_2,\, x_1) = d_{\vec{G}_2}(y_2,\, y_1)$, then the graph $\vec{G} = M^{x_1=y_1}_{x_2=y_2}(\vec{G}_1,\ \vec{G}_2)$ is also d-convex simple.*

Proof: We are going to prove this theorem using the same approach as in the last theorem. We consider that $\vec{G}_{i-1} = \vec{G}_1$ and $\vec{G}_i = \vec{G}$, and will show that $\vec{G}$ can be obtained from $\vec{G}_1$ with respect of condition II of description of the class $\mathfrak{D}$. From the condition that the selected pairs of vertexes are nonadjacent and construction of the graph $\vec{G} = M^{x_1=y_1}_{x_2=y_2}(\vec{G}_1,\ \vec{G}_2)$, it results that the vertexes of graph $\vec{G}_1$ were not joined with new arcs. More than that, the following relations are true:

a) for any $y_j \in X_{\vec{G}_2}$, there exist vertexes $x_1 = y_1,\ x_2 = y_2 \in X_{\vec{G}_1}$, such that: $y_j \in d - conv(\{y_1,\ y_2\}) = d - conv(\{x_1,\ x_2\})$;

b) for any two adjacent vertexes $a,\ b \in X_{\vec{G}}$, there exist two distinct vertexes $x_1,\ x_2 \in X_{\vec{G}_1}$, such that:

   0. $x_1 = y_1,\ x_2 = y_2$;
   1. $x_1,\ x_2 \in d - conv(\{a,\ b\})$;
   2. $d_{\vec{G}_1}(x_1,\ x_2) = d_{\vec{G}_2}(y_1,\ y_2) = d_{\vec{G}}(y_1,\ y_2)$;
   3. $d_{\vec{G}_1}(x_2,\ x_1) = d_{\vec{G}_2}(y_2,\ y_1) = d_{\vec{G}}(y_2,\ y_1)$.

   It follows that $\vec{G} = M^{x_1=y_1}_{x_2=y_2}(\vec{G}_1,\ \vec{G}_2) \in \mathfrak{D}$.

According to theorem 1.2, this means the graph $\vec{G}$ is d-convex simple graph. □

From the theorem 2.3 it results that the operation M, introduced above and being applied to the pairs of vertexes, that are at the same distances in the different d-convex simple graphs, is an *algebraic operation* on the set of the directed d-convex simple graphs $\mathfrak{D}$.

Definition 2.1 *Two vertexes u and v of a graph $\vec{G} = (X,\ \vec{U})$ are called to be* ***copies*** *vertexes in $\vec{G}$ if there are equalities:*

$$\Gamma^+(u) = \Gamma^+(v) \text{ and } \Gamma^-(u) = \Gamma^-(v).$$

In case of

$$\Gamma^+(u) = \Gamma^-(v) \text{ and } \Gamma^-(u) = \Gamma^+(v),$$

the vertexes u and v are called ***anti-copies****.*

Let us observe, that if $\vec{G} = (X, \vec{U})$ is a directed and strongly connected graph, where there exists a pair of vertexes anti-copies u and v, then $d_{\vec{G}}(u, v) = d_{\vec{G}}(v, u) = 2$. Indeed, from the fact that the graph $\vec{G}$ is strongly connected it results that any vertex has as predecessors as successors. So, no one of the sets $\Gamma^+(u)$, $\Gamma^+(v)$, $\Gamma^-(u)$, $\Gamma^-(v)$ is empty. Since u and v are vertexes anti-copies and the relations $\Gamma^+(u) = \Gamma^-(v)$ and $\Gamma^-(u) = \Gamma^+(v)$ are true, then it results:

a) u and v are not adjacent, i. e. neither u nor v is from the mentioned sets (otherwise the graph $\vec{G}$ will contain loops);

b) in $\vec{G}$ exist paths of length two that join u with v and v with u.

From a) we obtain that $d_{\vec{G}}(u, v)$ and $d_{\vec{G}}(v, u)$ are not equal to 1, and from b) we have that $d_{\vec{G}}(u, v) = d_{\vec{G}}(v, u) = 2$. So, circuit of minimal length that contains the vertexes u and v is of length 4.

From the above and theorem 2.3 we have:

Corollary 2.2 *If $\vec{G}_1$ and $\vec{G}_2$ are two d-convex simple graphs, where there exist the vertexes anti-copies $x_1, x_2 \in \vec{G}_1$ and $y_1, y_2 \in \vec{G}_2$, then the graph $\vec{G} = M^{x_1=y_1}_{x_2=y_2}(\vec{G}_1, \vec{G}_2)$ is also d-convex simple.*

Let us observe that if $\vec{G} = (X, \vec{U})$ is a directed and strongly connected graph, where there exists a pair of vertexes copies u and v, then $d_{\vec{G}}(u, v) = d_{\vec{G}}(v, u)$. Indeed, if we suppose that, the following inequality is true:

$$d_{\vec{G}}(u, v) < d_{\vec{G}}(v, u),$$

and $[u, x_1, x_2, \ldots, x_r, v]$ is one path of minimal length that joins the vertex u with vertex v, then, since u and v are vertexes copies we have $\Gamma^+(u) = \Gamma^+(v)$, that implies $(v, x_1) \in \vec{U}$, and $\Gamma^-(u) = \Gamma^-(v)$, that implies $(x_r, u) \in \vec{U}$. It follows that in $\vec{G}$ there exists the path $[v, x_1, x_2, \ldots, x_r, u]$, the fact that contradicts the assumption that the distance from v to u is longer than the distance from u to v. So, assumption is false and $d_{\vec{G}}(u, v) = d_{\vec{G}}(v, u)$.

From the above and theorem 2.3 we have:

Corollary 2.3 *If $\vec{G}_1$ and $\vec{G}_2$ are two d-convex simple graphs, where there exist the vertexes copies $x_1,\ x_2 \in \vec{G}_1$ and $y_1,\ y_2 \in \vec{G}_2$, which satisfy the condition $d_{\vec{G}_1}(x_1,\ x_2) = d_{\vec{G}_2}(y_1,\ y_2)$, then the graph $\vec{G} = M^{x_1=y_1}_{x_2=y_2}(\vec{G}_1,\ \vec{G}_2)$ is also d-convex simple.*

Now, let $\vec{G} = (X, \vec{U})$ be a d-convex simple graph and v any vertex of $\vec{G}$. We form the graph $\vec{G}^{++}$, which is obtained from the graph $\vec{G}$ by adding one vertex copy for v, which is denoted by $\tilde{v}$.

Theorem 2.4 *If $\vec{G} = (X, \vec{U})$, $X \geq 3$, is a d-convex simple graph, then $\vec{G}^{++}$ is also d-convex simple graph.*

Proof: Let $\vec{G} = (X, \vec{U})$ be a d-convex simple graph. By the theorem 1.3 this graph is from $\mathfrak{D}$. We choose an arbitrary vertex $v \in X$ and, according to the above, form the graph $\vec{G}^{++}$. Let $x,\ y$ be two vertexes from $X^{++}_{\vec{G}}$ and let $d - conv_{\vec{G}^{++}}(\{x,\ y\})$ be d-convex hull of these vertexes. It is easy to see that if $v \in d - conv_{\vec{G}^{++}}(\{x,\ y\})$ then we immediately have $\tilde{v} \in d - conv_{\vec{G}^{++}}(\{x,\ y\})$, the reverse assertion is also true. More than that, if $x,\ y$ are two different vertexes from $X_{\vec{G}^{++}}$ and they are different from the vertexes $v,\ \tilde{v}$, then $v \in d - conv_{\vec{G}^{++}}(\{x,\ y\})$, because graph $\vec{G}$ is d-convex simple. It follows that $\tilde{v} \in d - conv_{\vec{G}^{++}}(\{x,\ y\})$. But the set $\langle\overrightarrow{v,\ \tilde{v}}\rangle_{\vec{G}^{++}} \cup \langle\overrightarrow{\tilde{v},\ v}\rangle_{\vec{G}^{++}}$ contains all vertexes of at least one circuit of minimal length that pass trough vertex v, because the graph $\vec{G}$ is d-convex simple and this determine it to be strongly connected. This circuit contains vertexes from $\vec{G}$, which had kept the same distances among them in the graph $\vec{G}^{++}$ like in $\vec{G}$. The d-convex hull of these last vertexes contains all vertexes from $\vec{G}$. From this it results:

$$d - conv_{\vec{G}^{++}}(\{v,\ \tilde{v}\}) = X_{\vec{G}^{++}}.$$

By this way we obtained that d-convex hull of any two distinct vertexes from $\vec{G}^{++}$, which are different from v and $\tilde{v}$, contains all vertexes from $\vec{G}^{++}$.

Let y be any vertex from $\vec{G}^{++}$, different from vertexes v and $\tilde{v}$. D-convex hull of the set $\{y,\ v\}$ in $\vec{G}$ contains all vertexes of at least one circuit of minimal length that pass through v and y, which contains vertexes from $\vec{G}$, which had kept the same distances in the graph $\vec{G}^{++}$. So, $d - conv_{\vec{G}^{++}}(\{v,\ y\}) = X_{\vec{G}^{++}}$. And because the vertexes v and $\tilde{v}$ are vertexes copies then there are the equalities $\langle\overrightarrow{v,\ y}\rangle = \langle\overrightarrow{\tilde{v},\ y}\rangle$ and $\langle\overrightarrow{y,\ v}\rangle = \langle\overrightarrow{y,\ \tilde{v}}\rangle$, from where it results

$$d - conv_{\vec{G}^{++}}(\{v,\ y\}) = d - conv_{\vec{G}^{++}}(\{\tilde{v},\ y\}) = X_{\vec{G}^{++}}.$$

This means that the graph $\vec{G}^{++}$ is d-convex simple. □

From the theorem 2.4 it results that in directed d-convex simple graphs, like in undirected d-convex simple graphs, we can multiply any vertex as many times as we need, and the obtained graphs will be also d-convex simple.

Let $\vec{G} = (X,\ \vec{U})$ be a directed d-convex simple graph, where there exist three vertexes copies v_1, v_2 and v_3. We denote by $\vec{G}^{--}$ the graph that is obtained by the graph $\vec{G}$ as the result of elimination of one of them, for example, the vertex v_3.

Theorem 2.5 *If $\vec{G}$ is d-convex simple graph and v_1, v_2 and v_3 are three vertexes copies of it, then the graph $\vec{G}^{--}$, where one of them is missing, is also d-convex simple graph.*

Proof: Let $\vec{G} = (X,\ \vec{U})$ be a directed d-convex simple graph, where there exist three vertexes copies v_1, v_2 and v_3, and let $\vec{G}^{--}$ be the graph that is obtained by the graph $\vec{G}$ as the result of elimination of one of them, for example, of vertex v_3. Obviously, from the fact that the vertexes v_1, v_2 and v_3 are vertexes copies in $\vec{G}$, it results that any d-segment that contains one of these three vertexes, will immediately contain the others as well. This property is true for the vertexes v_1 and v_2 in graph $\vec{G}^{--}$, too. From the same considerations, for any two vertexes $x,\ y \in X_{\vec{G}^{--}}$ the following equality holds:

$$d_{\vec{G}}(x,\ y) = d_{\vec{G}^{--}}(x,\ y). \qquad (*)$$

More than that, since $\vec{G}$ is d-convex simple, then in $\vec{G}$ there are equalities $\langle\overrightarrow{v_1,\ v_2}\rangle = \langle\overrightarrow{v_1,\ v_3}\rangle$ and $\langle\overrightarrow{v_2,\ v_1}\rangle = \langle\overrightarrow{v_3,\ v_1}\rangle$, from where it results:

$$d - conv_{\vec{G}}(\{v_1,\ v_2\}) = d - conv_{\vec{G}}(\{v_1,\ v_3\}) = X_{\vec{G}}.$$

But the last implies:

$$d - conv_{\vec{G}^{--}}(\{v_1,\ v_2\}) = X_{\vec{G}^{--}}. \qquad (**)$$

Now, we choose two distinct vertexes $a,\ b \in X_{\vec{G}^{--}}$. Because initial graph $\vec{G}$ is d-convex simple, it follows, that d-convex hull of the set $\{a,\ b\}$ in this graph can be built by the sequence of sets:

$$B_0 = \{a,\ b\},\ B_1 = P(B_0),\ B_2 = P(B_1),\ \ldots,$$

$$B_i = P(B_{i-1}),\ \ldots,\ d - conv(\{a,\ b\}) = X_{\vec{G}}.$$

We consider that, for example, B_i is the first set of this sequence, which contains the vertex v_3. Obviously, the vertexes v_1 and v_2 belong to this set, too.

Because of the relation (*), for the set $\{a,\ b\}$ in the graph $\vec{G}^{--}$ we can build the sequence:

$$B_0^{--} = \{a,\ b\},\ B_1^{--} = P(B_0^{--}),\ B_2^{--} = P(B_1^{--}),\ \ldots,\ B_i^{--} =$$

$$= P(B_{i-1}^{--}),\ \ldots,$$

such that $B_j = B_j^{--}$, for all $0 \leq j \leq i-1$. In these conditions we obtain that $B_i \backslash B_i^{--} = \{v_3\}$, but the vertexes v_1 and v_2 belong to the set B_i^{--}, that means, according to the (**), that $d - conv_{\vec{G}^{--}}(\{a,\ b\}) = X_{\vec{G}^{--}}$. □

From the theorem 2.5 it results that in the directed d-convex simple graphs, like in the undirected d-convex simple graphs, we can eliminate the vertexes copies of the vertex v, keeping only one copy for v, and the obtained graph will be d-convex simple, too.

Let $\vec{G} = (X,\ \vec{U})$ be a directed graph. We form the graph $\vec{G}^t = (X,\ \vec{U}^t)$ that is obtained from $\vec{G}$ by redirecting of all arcs of it. It is easy to see, that the adjacent matrix of the graph $\vec{G}^t$ is the transpose of the adjacent matrix of the graph $\vec{G}$.

Theorem 2.6 *If $\vec{G}$ is d-convex simple graph, then the graph $\vec{G}^t$ is also d-convex simple graph.*

Proof: Let $\vec{G} = (X,\ \vec{U})$ be a d-convex simple graph, and $\vec{G}^t = (X,\ \vec{U}^t)$ the graph formed like it is described above. From the construction it results that for any two distinct vertexes x and y from X the following equalities are true:

$$\langle\overrightarrow{x,\ y}\rangle_{\vec{G}} = \langle\overrightarrow{y,\ x}\rangle_{\vec{G}^t} \text{ and } \langle\overrightarrow{y,\ x}\rangle_{\vec{G}} = \langle\overrightarrow{x,\ y}\rangle_{\vec{G}^t}.$$

It follows that we have $d - conv_{\vec{G}}(\{x,\ y\}) = d - conv_{\vec{G}^t}(\{y,\ x\})$. But the initial graph $\vec{G}$ is d-convex simple and the order of elements is not important in sets. It results that $\vec{G}^t$ is d-convex simple, too. □

Let $\vec{G} = (X,\ \vec{U})$, $|X| \geq 4$ be a directed graph. We choose in $\vec{G}$ four vertexes x_1, x_2, y_1 and y_2, that are nonadjacent two by two in $\vec{G}$. We denote by $W^{x_1=y_1}_{x_2=y_2}(\vec{G})$ the graph obtained from $\vec{G}$ as the result of joining the vertexes x_1 with y_1 and x_2 with y_2.

Theorem 2.7 *If $\vec{G} = (X,\ \vec{U})$, $|X| \geq 4$ is d-convex simple graph, where there exist four vertexes x_1, x_2, y_1 and y_2, which satisfy conditions:*

1. *the vertexes x_1, x_2, y_1 and y_2 are nonadjacent two by two;*
2. $d(x_1,\ x_2) = d(y_1,\ y_2),\ d(x_2,\ x_1) = d(y_2,\ y_1)$;
3. $\min\{d(x_1,\ y_1),\ d(y_1,\ x_1),\ d(x_2,\ y_2),\ d(y_2,\ x_2)\} \geq d(x_1,\ x_2) + d(x_2,\ x_1)$;
4. $\min\{d(x_1,\ y_2),\ d(y_2,\ x_1),\ d(x_2,\ y_1),\ d(y_1,\ x_2)\} \geq d(x_1,\ x_2) + d(x_2,\ x_1)$,

then the graph $\vec{H} = W^{x_1=y_1}_{x_2=y_2}(\vec{G})$ is also d-convex simple.

Proof: Let $\vec{G} = (X,\ \vec{U})$ be a directed d-convex simple graph that satisfies the conditions of the theorem. Let $\vec{H} = W^{x_1=y_1}_{x_2=y_2}(\vec{G})$ be the graph constructed as it is shown above. We remind that in this work we study the directed graphs, without loops or multiple arcs. Because the initial graph $\vec{G}$ is d-convex simple, and the set of selected vertexes $\{x_1,\ x_2,\ y_1,\ y_2\}$ satisfies conditions 1 of the theorem, it results that in graph $\vec{H}$ there are no loops or multiple arcs, too. More, from the last we have that for any two vertexes $s,\ t \in X_{\vec{H}}$ there exist in $\vec{H}$ at most one of the two possible arcs $s,\ t$ and $t,\ s$.

Let us suppose that $\vec{H}$ is not d-convex simple. This means that in this graph there exists a subset of d-convex vertexes B, $1 < |B| < |X_{\vec{H}}|$. Thus, initial graph $\vec{G}$ is d-convex simple, we have that d-convex hull of the set B in the graph $\vec{G}$ is $d-conv_{\vec{G}}(B) = X_{\vec{G}}$. This d-convex hull can be formed in $\vec{G}$ as follows:

$$B_0 = B,\ B_1 = P_{\vec{G}}(B_0),\ B_2 = P_{\vec{G}}(B_1),\ \ldots,\ B_i = d-conv_{\vec{G}}(B) = X_{\vec{G}}.$$

Let us form the d-convex hull of the set B in the graph $\vec{H}$. By using the same operation we have:

$$A_0 = B,\ A_1 = P_{\vec{H}}(A_0),\ A_2 = P_{\vec{H}}(A_1),\ \ldots,\ A_i = P_{\vec{H}}(A_{i-1})\ldots$$

From conditions 2, 3 and 4 of the theorem, we can have that:

$$B_j \backslash \{x_1,\ x_2,\ y_1,\ y_2\} = A_j \backslash \{x_1 = y_1,\ x_2 = y_2\},\ \forall j \in \mathbb{N}.$$

The last implies that for $j = i$ we have

$$X_{\vec{G}} \backslash \{x_1,\ x_2,\ y_1,\ y_2\} = A_j \backslash \{x_1 = y_1,\ x_2 = y_2\}.$$

But the vertexes $x_1,\ x_2,\ y_1,\ y_2$ are in $B_i = X_{\vec{G}}$, it results that $x_1 = y_1, x_2 = y_2 \in A_i$. But $[X_{\vec{G}} \backslash \{x_1, x_2, y_1, y_2\}] \cup \{x_1 = y_1, x_2 = y_2\} = X_{\vec{H}}$. So, we have

$$X_{\vec{H}} \subseteq A_i \subseteq d-conv_{\vec{H}}(B),$$

that contradicts the assertion that $d-conv_{\vec{H}}(B) \neq X_{\vec{H}}$. The assumption is false, the graph $\vec{H}$ is d-convex simple. $\square$

Corollary 2.4 *If $\vec{G} = (X,\ \vec{U})$, $|X| \geq 4$ is d-convex simple graph, where there exist two pairs of vertexes anti-copies $x_1,\ x_2$ and $y_1,\ y_2$, which satisfy conditions:*

a. $\min\{d(x_1,\ y_1),\ d(y_1,\ x_1),\ d(x_2,\ y_2),\ d(y_2,\ x_2)\} \geq 4$;

b. $\min\{d(x_1,\ y_2),\ d(y_2,\ x_1),\ d(x_2,\ y_1),\ d(y_1,\ x_2)\} \geq 4$, *then the graph* $\vec{H} = W^{x_1=y_1}_{x_2=y_2}(\vec{G})$ *is also d-convex simple graph.*

Corollary 2.5 *If* $\vec{G} = (X,\ \vec{U})$, $|X| \geq 4$ *is d-convex simple graph, where there exist two pairs of vertexes copies* $x_1,\ x_2$ *and* $y_1,\ y_2$, *which satisfy conditions:*

a. $d(x_1,\ x_2) = d(y_1,\ y_2)$;

b. $\min\{d(x_1,\ y_1),\ d(y_1,\ x_1),\ d(x_2,\ y_2),\ d(y_2,\ x_2)\} \geq 2 \cdot d(x_1,\ x_2)$;

c. $\min\{d(x_1,\ y_2),\ d(y_2,\ x_1),\ d(x_2,\ y_1),\ d(y_1,\ x_2)\} \geq 2 \cdot d(x_1,\ x_2)$,

then the graph $\vec{H} = W^{x_1=y_1}_{x_2=y_2}(\vec{G})$ *is also d-convex simple graph.*

Further, we will use the notions of chain and cycle for directed graphs, defined in [9]. The number of arcs that belong to a chain (cycle) is called *length* of it. For example, the chain $l = (x_{i_1},\ x_{i_2},\ \ldots,\ x_{i_t})$ has length equal to $t - 1$. For simplicity, further, the cycle of length three, will be called *triangle*. The directed graph $\vec{G} = (X,\ \vec{U})$ is called *weakly connected*, if any two vertexes of it are joined by a chain.

We define for the directed graphs a special operation, denoted by L_2, which, to tell the truth, is defined analogically to the case of undirected graphs [7, 8]. Let us denote by $X = \{x_1,\ x_2,\ \ldots,\ x_n\}$ the set of vertexes of the directed graph $\vec{G} = (X,\ \vec{U})$. Let $\vec{G}_1,\ \vec{G}_2$ be two copies of the graph $\vec{G}$ with the sets of vertexes $X_{\vec{G}_1} = \{x^1_1,\ x^1_2,\ \ldots,\ x^1_n\}$ and $X_{\vec{G}_2} = \{x^2_1,\ x^2_2,\ \ldots,\ x^2_n\}$ respectively. The vertexes x^1_j and x^2_j, $j = 1,\ 2,\ \ldots,\ n$, are called correspondent vertexes to the vertex x_j, $1 \leq j \leq n$, in conditions when $(x_i, x_k) \in \vec{U}$ if and only if $(x^p_i, x^p_k) \in \vec{U}_{\vec{G}_p}$, where $p = 1,\ 2$. By $L_2(\vec{G})$ we define the graph that is obtained from $\vec{G}_1$ and $\vec{G}_2$ by adding the following arcs: for any vertex x^1_j, $1 \leq j \leq n$, from $\vec{G}_1$, we add arcs to all vertexes which are adjacent to the vertex x^2_j in $\vec{G}_2$, only that they are of the opposite directions. It is obvious that if $|X| = n$ and $|\vec{U}| = r$, then the graph $L_2(\vec{G})$ will have $2n$ vertexes and $4r$ arcs. More than that, in the $L_2(\vec{G})$, the vertexes x^1_j and x^2_j are vertexes anti-copies, for any $j = 1,\ 2,\ \ldots,\ n$.

Theorem 2.8 *If $\vec{G}$ is a directed, weakly connected, antisymmetric graph, without triangles, then the graph $L_2(\vec{G})$ is d-convex simple graph.*

Proof: We will prove first that if the graph $\vec{G}$ is a graph that satisfies the conditions of the theorem, then the graph $L_2(\vec{G})$ is strongly connected. Let u and v are two vertexes from $L_2(\vec{G})$. There are possible two cases: both of these vertexes belong to one of the two copies of graph $\vec{G}$, $\vec{G}_1$ and $\vec{G}_2$; or, for example, u is from $\vec{G}_1$ and v is from $\vec{G}_2$.

a. Let us suppose that both vertexes u and v are, for example, from $\vec{G}_1$. The fact that the graph $\vec{G}$ is weakly connected results that in $\vec{G}_1$ there exists the chain $l = (u = x^1_{i_1},\ x^1_{i_2},\ \ldots,\ x^1_{i_t} = v)$, that joins the vertexes u and v. The arcs, that joined the vertexes $x^1_{i_k}$ and $x^1_{i_{k+1}}$, $1 \leq k \leq t-1$, are of the arbitrary direction (from the vertex $x^1_{i_k}$ to the $x^1_{i_{k+1}}$, or reverse). From this chain we will build a path from u to v in the graph $L_2(\vec{G})$. Let the arc $(x^1_{i_{s+1}},\ x^1_{i_s})$ be the first from this chain that is directed from v to u. Then in our path from $L_2(\vec{G})$ this arc will be replaced with next three arcs: $(x^1_{i_s},\ x^2_{i_{s+1}})$, $(x^2_{i_{s+2}},\ x^2_{i_s})$, $(x^2_{i_s},\ x^1_{i_{s+1}})$, where $x^2_{i_s}$, $x^2_{i_{s+1}}$ are the vertexes anti-copies corresponding to $x^1_{i_s}$, $x^1_{i_{s+1}}$ respectively. We will do the same thing with all arcs from l, similar to arc $(x^1_{i_{s+1}},\ x^1_{i_s})$, and finally we obtain the wanted path from u to v in $L_2(\vec{G})$.

b. Let us suppose now that u is from $\vec{G}_1$ and v is from $\vec{G}_2$. By analogy with the case a we can, first, build a path from u to $\tilde{v} \in X_{\vec{G}_1}$, that is anti-copy of the vertex v from graph $\vec{G}_1$. Let $(x,\ \tilde{v})$ be the last arc from this path. We extend the path from u to $\tilde{v}$ with arcs $(\tilde{v},\ \tilde{x})$, $(\tilde{x},\ v)$, where $\tilde{x} \in X_{\vec{G}_2}$ is the anti-copy of vertex x, and obtain by this way a path in $L_2(\vec{G})$ from u to v.

From investigated cases a and b it results that for any pair of ordered vertexes $(u,\ v)$ from $L_2(\vec{G})$ there is a path from u to v. It follows that $L_2(\vec{G})$ is a strongly connected graph.

Let us prove, now, that graph $L_2(\vec{G})$ is d-convex simple. Let u and v be two adjacent vertexes in $L_2(\vec{G})$ and $\tilde{u}$ and $\tilde{v}$ be anti-copies of these vertexes respectively (the existence of the anti-copies $\tilde{u}$ and $\tilde{v}$ result from the construction of the graph $L_2(\vec{G})$). From the fact that the graph $\vec{G}$ is antisymmetric, without triangles, and from the construction of the graph $L_1(\vec{G})$ we obtain that the graph $L_2(\vec{G})$ is antisymmetric and without triangles, too. So it follows that the cycles and circuits of minimal length in this graph have at least four arcs. Such a circuit of minimal length is determined by vertexes u, v, $\tilde{u}$ and $\tilde{v}$. We obtain that d-convex hull of any two adjacent vertexes u, v contains at least one pair of vertexes anti-copies. Let z and $\tilde{z}$ be a pair of vertexes anti-copies from d-convex hull of any two adjacent vertexes $a,\ b \in X_{L_2(\vec{G})}$. We form in $L_2(\vec{G})$ the sequence of sets:

$$B_0 = \{z,\ \tilde{z}\} \subset d - conv(\{a,\ b\}),\ B_1 = P(B_0),\ \ldots,\ d - conv(B_0).$$

From the construction of the graph $L_2(\vec{G})$, and the condition of theorem that the graph $\vec{G}$ is weakly connected, the construction of the set B_{i+1} from the set B_i is always followed by addition of at least one pair of the new vertexes copies. This means, that in the last we will cover all the vertexes, and the set $d - conv(B_0)$ will coincide with $X_{L_2(\vec{G})}$. So we have that $L_2(\vec{G})$ is a d-convex simple graph. $\square$

3 Relation between Directed d-Convex Simple Graphs and Undirected d-Convex Simple Graphs.

Let $G = (X,\ U)$ be an undirected graph. This means that it is an directed graph, completely symmetric, where each edges $u = (x, y) \in U$ are considered as two arcs $(x,\ y)$ and $(y,\ x)$. Let us eliminate from each edge of the graph G one and only one of these two arcs. The obtained graph is antisymmetric, and it will be called *direction* graph of the initial graph G, and denote by $\vec{G}$. Of course, in dependence of what arcs are eliminated, for the graph G, we can obtain several its direction graphs. In this section we are going to show, that for any undirected, d-convex simple graph, the structure of which is known, there is at least one directed d-convex simple graph, that corresponds to that undirected, and the correspondence we will search in the set of direction graphs of the initial undirected graph.

Let us consider the class $\mathcal{A}$ of the undirected d-convex simple graphs from [7].

Theorem 3.1 *If G is an undirected d-convex simple graph from the class $\mathcal{A}$ then there exists at least one direction of the G, that is a directed d-convex simple graph.*

Proof: We are going to prove this theorem, by giving a way of construction of one directed d-convex simple graph, we need.

Let $G \in \mathcal{A}$ be an undirected d-convex simple graph. Then from [7] we have that $G = L(\Gamma, \Gamma_0)$, where Γ is a connected graph without triangles and Γ_0 is its atom. In order to construct Γ_0 first we have to determine the sets:

$$S = \{x \in X \mid \forall y \in X \Rightarrow \Delta(x) \nsubseteq \Delta(y)\};$$

$$R = \{x \in X \backslash S \mid \forall y \in X \Rightarrow \Delta(x) \not\subset \Delta(y)\},$$

Then, for any $x \in R$ we form the set $R(x) = \{x\} \cup \{y \in R \mid \Delta(x) = \Delta(y)\}$. By this way the set R is divided in classes of equivalence. Γ_0 is an induced subgraph of graph G, the set of vertexes of which is formed from the set S and by one vertex from each class of equivalence. It is easy to see from construction that the graph Γ_0 is also a connected graph, without triangles (see [7]).

Let us consider any direction of the graph Γ_0 and denote it by $\vec{\Gamma}_0$. The graph $\vec{\Gamma}_0$ is a directed, weakly connected, antisymmetric graph, without triangles. According to the theorem 2.8 the graph $L_2(\vec{\Gamma}_0)$ is a directed d-convex simple graph. Any vertex of the graph $L_2(\vec{\Gamma}_0)$ can be multiplied, as many times as we need, according to the theorem 2.4. Let us build now a new graph, denoted by $\vec{G}$, which is formed from the graph $L_2(\vec{\Gamma}_0)$ where those vertexes were multiplied, that satisfy the following condition: for any vertex from R, there will be a correspondence vertex in the graph $\vec{G}$. The new graph $\vec{G}$ is a directed d-convex simple graph, and it is a direction of the initial graph G. □

From the last proof, we have that if G is undirected, d-convex simple graph and $\vec{G}$ is its direction d-convex simple graph, then in $\vec{G}$ any vertex has a anti-copy. So, the pairs of vertexes copies from G, have become pairs of vertexes anti-copies in $\vec{G}$. The last means that these graphs can participate in operations M and W with any graph which has a pair of vertexes anti-copies.

The directed and d-convex simple graph from the fig.1 is a direction from the graph J_1 from [2, 3], denoted $\vec{J_1}$. It has the property that if we add a vertex anti-copy to any its vertex, the new graph will be also d-convex simple.

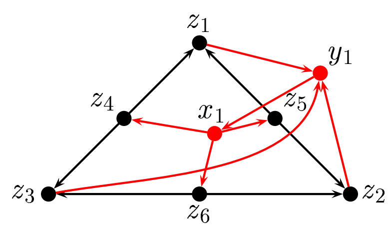

Fig. 1. The graph $\vec{J_1}$.

By this way, we obtain that all undirected d-convex simple graphs G from [2, 3], has at least one direction graph $\vec{G}$, that is a directed, d-convex simple graph. This means that the set of directed d-convex simple graphs contains, in this sense, the set of undirected d-convex simple graphs.

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Minimum d-convex partition of a multidimensional polyhedron with holes*

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Abstract

In a normed space $\mathcal{R}^n$ over the field of real numbers $\mathbb{R}$, which is an α-space [36, 39], one derives the formula expressing the minimum number of d-convex pieces into which a geometric n-dimensional polyhedron with holes can be partitioned. The problem of partitioning a geometric n-dimensional polyhedron has many theoretical and practical applications in various fields such as computational geometry, image processing, pattern recognition, computer graphics, VLSI engineering, and others [5, 10, 11, 19, 21, 28, 29, 31, 43].

Key Words: geometric n-dimensional polyhedron, d-convexity, CW complex, dividing

Mathematics Subject Classification: 68U05, 52A30, 57Q05

1. Introduction

Let (X, d) be a metric space, and let $x_1, x_2 \in X$ be two arbitrary points of (X, d). By analogy with the classical definition of a convex set one introduces the notion of metric convexity depending on d [6, 8, 20, 36]. The set of points, denoted by $\langle x_1, x_2 \rangle$ and defined by $\langle x_1, x_2 \rangle = \{x \mid d(x_1, x_2) = d(x_1, x) + d(x, x_2)\}$, is called a *metric segment* joining the points x_1 and x_2. A set $M \subset X$ is said to be *d-convex* if for any two points $x_1, x_2 \in M$ the metric segment $\langle x_1, x_2 \rangle \subset M$. It is easy to see that the intersection of two d-convex sets is a d-convex set.

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For a given set $M \subset X$, the d-convex hull of the set M, denoted by d-$conv\, M$, is defined as the intersection of all d-convex sets containing M. In case that (X, d) is a normed space $\mathcal{R}^n$ over the field of real numbers $\mathbb{R}$ with $d(x_1, x_2) = \|x_1 - x_2\|$ every d-convex set is also a convex set, but not always conversely. Convexity and d-convexity in $\mathcal{R}^n$ coincide if and only if the closed unit ball of $\mathcal{R}^n$ is strictly convex [6, 8, 20, 36]. Thus this notions coincide in the Euclidean space $\mathbb{E}^n$. For a bounded set $N \subset \mathcal{R}^n$ it can happen that d-$conv\, N = \mathcal{R}^n$ [6, 36]. We will only consider those normed spaces such that d-$conv\, N$ is bounded, that is, so-called α-spaces [36, 39].

In the papers [1, 7, 24, 26, 32, 38, 40, 42] it is given sufficient information on the problem of partitioning a polygon with holes or a 3-dimensional geometric polyhedron with holes into the minimum number of d-convex pieces.

Let $\mathcal{R}^2$ be a normed plane, and let P^2 be an open polygon with g holes of dimension $d \in \{0, 1, 2\}$ all of whose edges are d-convex.

In the paper [24] $\mathcal{R}^2$ coincides with the Euclidean plane $\mathbb{E}^2$, and the edges of the polygon P^2 are only parallel to two perpendicular directions while all the holes are of dimension 2. In this case it is shown that the minimum number $q\left(P^2\right)$ of rectangles partitioning the polygon P^2 is

$$q\left(P^2\right) = \frac{s}{2\pi} - h, \tag{1.1}$$

where s is the total sum of interior angles of the polygon P^2, measured in radians, and h is the maximum number of mutually disjoint segments that can be drawn within the closure of the polygon P^2, parallel to the edges of P^2, and with the endpoints at the concave vertices. This formula was generalized by Soltan and Gorpinevich [42] for the case of a rectilinear polygonal domain with possible degenerate holes. That problem appeared in VLSI engineering [35].

In the papers [7, 26, 32] the problem to partition the polygon P^2 into a minimum number of d-convex pieces is completely solved. The respective formula is shown to be

$$q\left(P^2\right) = m + 1 - g - h, \tag{1.2}$$

where m, g are the total sum of all measures of local nonconvexity of points of local nonconvexity [7, 26], the number of holes of the polygon P^2, respectively, and h is the number of elements of a maximum concordant system of dividing trees [7, 26]. Considering $\mathcal{R}^2$ with the norm $\|x\| = |x_1| + |x_2|$, it is easy to obtain that q in (1.1) and q in (1.2) are the same number for the case from [25].

Let P^3 be an open polyhedron in the Euclidean space $\mathbb{E}^3$ with polyhedral holes and the edges parallel to the coordinate axes of $\mathbb{E}^3$, where the holes can be of dimension $3, 2, 1, 0$. A formula expressing the minimum number of parallelepipeds $q\left(P^3\right)$ into which the polyhedron P^3 can be partitioned is proposed in the paper [1]. The researches done in this paper led to the fact that the minimum estimation of $q\left(P^3\right)$ required the methods of algebraic topology to be applied, as it would see below.

2. Auxiliary elements

In a normed space $\mathcal{R}^n$ it is possible to define the notion of a geometric n-dimensional polyhedron in a simpler or more complicated way. We will introduce a more natural notion of a polyhedron as the (simple) polygon [14] in $\mathcal{R}^2$ and the (simple) geometric polyhedron [14] in $\mathcal{R}^3$ are defined.

By analogy with [9] we propose

Definition 2.1. *A compact n-dimensional PL manifold* [15, 18, 27, 34] *with boundary in the normed space $\mathcal{R}^n$ which admits a decomposition into q handles* [16, 34] *of index* 1 *is said to be a* ***geometric n-dimensional polyhedron of genus q*** *in $\mathcal{R}^n$. It is denoted by P_q^n.*

For a geometric polyhedron P_q^n as a topological subspace of the space $\mathcal{R}^n$, we will denote by $bd\, P_q^n, int\, P_q^n, \overline{P_q^n}$ the boundary, the interior and the closure of P_q^n, respectively. By $B^n(x, \varepsilon) \subset \mathcal{R}^n$ we denote the closed ball with center at x and radius ε. We will use B^n and S^{n-1} as notations for the closed unit ball and the unit sphere of $\mathcal{R}^n$, respectively.

Definition 2.2. *Let x be a boundary point of a geometric polyhedron P_q^n. We will denote by aff $\left(x, P_q^n\right)$ the union of x and all lines l in $\mathcal{R}^n$ through the point x such that the intersection $l \bigcap bd\, P_q^n$ contains an open line segment which includes x. The connection component of aff $\left(x, P_q^n\right) \bigcap bd\, P_q^n$ which contains the point x is called a* ***face*** *of x of P_q^n. By convention, $\emptyset$ and P_q^n are called* ***improper faces*** *of P_q^n.*

Definition 2.3. *By a* ***m-dimensional face*** *F^m of P_q^n we mean a face of dimension m (the dimension of the affine hull of F^m). We call F^m*

1) a ***vertex*** *of P_q^n, if $m = 0$;*

2) an ***edge*** *of P_q^n, if $m = 1$;*

3) a ***facet*** *of P_q^n, if $m = n - 1$.*

The dimension of the empty face is set to -1 by convention.

Definition 2.4. [36, 39] *A normed space $\mathcal{R}^n$ is called an* ***α-space*** *if for every bounded set $N \subset \mathcal{R}^n$ the d-convex hull of N is bounded.*

Let X be a Hausdorff space.

Definition 2.5. *A subset e of the space X is called an* ***m-dimensional open polyhedral cell****, or* ***m-dimensional polyhedral cell*** *in X if it is PL homeomorphic with an m-dimensional open convex polytope* [13, 25, 33] *in the space $\mathcal{R}^n$.*

Definition 2.6. *A subset e of the space X is called an* ***m-dimensional closed polyhedral cell*** *in X if it is PL homeomorphic with an m-dimensional convex polytope in the space $\mathcal{R}^n$.*

Definition 2.7. *A subset e of the space X is called an* ***m-dimensional open cell****, or* ***m-dimensional cell*** *in X if it is homeomorphic to the open unit ball of the space $\mathcal{R}^m$. Let $\overline{e}$ be the closure of e in X, and let $\dot{e} = \overline{e} \setminus e$.*

Definition 2.8. [12, 18, 22] *A set $\mathcal{E} = \{e_\lambda \mid \lambda \in \Lambda\}$ of cells in the Hausdorff space X is called a* ***cellular decomposition*** *of X if the following three conditions are satisfied:*

1) $e_\lambda \bigcap e_\mu$ *is empty if* $\lambda \neq \mu$*;*

2) $X = \bigcup_{\lambda \in \Lambda} e_\lambda$*;*

3) for every m*-dimensional cell* $e_\lambda \in \mathcal{E}$ *there is a continuous mapping*

$$\varphi\colon \left(B^m, S^{m-1}\right) \to \left(X^{m-1} \bigcup e_\lambda, X^{m-1}\right)$$

such that $\varphi\left(B^m \setminus S^{m-1}\right) \to e_\lambda$ *is a homeomorphism, where* X^{m-1} *is the union of all the cells* $e_\mu \in \mathcal{E}$*, whose dimensions are not greater than* $m-1$*.*

Definition 2.9. [12, 18, 22] *A Hausdorff space* X *together with its cellular decomposition* $\mathcal{E} = \{e_\lambda \mid \lambda \in \Lambda\}$ *is called a* ***cell complex****. A cell complex* X *is said to be* ***finite*** *if the set* $\mathcal{E}$ *is finite.*

Definition 2.10. *The* ***dimension*** $\dim X$ *of a cell complex* X *is* m *if* X *contains an* m*-dimensional cell but no* $(m+1)$*-dimensional cell, and* ∞ *if* X *contains* m*-dimensional cells for all* $m \geqslant 0$*.*

Definition 2.11. [12, 18, 22] *A cell complex* X *is said to be* ***closure finite*** *if the closure of each cell meets only finitely many other cells, and* X *is said to have the* ***weak topology*** *if a subset* $A \subset X$ *is closed iff* $A \bigcap \overline{e}$ *is closed in* $\overline{e}$ *for each cell* e *of* X*.*

Definition 2.12. *We call a cell complex a* ***CW complex*** *if it is closure finite and has the weak topology.*

It is easy to see that a finite cell complex is a CW complex. A geometric polyhedron is also a finite CW complex.

Theorem 2.1. [12, 18, 22] *If* X *is a* CW *complex, then the* m*th integral cellular homology group of* X *is isomorphic to the* m*th integral singular homology group of* X*.*

Definition 2.13. *If* X *is a finite* CW *complex and* β_m *denotes the rank of the* m*th integral singular homology group of* X*, then the number* $\chi(X) = \sum_{m=0}^{\dim X} (-1)^m \beta_m$ *is called the* ***Euler-Poincaré characteristic*** [12, 17, 18, 22] *of* X*.*

Theorem 2.2 (Euler-Poincaré). [12, 18, 22, 23, 30, 44] *For a finite CW complex X it holds that*

$$\chi(X) = \sum_{m=0}^{dim\,X} (-1)^m \beta_m = \sum_{m=0}^{dim\,X} (-1)^m \alpha_m,$$

where α_m denotes the number of m-dimensional cells of X.

It is evident from the definition of the integral singular homology group and the theorems above that the Euler-Poincaré characteristic $\chi(X)$ is an integer topological invariant for a finite CW complex X. Moreover, $\chi(X)$ depends only on the homotopy type of X. In particular, given any CW decomposition of X, we always will get the same integer $\chi(X)$.

In that follows, a subscript in the name of mathematical objects denotes their dimension.

3. Main theorem

Let $\mathcal{R}^n$ be an α-space, and let $P_q^n \subset \mathcal{R}^n$ be a geometric n-polyhedron of genus q all of whose facets belong to the d-convex hyperplanes in $\mathcal{R}^n$. Consider this polyhedron P_q^n having also a finite number of holes, mutually disjoint open geometric polyhedrons of genus 0 of dimension $n, n-1, \ldots, 0$, the facets of which belong to the d-convex linear manifolds in $\mathcal{R}^n$. Suppose also that $\mathcal{R}^n \setminus U$ is an n-cell in $\mathcal{R}^n$, where U is the unbounded connection component of the complement $\mathcal{R}^n \setminus int\, P_q^n$. When speaking of a face of P_q^n, we will mean either a face of the polyhedron P_q^n without holes or a face of a certain hole of P_q^n. Since P_q^n is compact, the set of faces of P_q^n is finite.

Definition 3.1. *A point $x \in bd\, P_q^n$ is called a* ***point of local non-d-convexity*** [6, 7, 26, 32, 40] *of P_q^n if, for any sufficiently small $\varepsilon > 0$, there exists at least one non-d-convex connection component of the intersection d-conv $B^n(x, \varepsilon) \bigcap int\, P_q^n$.*

Let R be the set of all points of local non-d-convexity of the polyhedron P_q^n. We will always assume that the set R is not empty.

Definition 3.2. *The geometric polyhedron P_q^n is called* ***partitioned into d-convex pieces*** *$Q_1, Q_2, \dots, Q_r$ if*

1) $int\, Q_i \neq \emptyset,\ i = 1, 2, \dots, r;$

2) $\bigcup_{i=1}^{r} int\, Q_i \subset int\, P_q^n \subset \bigcup_{i=1}^{r} Q_i;$

3) $int\, Q_i \bigcap int\, Q_j = \emptyset,\ i \neq j.$

It is obvious that P_q^n has at least one d-convex partition. We will denote by $p\left(P_q^n\right)$ the minimum number of d-convex pieces into which P_q^n can be partitioned. Since the number of d-convex pieces of a d-convex partition $Q_1, Q_2, \dots, Q_r$ of the geometric polyhedron P_q^n is equal to the number of d-convex pieces of the d-convex partition $\overline{Q_1}, \overline{Q_2}, \dots, \overline{Q_r}$, without losing generality, we will assume that the d-convex pieces into which P_q^n can be partitioned are closed.

By D^{n-1} and $\left|D^{n-1}\right|$ we will denote a finite set of polyhedral cells of dimension $\leq n-1$ in the α-space $\mathcal{R}^n$, belonging to the interior of P_q^n, and the set of all points of the cells, respectively.

Definition 3.3. *A set D^{n-1} is called a* ***dividing*** [1 – 4, 37] *of the polyhedron P_q^n if D^{n-1} satisfies the following condition: for every $x \in R \bigcup \overline{|D^{n-1}|}$ there exists an $\varepsilon > 0$ such that the intersection $\left(int\, P_q^n \setminus \overline{|D^{n-1}|}\right) \bigcap d\text{-}conv\, B^n(x, \varepsilon)$ consists only of d-convex connection components;*

The set $dvz\, P_q^n$ of all dividings of the polyhedron P_q^n is not empty. This assertion relies on the existence of a d-convex partition of P_q^n. Inverse, any d-convex partition of P_q^n can be obtained by a certain dividing of P_q^n.

Definition 3.4. *The number $\chi\left(D^{n-1}\right) = \sum_{i=0}^{n-1} (-1)^i \alpha_i$ will be called the* ***Euler-Poincaré characteristic of the dividing*** *D^{n-1}, where α_i is the number of polyhedral cells of dimension i of D^{n-1}.*

The Euler-Poincaré characteristic is an integer topological invariant for $\left|D^{n-1}\right|$. This fact results from the definition of the dividing. The notion of a dividing is not other than a generalization of the notion of a concordant system of dividing trees $[6, 26]$ for a polygonal domain.

Theorem 3.1. *The Euler-Poincaré characteristic of a polyhedron* $P_q^n \subset \mathcal{R}^n$, $n \geq 3$, *satisfies the property:*

$$\chi\left(bd\, P_q^n\right) - \chi\left(\overline{P_q^n}\right) = (-1)^{n-1}(1-q) + g, \tag{3.1}$$

where g *is the number of bounded connection components of the complement* $\mathcal{R}^n \setminus int\, P_q^n$.

Proof. First we consider that $q = 0$. We will prove the statement of the theorem by induction on g.

The case $g = 0$ is trivial: $\chi\left(\overline{P_0^n}\right) = \chi\left(bd\, P_0^n\right) + (-1)^n$.

For the case $g = 1$, let Q denote the bounded connection component of the complement $\mathcal{R}^n \setminus int\, P_0^n$. Let z be a vertex of Q and let H be a hyperplane through the point z such that $H \bigcap Q = z$. The existence of such a point z results from the facts that the set of holes is finite and the holes are geometric polyhedrons. Consider that the connection component of the intersection $H \bigcap P_0^n$ which contains the point z is a closed polyhedral $(n-1)$-cell in the space $\mathcal{R}^n$. This connection component divides $int\, P_0^n$ into two polyhedral n-cells in $\mathcal{R}^n$. The polyhedron P_0^n can be PL homeomorphically deformed in order to pass such a hyperplane H. Therefore

$$\chi\left(\overline{P_0^n}\right) = \left(\chi\left(bd\, P_0^n\right) - 1\right) + (-1)^{n-1} + (-1)^n \cdot 2.$$

As a result,

$$\chi\left(bd\, P_0^n\right) - \chi\left(\overline{P_0^n}\right) = (-1)^{n-1}(1-0) + 1.$$

Assume the equality (3.1) holds for all polyhedrons M_0^n with at most $l-1$, $l \geqslant 2$, bounded connection components of the complement $\mathcal{R}^n \setminus int\, M_0^n$, and let P_0^n be a polyhedron with l bounded connection components of the complement $\mathcal{R}^n \setminus int\, P_0^n$. Choose a hyperplane H

such that $H\bigcap int\, P_0^n \neq \emptyset$ and regard a connection component C of the intersection $H\bigcap P_0^n$. Suppose that the connection component C is a closed polyhedral $(n-1)$-cell in the space $\mathcal{R}^n$ and $C\bigcap int\, P_0^n$ is an open polyhedral $(n-1)$-cell. The connection component C determines two n-polyhedrons P_1 and P_2. Consider that each of these polyhedrons contains at least one bounded connection component of its complement. The polyhedron P_0^n can be PL homeomorphically deformed in order to pass such a hyperplane H. By inductive assumption,

$$\chi\left(bd\, P_1\right) - \chi\left(\overline{P_1}\right) = (-1)^{n-1}(1-0) + g_1$$

and

$$\chi\left(bd\, P_2\right) - \chi\left(\overline{P_2}\right) = (-1)^{n-1}(1-0) + g_2,$$

where $g_1,\ g_2$ are the bounded connection components of the complements $\mathcal{R}^n \setminus int\, P_1$ and $\mathcal{R}^n \setminus int\, P_2$, respectively. It is easily seen, that $g_1 + g_2 = g$. Since $\chi\left(S^{n-1}\right) = 1 + (-1)^{n-1}$ for the unit sphere S^{n-1} in $\mathcal{R}^n$, we have

$$\begin{aligned} \chi\left(bd\, P_0^n\right) &= \chi\left(bd\, P_1\right) + \chi\left(bd\, P_2\right) - (-1)^{n-1}\cdot 2 - \left(1 + (-1)^{n-2}\right), \\ \chi\left(\overline{P_0^n}\right) &= \chi\left(\overline{P_1}\right) + \chi\left(\overline{P_2}\right) - (-1)^{n-1} - \left(1 + (-1)^{n-2}\right). \end{aligned}$$

Hence

$$\chi\left(bd\, P_0^n\right) - \chi\left(\overline{P_0^n}\right) = (-1)^{n-1}(1-0) + g_1 + (-1)^{n-1}(1-0) + g_2 - (-1)^{n-1}.$$

Whence

$$\chi\left(bd\, P_0^n\right) - \chi\left(\overline{P_0^n}\right) = (-1)^{n-1}(1-0) + g.$$

Now assume that $q > 0$. Choose for the handles in P_q^n two secant balls each. Suppose that each of these secant balls intersects the polyhedron P_q^n and the interior of P_q^n by a closed polyhedral $(n-1)$-cell and by an open polyhedral $(n-1)$-cell in the space $\mathcal{R}^n$, respectively. The polyhedron P_q^n can be PL homeomorphically deformed in order to pass such balls. The balls separate P_q^n into $q+1$ polyhedrons P_i of dimension n of genus 0, $i = 1, 2, \ldots, q+1$. Reasoning as above, we obtain

$$\begin{aligned}
\chi\left(bd\,P_i\right)-\chi\left(\overline{P_i}\right) &= (-1)^{n-1}(1-0)+g_i,\ i=1,2,\ldots,q+1,\\
\sum_{i=1}^{q+1} g_i &= g,\\
\chi\left(bd\,P_q^n\right) &= \sum_{i=1}^{q+1}\chi\left(bd\,P_i\right)-(-1)^{n-1}\cdot 4q-\left(1+(-1)^{n-2}\right)\cdot 2q,\\
\chi\left(\overline{P_q^n}\right) &= \sum_{i=1}^{q+1}\chi\left(\overline{P_i}\right)-(-1)^{n-1}\cdot 2q-\left(1+(-1)^{n-2}\right)\cdot 2q,
\end{aligned}$$

where g_i is the bounded connection components of the complement $\mathcal{R}^n\setminus int\,P_i,\ i=1,2,\ldots,q+1$. It follows immediately that

$$\chi\left(bd\,P_q^n\right)-\chi\left(\overline{P_q^n}\right)=\sum_{i=1}^{q+1}\left((-1)^{n-1}(1-0)+g_i\right)-(-1)^{n-1}\cdot 2q.$$

Therefore we get

$$\chi\left(bd\,P_q^n\right)-\chi\left(\overline{P_q^n}\right)=(-1)^{n-1}(1-q)+g.$$

This completes the proof. □

Theorem 3.2 (Main Theorem). *For a polyhedron* $P_q^n\subset\mathcal{R}^n$, $n\geq 3$, *the equality*

$$p\left(P_q^n\right)=(-1)^{n-1}\left(\chi\left(bd\,P_q^n\right)-\chi\left(\overline{P_q^n}\right)\right)+\min_{D^{n-1}\in dvz\,P_q^n}\left|\chi\left(D^{n-1}\right)\right|.$$

holds.

Proof. Let D^{n-1} be a dividing of the polyhedron P_q^n. This dividing determines a finite CW n-complex K^n representing the polyhedron P_q^n and whose n-cells are open d-convex polytopes. Indeed, from the definition of the dividing, the set of all points belonging to the closures of polyhedral cells of the dividing D^{n-1} is a finite CW $(n-1)$-complex

M^{n-1}. Moreover, the set of the cells of M^{n-1}, each of which belongs to the boundary of P_q^n, determines a decomposition of the boundary into polyhedral cells in the space $\mathcal{R}^n$. Denote by L^{n-1} the CW $(n-1)$-complex formed by this decomposition. The finite CW complex $M^{n-1} \bigcup L^{n-1}$ together with the connection components C_i of the set $int\, P_q^n \setminus \left(M^{n-1} \bigcup L^{n-1}\right)$ forms the required cell decomposition. The connection components C_i are open, local d-convex, so they are also d-convex [41]. The closures of these connection components partition P_q^n into d-convex pieces. We have $\chi\left(\overline{P_q^n}\right) = \chi\left(K^n\right)$ and $\chi\left(bd\, P_q^n\right) = \chi\left(L^{n-1}\right)$. From the theorem 2.2, it is clear that

$$\chi\left(K^n\right) = \sum_{i=0}^{n} (-1)^i \alpha_i, \tag{3.2}$$

where α_n is the number of d-convex pieces into which P_q^n is partitioned, and α_i represents the number of polyhedral i-cells of K^n, $i = 0, 1, \ldots, n-1$. Rewrite (3.2) as follows

$$(-1)^n \alpha_n = \chi\left(K^n\right) - \sum_{i=0}^{n-1} (-1)^i \alpha_i.$$

Whence

$$(-1)^n \alpha_n = \chi\left(K^n\right) - \sum_{i=0}^{n-1} (-1)^i \alpha_i' - \sum_{i=0}^{n-1} (-1)^i \alpha_i'',$$

where α_i' is the number of polyhedral i-cells belonging to the boundary of P_q^n, and α_i'' is the number of polyhedral i-cells belonging to the dividing D^{n-1}. Therefore we get

$$(-1)^n \alpha_n = \chi\left(K^n\right) - \chi\left(L^{n-1}\right) - \chi\left(D^{n-1}\right). \tag{3.3}$$

Thus, both sides of the equality (3.3) being multiplied by $(-1)^n$, we obtain

$$\alpha_n = (-1)^n \chi\left(K^n\right) + (-1)^{n-1} \chi\left(L^{n-1}\right) + (-1)^{n-1} \chi\left(D^{n-1}\right).$$

Whence

$$\alpha_n = (-1)^{n-1}\left(\chi\left(bd\,P_q^n\right) - \chi\left(\overline{P_q^n}\right)\right) + (-1)^{n-1}\chi\left(D^{n-1}\right). \qquad (3.4)$$

The Euler-Poincaré characteristic of the dividing D^{n-1} is nonnegative for odd n and is nonpositive for even n in view of the facts that the relations (3.1), (3.4) and the inequality $\alpha_n > g$ hold. Therefore we get $\left|\chi\left(D^{n-1}\right)\right| = (-1)^{n-1}\chi\left(D^{n-1}\right)$. If the dividing D^{n-1} is chosen such that the value of $\left|\chi\left(D^{n-1}\right)\right|$ to be minimum, then α_n is minimum, too. Hence we obtain

$$p\left(P_q^n\right) = (-1)^{n-1}\left(\chi\left(bd\,P_q^n\right) - \chi\left(\overline{P_q^n}\right)\right) + \min_{D^{n-1}\in dvz\,P_q^n}\left|\chi\left(D^{n-1}\right)\right|,$$

and the theorem is proved. □

Corollary 3.1. *Let $P_q^n \subset \mathcal{R}^n$, $n \geq 3$, be a geometric n-polyhedron. Then*

$$p\left(P_q^n\right) = 1 - q + (-1)^{n-1}\cdot g + \min_{D^{n-1}\in dvz\,P_q^n}\left|\chi\left(D^{n-1}\right)\right|,$$

where g is the number of bounded connection components of the complement $\mathcal{R}^n \setminus int\,P_q^n$.

Corollary 3.2. *Let $P_q^2 \subset \mathcal{R}^2$ be a geometric 2-polyhedron (a polygonal domain). Then*

$$p\left(P_q^2\right) = 1 - g + \min_{D^1\in dvz\,P_q^2}\left|\chi\left(D^1\right)\right|,$$

where g is the number of bounded connection components of the complement $\mathcal{R}^2 \setminus int\,P_q^2$.

To prove the corollary, it suffices to analyse the proof of the theorem 3.1.

If the norm of $\mathcal{R}^n$ is defined by $\|x\| = \sum_{i=1}^{n}|x_i|$, then for the polyhedrons P^2 and P^3 in Figure 3.1 and Figure 3.2, respectively, we obtain $p\left(P^2\right) = 1-5+|1-18| = 13$ and $p\left(P^3\right) = 1-0+2+|0-22+34| = 15$.

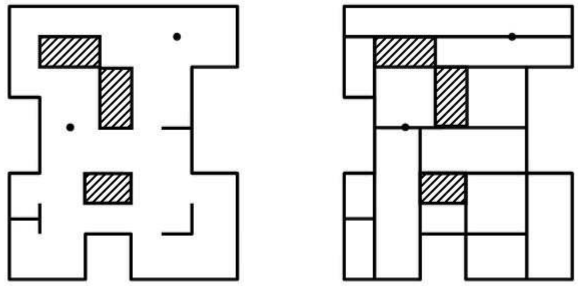

Figure 3.1.

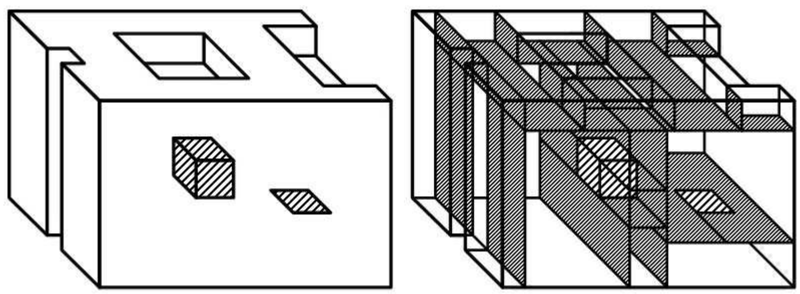

Figure 3.2.

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About Precise Characterization of Languages Generated by Hybrid Networks of Evolutionary Processors with One Node *

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Abstract

A hybrid network of evolutionary processors (an HNEP) is a graph where each node is associated with an evolutionary processor (a special rewriting system), a set of words, an input filter and an output filter. Every evolutionary processor is given with a finite set of one type of point mutations (an insertion, a deletion or a substitution of a symbol) which can be applied to certain positions of a string over the domain of the set of these rewriting rules. The HNEP functions by rewriting the words that can be found at the nodes and then re-distributing the resulting strings according to a communication protocol based on a filtering mechanism. The filters are defined by certain variants of random-context conditions. In this paper we complete investigation of HNEPs with one node and present a precise description of languages generated by them.

1 Introduction

Insertion, deletion, and substitution are fundamental operations in formal language theory, their power and limits have obtained much attention during the years. Due to their simplicity, language generating mechanisms based on these operations are of particular interest. *Networks of evolutionary processors* (NEPs, for short), introduced in [8],

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are proper examples for distributed variants of these constructs. In this case, an evolutionary processor (a rewriting system which is capable to perform an insertion, a deletion, and a substitution of a symbol) is located at every node of a virtual graph which may operate over sets or multisets of words. The system functions by rewriting the collections of words present at the nodes and then re-distributing the resulting strings according to a communication protocol defined by a filtering mechanism. The language determined by the network is defined as the set of words which appear at some distinguished node in the course of the computation. These architectures also belong to models inspired by cell biology, since each processor represents a cell performing point mutations of DNA and controlling its passage inside and outside the cell through a filtering mechanism. The evolutionary processor corresponds to the cell, the generated word to a DNA strand, and the operations insertion, deletion, and substitution of a symbol to the point mutations. It is known that, by using an appropriate filtering mechanism, NEPs with a very small number of nodes are computationally complete computational devices, i.e. they are as powerful as the Turing machines (see, for example [5, 6]).

Particularly interesting variants of these devices are the so-called *hybrid networks of evolutionary processors* (HNEPs), where each language processor performs only one of the above operations on a certain position of the words in that node. Furthermore, the filters are defined by some variants of random-context conditions, i.e., they check the presence/absence of certain symbols in the words. These constructs can be considered both language generating and accepting devices, i.e., generating HNEPs (GHNEPs) and accepting HNEPS (AHNEPs). The notion of an HNEP, as a language generating device, was introduced in [16] and the concept of an AHNEP was defined in [15].

In [9] it was shown that, for an alphabet V, GHNEPs with $27 + 3 \cdot card(V)$ nodes are computationally complete. A significant improvement of the result can be found in [1], where it was proved that GHNEPs with 10 nodes (irrespectively of the size of the alphabet) obtain the universal power. For accepting HNEPs, in [13] it was shown that for any recursively enumerable language there exists a recognizing

AHNEP with 31 nodes; the result was improved significantly in [14] where the number of necessary nodes was reduced to 24. Furthermore, in [14] the authors demonstrated a method to construct for any NP-language L an AHNEP with 24 nodes which decides L in polynomial time.

At last in [2] it was proved that any recursively enumerable language can be generated by a GHNEP having 7 nodes (thus, the result from [1] is improved) and in [3] the same authors showed that any recursively enumerable language can be accepted by an AHNEP with 7 nodes (thus, the result from [14] is improved significantly). In [3] also it was showed that the families of GHNEPs and AHNEPs with 2 nodes are not computationally complete.

In [9] it was demonstrated that a GHNEP with one node can generate only regular language, but now in this paper we present a precise form of the generated language and consider one case omitted in the previous proof. Tasks of characterization of languages generated by a GHNEP with two nodes and languages accepting by an AHNEP with two nodes are still opened.

2 Prerequisites

We first recall some basic notions from formal language theory that we shall use in the paper. An alphabet is a finite and non-empty set of symbols. The cardinality of a finite set A is denoted by $card(A)$. A sequence of symbols from an alphabet V is called a word (or a string) over V. The set of all words over V is denoted by V^*; the empty word is denoted by ε; and we define $V^+ = V^* \setminus \{\varepsilon\}$. The length of a word x is denoted by $|x|$, and we designate the number of occurrences of a letter a in a word x by $|x|_a$. For each non-empty word x, $alph(x)$ denotes the smallest alphabet Σ such that $x \in \Sigma^*$.

The shuffle operation defined on two words $x, y \in V^*$ by

$$\amalg\!\amalg(x,y) = \{x_1y_1x_2y_2 \dots x_ny_n \mid n \geq 1,\ x_i, y_y \in V^*,\\ x = x_1x_2 \dots x_n,\ y = y_1y_2 \dots y_n\}.$$

Let $L_1, L_2 \in V^*$ are two languages. Then

$$\coprod\!\!\coprod(L_1, L_2) = \bigcup_{x \in L_1, y \in L_2} \coprod\!\!\coprod(x, y).$$

A *type-0 generative grammar* is a quadruple $G = (N, T, S, P)$, where N and T are disjoint alphabets, called the nonterminal and terminal alphabet, respectively, $S \in N$ is the start symbol or the axiom, and P is a finite set of productions or rewriting rules of the form $u \to v$, where $u \in (N \cup T)^* N (N \cup T)^*$ and $v \in (N \cup T)^*$. For two strings x and y in $(N \cup T)^*$, we say that x directly derives y in G, denoted by $x \Longrightarrow_G v$, if there is a production $u \to v$ in P such that $x = x_1 u x_2$ and $y = x_1 v x_2$, $x_1, x_2 \in (N \cup T)^*$ holds. The transitive and reflexive closure of $\Longrightarrow_G$ is denoted by $\Longrightarrow_G^*$. The language $L(G)$ generated by G is defined by $L(G) = \{w \in T^* \mid S \Longrightarrow_G^* w\}$.

We recall now a concept dual to a type-0 generative grammar, called a *type-0 analytic grammar* [17]. A type-0 analytic grammar $G = (N, T, S, P)$ is a quadruple where N, T, S are defined in the same way as for a generative grammar, and P is a finite set of productions of the form $u \to v$, where $u \in (N \cup T)^*$ and $v \in (N \cup T)^* N (N \cup T)^*$. The derivation relation is defined for a type-0 analytic grammar analogously to the derivation relation for a type-0 generative grammar. The language $L(G)$ recognized or accepted by a type-0 analytic grammar $G = (N, T, S, P)$ is defined as $L(G) = \{w \in T^* \mid w \Longrightarrow_G^* S\}$.

It is well-known that for the type-0 analytic grammar G' obtained from a type-0 generative grammar G with interchanging the left and the right hand sides of the productions in G, it holds that $L(G') = L(G)$.

In the sequel, following the terminology in [9], we recall the necessary notions concerning evolutionary processors and their hybrid networks. These language processors use so-called evolutionary operations, simple rewriting operations which abstract local gene mutations.

For an alphabet V, we say that a rule $a \to b$, with $a, b \in V \cup \{\varepsilon\}$ is a *substitution rule* if both a and b are different from ε; it is a *deletion rule* if $a \neq \varepsilon$ and $b = \varepsilon$; and, it is an *insertion rule* if $a = \varepsilon$ and $b \neq \varepsilon$. The set of all substitution rules, deletion rules, and insertion rules over an alphabet V is denoted by Sub_V, Del_V, and Ins_V, respectively. Given such rules π, ρ, σ, and a word $w \in V^*$, we define the following *actions* of σ on w: If $\pi \equiv a \to b \in Sub_V$, $\rho \equiv a \to \varepsilon \in Del_V$, and

$\sigma \equiv \varepsilon \to a \in Ins_V$, then

$$\begin{aligned}
\pi^*(w) &= \begin{cases} \{ubv: & \exists u,v \in V^*(w=uav)\}, \\ \{w\}, & \text{otherwise} \end{cases} & (1)\\
\rho^*(w) &= \begin{cases} \{uv: & \exists u,v \in V^*(w=uav)\}, \\ \{w\}, & \text{otherwise} \end{cases} & (2)\\
\rho^r(w) &= \begin{cases} \{u: & w=ua\}, \\ \{w\}, & \text{otherwise} \end{cases} & (3)\\
\rho^l(w) &= \begin{cases} \{v: & w=av\}, \\ \{w\}, & \text{otherwise} \end{cases} & (4)\\
\sigma^*(w) &= \{uav: \exists u,v, \in V^*(w=uv)\}, & (5)\\
\sigma^r(w) &= \{wa\},\ \sigma^l(w)=\{aw\}. & (6)
\end{aligned}$$

Symbol $\alpha \in \{*, l, r\}$ denotes the way of applying an insertion or a deletion rule to a word, namely, at any position ($a = *$), in the left-hand end ($a = l$), or in the right-hand end ($a = r$) of the word, respectively. Note that a substitution rule can be applied at any position. For every rule σ, action $\alpha \in \{*, l, r\}$, and $L \subseteq V^*$, we define the $\alpha - action$ of σ on L by $\sigma^\alpha(L) = \bigcup_{w\in L} \sigma^\alpha(w)$. For a given finite set of rules M, we define the $\alpha - action$ of M on a word w and on a language L by $M^\alpha(w) = \bigcup_{\sigma\in M} \sigma^\alpha(w)$ and $M^\alpha(L) = \bigcup_{w\in L} M^\alpha(w)$, respectively.

An evolutionary processor consists of a set of evolutionary operations and a filtering mechanism.

For two disjoint subsets P and F of an alphabet V and a word over V, predicates $\varphi^{(1)}$ and $\varphi^{(2)}$ are defined as follows:

$$\varphi^{(1)}(w; P, F) \equiv P \subseteq alph(w) \wedge F \cap alph(w) = \emptyset$$

and

$$\varphi^{(2)}(w; P, F) \equiv alph(w) \cap P \neq \emptyset \wedge F \cap alph(w) = \emptyset.$$

The construction of these predicates is based on *random-context conditions* defined by the two sets P (*permitting contexts*) and F (*forbidding contexts*).

For every language $L \subseteq V^*$ we define $\varphi^i(L, P, F) = \{w \in L \mid \varphi^i(w; P, F)\}$, $i = 1, 2$.

An *evolutionary processor over* V is a 5-tuple (M, PI, FI, PO, FO) where:

- Either $M \subseteq Sub_V$ or $M \subseteq Del_V$ or $M \subseteq Ins_V$. The set M represents the set of evolutionary rules of the processor. Notice that every processor is dedicated to only one type of the evolutionary operations.

- $PI, FI \subseteq V$ are the *input* permitting/forbidding contexts of the processor, while $PO, FO \subseteq V$ are the *output* permitting/forbidding contexts of the processor.

The set of evolutionary processors over V is denoted by EP_V.

Definition 1 *A* hybrid network of evolutionary processors *(an HNEP, shortly) is a 7-tuple* $\Gamma = (V, H, \mathcal{N}, C_0, \alpha, \beta, i_0)$*, where the following conditions hold:*

- V *is an alphabet, the alphabet of the network.*

- $H = (X_H, E_H)$ *is an undirected graph with set of vertices or nodes* X_H *and set of edges* E_H*.* H *is called the underlying graph of the network.*

- $\mathcal{N} : X_H \longrightarrow EP_V$ *is a mapping which associates with each node* $x \in X_H$ *the evolutionary processor* $\mathcal{N}(x) = (M_x, PI_x, FI_x, PO_x, FO_x)$*.*

- $C_0 : X_H \longrightarrow 2^{V^*}$ *is a mapping which identifies the initial configuration of the network. It associates a finite set of words with each node of the graph* H*.*

- $\alpha : X_H \longrightarrow \{*, l, r\}$*;* $\alpha(x)$ *defines the action mode of the rules performed in node* x *on the words occurring in that node.*

- $\beta : X_H \longrightarrow \{(1), (2)\}$ *defines the type of the input/output filters of a node. More precisely, for every node,* $x \in X_H$*, we define the following filters: the input filter is given as* $\mu_x(\cdot) = \varphi^{\beta(x)}(\cdot; PI_x, FI_x)$*, and the output filter is defined as* $\tau_x(\cdot) = \varphi^{\beta(x)}(\cdot, PO_x, FO_x)$*. That is,* $\mu_x(w)$ *(resp.*τ_x*) indicates whether or not the word* w *can pass the input (resp. output) filter of* x*. More generally,* $\mu_x(L)$ *(resp.* $\tau_x(L)$*) is the set of words of* L *that can pass the input (resp. output) filter of* x*.*

- $i_0 \in X_H$ *is the output node of* Γ*.*

We say that $card(X_H)$ is the size of Γ. An HNEP is said to be a complete HNEP, if its underlying graph is a complete graph.

A configuration of an HNEP Γ, as above, is a mapping $C : X_H \longrightarrow 2^{V^*}$ which associates a set of words with each node of the graph. A component $C(x)$ of a configuration C is the set of words that can be found in the node x in this configuration, hence a configuration can be considered as the sets of words which are present in the nodes of the network at a given moment.

A configuration can change either by an evolutionary step or by a communication step. When it changes by an evolutionary step, then each component $C(x)$ of the configuration C is altered in accordance with the set of evolutionary rules M_x associated with the node x and the way of applying these rules, $\alpha(x)$. Formally, the configuration C' is obtained in one evolutionary step from the configuration C, written as $C \Longrightarrow C'$, iff $C'(x) = M_x^{\alpha(x)}(C(x)) \quad \text{for all } x \in X_H$.

When the configuration changes by a communication step, then each language processor $\mathcal{N}(x)$, where $x \in X_H$, sends a copy of its each word to every node processor where the node is connected with x provided that this word is able to pass the output filter of x, and receives all the words which are sent by processors of nodes connected with x provided that these words are able to pass the input filter of x. Those words which are not able to pass the respective output filter, remain at the node. Formally, we say that configuration C' is obtained in one communication step from configuration C, written as $C \vdash C'$, iff $C'(x) = (C(x) - \tau_x(C(x))) \bigcup_{\{x,y\} \in E_G} (\tau_y(C(y)) \cap \mu_x(C(y)))$ holds for all $x \in X_H$.

For an HNEP Γ, the computation in Γ is a sequence of configurations C_0, C_1,$C_2, \ldots,$ where C_0 is the initial configuration of Γ, $C_{2i} \Longrightarrow C_{2i+1}$ and $C_{2i+1} \vdash C_{2i+2}$, for all $i \geq 0$.

HNEPs can be considered both language generating devices (generating hybrid networks of evolutionary processors or GHNEPs) and language accepting devices (accepting hybrid networks of evolutionary processors or AHNEPs).

In the case of GHNEPs we define the generated language as the set of all words which appear in the output node at some step of the computation. Formally, the language generated by a generating hybrid network of evolutionary processors Γ is $L(\Gamma) = \bigcup_{s\geq 0} C_s(i_0)$.

In the case of AHNEPs, in addition to the components above, we distinguish an input alphabet and a network alphabet, V and U, where $V \subseteq U$, and instead of an initial configuration, we indicate an input node i_I. Thus, for an AHNEP, we use the notation $\Gamma = (V, U, H, \mathcal{N}, i_I, \alpha, \beta, i_0)$.

The computation by an AHNEP Γ for an input word $w \in V^*$ is a sequence of configurations $C_0^{(w)}$, $C_1^{(w)}$,$C_2^{(w)}, \ldots,$ where $C_0^{(w)}$ is the initial configuration of Γ, with $C_0^{(w)}(i_I) = \{w\}$ and $C_0^{(w)}(x) = \emptyset$, for $x \in G$, $x \neq i_I$, and $C_{2i}^{(w)} \Longrightarrow C_{2i+1}^{(w)}$, $C_{2i+1}^{(w)} \vdash C_{2i+2}^{(w)}$, for all $i > 0$.

A computation as above is said to be accepting if there exists a configuration in which the set of words that can be found in the output node i_o is non-empty. The language accepted by Γ is defined by

$$L(\Gamma) = \{w \in V^* \mid \text{ the computation by } \Gamma \text{ on } w \text{ is an accepting one}\}.$$

3 Result

The following theorem states the regularity result for GHNEPs with one node. Although this has already been stated in [9], their proof is certainly incomplete. They stated that while GHNEPs without insertion only generate finite languages, GHNEPs with one insertion node only generate languages I^*C_0, C_0I^*, $C_0 \coprod I^*$, for the mode $l, r, *$, respectively. In the theorem below we present a precise characterization of languages generated by GHNEP with one node and consider the case omitted in [9] then the underlying graph G has a loop.

Theorem 1 *GHNEPs with one node only generate regular languages.*

Proof. As finite languages are regular, the statement holds for GHNEPs without insertion nodes. We now proceed with the case of one insertion node. Consider such a GHNEP $\Gamma = (V, G, N_1, C_0, \alpha, \beta, 1)$, where

$$N_1 = (M, PI, FI, PO, FO).$$

Let us introduce a few notations. Inserting a symbol from I in a language C yields a language $\mathsf{ins}_I(C)$. Depending on whether $\alpha = l$, $\alpha = r$ or $\alpha = *$, $\mathsf{ins}_I(C)$ is one of IC, CI, $C \coprod I$, respectively. For inserting an arbitrary number of symbols from a set I in a language C, $\mathsf{ins}^*_I(C)$ is one of I^*C, CI^*, $C \coprod I^*$. Clearly, ins^*_I preserves regularity.

We denote the set of symbols inserted in N_1 by $I = \{a \mid \lambda \to a \in M\}$. The configuration of N_1 after one step is $C_1 = \mathsf{ins}_I(C_0)$. Assume that $\beta = 2$ (a case then $\beta = 1$ can be considered analogously), then the conditions of passing permitting and forbidding output filter can be specified by regular languages $\pi = V^*POV^*$ and $\varphi = (V - FO)^*$, respectively. For instance, the set of words of C_1 that pass the forbidding output filter but do not pass the forbidding input filter is $C_1' = C_1 \cap \varphi \setminus \pi$. Notice that inserting symbols that belong to neither PO nor FO does not change the behavior of the filters; we denote the corresponding language by $B = \mathsf{ins}^*_{I\setminus(PO\cup FO)}(C_1)$.

Consider the case when the graph G consists of one node and no edges. Then, Γ generates the following language

$$\begin{aligned} L_1 = L_1(\Gamma) &= C_0 \cup C_1 \cup \mathsf{ins}^*_I(C_1 \setminus \varphi) \cup B \\ &\quad \cup \mathsf{ins}_{I\cap PO\setminus FO}(B) \cup \mathsf{ins}^*_I(\mathsf{ins}_{I\cap FO}(B)), \qquad (7) \\ B &= \mathsf{ins}^*_{I\setminus(PO\cup FO)}(C_1), \\ C_1 &= \mathsf{ins}_I(C_0). \end{aligned}$$

Indeed, this is a union of six languages:

1. initial configuration,
2. configuration after one insertion,
3. all words that can be obtained from a word from C_1 if it is trapped in N_1 by the forbidding filter,
4. B represents the words that pass the forbidding filter but not the permitting filter,
5. words obtained by inserting one permitting and not forbidden symbol into B, and

6. words obtained by inserting one forbidden symbol into B, and then by arbitrary insertions.

Consider the case when the graph G has a loop. The set of words leaving the node (for the first time) is $D = (C_1 \cap \varphi \cap \pi) \cap \mathsf{ins}_{I \cap PO \setminus FO}(B)$. The conditions of the permitting and forbidding input filters can be specified by regular languages $\pi' = V^* PIV^*$ and $\varphi' = (V - FI)^*$, respectively. Some of words from D return to N_1, namely $D \cap \pi' \cap \varphi'$. Notice that further insertion of symbols that belong neither to FO nor to FI causes the words to continuously exit and reenter N_1. The associated language is $B' = \mathsf{ins}^*_{I \setminus (FO \cup FI)}(D \cap \pi' \cap \varphi')$. Finally, we give the complete presentation of the language generated by Γ in this case:

$$\begin{aligned} L_1' = L_1(\Gamma) &= L_1 \cup B' \cup \mathsf{ins}^*_I(\mathsf{ins}_{I \cap FO}(B')) \cup \mathsf{ins}_{I \cap FI \setminus FO}(B'), \quad (8) \\ B' &= \mathsf{ins}^*_{I \setminus (FO \cup FI)}(D \cap \pi' \cap \varphi'), \\ D &= (C_1 \cap \varphi \cap \pi) \cap \mathsf{ins}_{I \cap PO \setminus FO}(B), \\ C_1 &= \mathsf{ins}_I(C_0). \end{aligned}$$

Indeed, this is a union of four languages:

1. words that never reenter N_1, as in the case when G has no edges,
2. B' represents the words that once leave and reenter N_1, and keep doing so after subsequent insertions,
3. words obtained by inserting a symbol from FO into B', and then by arbitrary insertions,
4. words obtained by inserting a symbol from $FI \setminus FO$ into B'.

□

4 Conclusion

In this paper we presented a precise form of the languages generated by an HNEP with one node and considered one case omitted in the previous proof. Thus we completed investigation of this class of HNEPs.

Tasks of characterization of languages generated by a GHNEP with two nodes and languages accepting by an AHNEP with two nodes are still opened.

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The Development of e-Government in the Republic of Moldova

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Abstract

Moldova has received very different rankings in different comparative studies on e-government development. This paper attempts to analyze the real state of e-government in the Republic of Moldova. We discuss the current situation, and evaluate that current situation using several accepted models. The most interesting aspect of this research is that e-government in Moldova is developing in the very complicated conditions of transition from a Soviet state with a centralized command system to a democratic state with a market economy, in parallel with a total reengineering of government institutions. In order to direct the strategy of the government of Moldova towards further development, we make a number of suggestions tailored to this complex environment.

Keywords: e-government, information and communication technology, Moldova

1 Introduction

The processes of integration in Europe and "globalization" throughout the world have led to countries becoming more and more interdependent. In this context, a significant challenge for researchers and practitioners is found in the processes occurring in the Newly Independent States, which appeared after the USSR's collapse, and where the period of transition from Soviet regimes with their totally centralized command economies to democratic societies with market economies, has proved to be much more complicated and multidimensional than

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expected. To what extent are these countries able to integrate into the world community when this requires new standards and principles in almost all fields of activity?

Rising public demand for government information is a common characteristic of modern societies, and it is an important dimension of democracy. This is especially important in the states of the former Soviet Union (FSU), including the Republic of Moldova, that suffered long decades under the totalitarian Soviet regime. E-government holds out a promise of help with this problem; however, experience in the field of e-government development, accumulated in the industrialized countries, cannot be directly transferred to these countries because they are also undergoing, in parallel, a total reengineering of public institutions, and because their governments are quite different from those in the industrialized west. The government of Moldova is democratically elected, but in many ways it still has highly centralized government functions and little autonomy for local governments. It is therefore necessary to carefully consider how to create and nurture e-government in this kind of environment.

This paper seeks to examine the current state of e-government development in the Republic of Moldova, and to compare it to other countries in order to direct the strategy of the government of Moldova in further development. Moldova's very different rankings in different comparative studies on e-government development served as a stimulus for this research. For example, each year since 2002, a report on Global e-government has been published, first by the Taubman Center for Public Policy (West, 2002-2007) and then by the Brookings Institute (West 2008). In addition to assessments of the general state of e-government services throughout the world, the report includes a comparative ranking of the e-governments of the countries of the world. In 2002, the Republic of Moldova was ranked 105th, and Russia was ranked 132nd. In 2005, Moldova had climbed to 57th, while Russia had only climbed to 110th. Moldova was the highest ranked country in the CIS, and indeed it outranked some of the former Warsaw states. In particular, Moldova was ranked slightly above Romania, the western state to which it has the closest cultural ties.

Table 1. Ranking of Moldova's e-government in the reports of the Taubman Center and Brookings Institute

Country	**Rank per year**			
	2005	2006	2007	2008
Moldova	57	132	181	126
Ukraine	93	21	115	89
Russia	110	43	133	95
...	...	...	...	...
Romania	84	61	98	146

However, in the context of the rankings in the following three years, this evaluation of Moldova's e-government looks too optimistic. Some selected data from these reports, related to Moldova, other countries of the FSU and Romania, are listed in the Table 1. In 2008 Moldova was ranked only 126th, changing its position from the highest ranked country in the CIS to a position behind all the CIS countries except Turkmenistan (183rd) and Tajikistan (tied at 126th).

Table 2. UN e-government readiness index

Country	**Rank per year**	
	2005	2008
Moldova	109	93
Ukraine	48	41
Russia	50	60
...		...
Romania	44	51

At the same time, Moldova's ranking in other studies is very different, both absolutely and comparatively (West, 2002-7; International Telecommunication Union, 2005; Sciadas 2005). For example, in Table 2 (data selected from two United Nations studies (United Nations 2003, 2008)), we see that Moldova's e-government readiness was ranked as

109th in 2005 - 10th in the CIS - and moved up to 93rd in 2008 - 5th in the CIS. These studies suggest change in the opposite direction from that suggested by the Brown/Brookings studies.

Rankings like these are widely reported in the media in Moldova, and are used by the government as assessments of the current situation. The wide variation in the rankings of the e-government development in the Republic of Moldova strongly suggests that additional research is needed.

2 Theoretical framework

Recently, interest in the *information society*, and more specifically in *e-government*, has increased dramatically, as has the number of publications in this field. A significant part of them are dedicated to e-government evaluation studies (Janssen, Rotthier & Snijkers, 2004; Kunstelj & Vintar, 2004). But the results are varied, and the same country in different studies may receive significantly different evaluations of its e-government. As noticed by Janssen et. al. (2004), "the boom in the amount of comparative studies produced in the years 2000-2001 has more than often resulted in a country scoring the high marks in one study, and ending at the bottom in another."

This recent research studying approaches to e-government evaluations has noted that different evaluation studies have covered different aspects of e-government, using different indicators. A crucial step for an objective and useful evaluation of the level of development of e-government is to choose the correct definition of the concept "e-government," the relevant criteria and indicators for the measurement of e-government, and an appropriate approach to monitoring, evaluating and benchmarking e-government development.

Without reiterating a detailed description (Janssen et. al. 2004) of the wide spectrum of definitions of e-government between the narrow (evaluating e-government solely in terms of online service delivery) and the broad (evaluating the broader use of information and communication technology (ICT) in the public sector), in this article we will use a broad definition, which we believe most apt for our search for appropriate future strategies.

2.1 Broad Goals

Most researchers in the field promote the idea that e-government should be customer focused, and we support the approach that government and organizational processes should be restructured while being automated: "Re-invent government, don't simply automate it" (Poon & Huang, 2002).

E-government is much more than offering electronic services online and it must be based on applying ICT to the full range of government functions, including relations with citizens, internal governmental functions and interactions between governmental bodies. Recently some countries have begun to move away from systems organized around government function, and towards more integrated organizations based on the customers' view of events (Kunstelj & Vintar, 2004).

An interesting example of successful implementation of citizen-focussed e-government in Hong Kong is described by Poon and Huang (2002). The ESDLife (from ESD, electronic service delivery) site asks users what they want to do, rather than asking them what governmental department they need to visit. The e-government coordinator attributes some of the success of the program to this novel organization that does not require citizens to understand the details of government structure.

Poon and Huang (2002) also observe that e-government has three large groups of functions: *e-service* (the delivery of services to the citizens), *e-democracy* (electronic communications between government and citizens, including voting), and *e-business* (activities involving government and business). Such a comprehensive definition of e-government requires the use of multiple evaluative criteria and approaches.

2.2 Specific Criteria

One evaluation of e-government in this article will be based on a widely used model (Kunstelj et. al, 2004; Teicher & Dow, 2002; National Office of Information Economy & DMR Consulting, 2003) describing the following four stages of e-government development:

1. *Web Presence* – providing basic information on the Internet.

2. *Interaction* – providing more information, online forms to fill out and print, facilitating electronic communication with government offices, providing site search and links to other sites.

3. *Transaction* – offering full online transactions using electronic payment if required and including online delivery of receipts and documents.

4. *Transformation* – the long-term objective of e-government, with integrated services in a "one-stop shop."

Note, however, that these criteria evaluate e-government mostly in terms of the services provided electronically by the government for the citizens. They fail to evaluate whether the citizens can access these services, including both questions of citizen knowledge and of technological availability. They also fail to evaluate whether the citizens are ready and willing to access e-government. Therefore, we will also use the "holistic" approach suggested by Kunstelj and Vintar (2004). They classified indicators into four main groups: *environmental maturity*, *back-office*, *front-office* and *impact*. Their approach suggests that assessment should be focused, when possible, on "how much value the service brings to the citizen."

In order to apply these models, we need an understanding of the current state of information technology and of e-government in Moldova.

3 General evaluation of current Moldovan e-government

The hierarchical structure of government in Moldova includes the central national government, municipality (cities with separate administration sectors) and county (raional) government and local (towns, villages) government.

The central public administration consists of the Presidency, Parliament, Government, and other institutions attached to the Parliament or Government (National Bank, Accounts Chamber, national agencies, etc.), in total 36 institutions.

There are 5 municipalities, 60 cities and 917 villages (communes) in Moldova. In each of these 982 localities, the public administration is represented by a mayoral office. To those municipalities and villages belong another 697 localities, for a total of 1679 (Statistical Yearbook, 2008). Of the 60 cities, 40 are county centers. The public administration of each county is represented by a County Council.

We performed a census and evaluation of government websites in Moldova (without the Trans-Dniester region). Our census, ending 12 November 2008, found 52 functioning government websites in Moldova, as follows:

- central public administration – 33 institutions (91.7%), including 100% of the high public administration (Presidency, Parliament and Government);
- municipality and county public administration – 18 institutions (51.4%);
- city and village public administration – 1 site (0.1%). There are another 23 localities with websites, but they don't represent the respective governments.

These websites were assessed using the following six criteria:

- What is the available range of information and services?
- Which of the 20 services recommended by the European Union's eEurope benchmarking project are provided by the site? (Communication from the commission, 2005)
- What forms of feedback or e-communication are provided for the users?
- What is the level of maintenance (updating)?

- Can the website be searched?
- Does the website provide access statistics?

In our assessment, more than one third of those 52 are rich in information - they contain a significant amount of interesting and useful information. Especially impressive is the database of legal documents that includes more than 43 thousand normative documents concerning national law. The most visited is the main Government website (www.gov.md) - 888381 visits as of 12 November 2008. However, the sites rarely contain forms for feedback from the population to public administrators, with the exception of a few surveys and the ability to initiate online contact with specific people in public institutions.

As mentioned above, for the first e-Europe benchmarking exercise (Communication from the commission, 2005), the European Union approved a set of 20 e-government services to be used as benchmarks: 12 for citizens and 8 for business. Of these 20 basic online services only two are offered in Moldova and even these only partially: job search and access to state information. The state has taken some actions to improve this situation. The implementation of each of these 20 services has been assigned to a specific government organization (Ministry of Information Development, 2008), but no concrete deadlines have been specified.

Government of Moldova decision nr. 668 from 19 June 2006 requires the creation of websites of municipality and county public administrations by 1 October 2006, and of mayoralty and villages ones by 1 January 2007. Thus, by the beginning of 2007 all public administration institutions of Moldova were required to be present on the Internet. It should be noted, however, that this decision provides neither funding, which is left to the localities, nor specifics about what is required. This decision was a positive step, but as of November 2008, it has been incompletely realized.

Using this data, representing some aspects of Moldova's e-government development, we will evaluate Moldova's e-government according to the models discussed above. First, we will assess the four stages of *Web Presence, Interaction, Transaction* and *Transformation.*

Based on the information in our census, we see that Moldova's e-government has reached the first two levels of development: *Web Presence* and *Interaction*. Some databases are rather significant (e.g. the large database of legal documents) and there are some good websites (those aforementioned websites relatively rich in information), but even at these stages the presence is severely limited below the national level.

All the information and services available for customers represent only front-office functions, a kind of government façade, without a link with back-office processes and information.

The stage of *Transactions* is only at the very beginning, with ongoing discussions at different levels of government about the necessity of online transaction services. There is no evidence of real results in moving toward the "last" stage of *Transformation*. However, there are some government initiatives as preconditions for this stage, for example: "Design of the Conception of an Integrated System for the Circulation of Electronic Documents (by 1 August 2006)"; "Design of the Conception of a Governmental Portal (by 1 October 2006)," and the creation of the model of this portal to provide public services online (by 20 October 2006). As of November 2008, this portal has not yet been launched.

Although many functions and internal operations (back-office processes) are computerized, and have a rather good level of informatization, with many applications, these back-office systems are oriented to the existing administrative structures, and are not integrated with front-office systems. The next stages of development (*Transactions* and *Transformation*) will require significant reengineering of government processes and information systems.

At the time of our census, governmental units below the county level had almost no web presence. As we have noted, government in Moldova tends to flow from the national level down to other levels and it is no surprise that the level of e-government development in the central (national level) government is much better than in the local governments. But we have seen that there are government decisions, which were designed to implement elements of e-government in all local governments by January 2007.

4 In-depth Evaluation

Recall the second major evaluative classification, in which Kunstelj and Vintar (2004) classified indicators into four main groups: *environment maturity*, *back-office*, *front-office* and *impact*. We will now use this classification to take a more in-depth look at conditions in Moldova. Since we have already discussed back-office and front-office applications, and the disconnect between them, we will focus on environmental maturity and impact.

4.1 Environment maturity

Certainly, one of the necessary conditions for e-government implementation is an adequate level of ICT infrastructure development. But e-government can't be interpreted only as the implementation of ICT and the providing of information on governmental sites; these should be taken only as necessary technological preconditions. For an efficient and successful e-government, it is necessary to have a range of additional conditions, linked with the quality of the environment. Kunstelj and Vintar (2004) give the following eight main indicators of what they call "***environment maturity***": *adoption and use of information infrastructure, adoption and content of strategic documents and opinions on strategic documents, trust and security issues, digital divide issues, knowledge and skills issues, motivation and barriers to implementing e-government and to using e-government, financing the development of e-government, motivation and barriers to the development of integrated services.*

In other words, environment maturity is e-readiness: the readiness of the government and customers (citizens and businesses) for e-government processes. We have further classified their eight indicators into two groups, those principally concerning the technological and financial issues involved in providing and accessing e-government and those more concerned with social issues. While we realize that this division is approximate and some indicators straddle the categories, we want to emphasize that there are two major issues, the readiness of the infrastructure and the readiness of the people, both outside and inside government, to use it.

Technological and Financial Issues

Adoption and use of information infrastructure

The current state of the ICT service market can be evaluated, roughly, by such indicators as phone penetration, the number of computers per 100 inhabitants, and the number of Internet users per 100 inhabitants.

According the National Regulatory Agency in Telecommunications and Informatics (ANRTI), in 2007 fixed phone penetration in Moldova was 30.3% (ANRTI, 2007). For comparison, fixed phone penetration in the United States exceeds 95%.

Mobile phone services began in 1998 (Voxtel). A second operator (Moldcell) entered the market in 2000, the third license was issued (to the state phone company MoldTelecom) in 2006, and a fourth operator (Eventis Mobile) entered the market in 2007. In 2007, mobile phone penetration reached 52.7%, having overtaken that of fixed phones several years before (Statistical Yearbook of the Republic of Moldova, 2008). But in comparison with Romania (106.7%), Ukraine (119.6%), Russia (114.6%), Belarus (61.4%) and Bulgaria (129.6%), the mobile phone sector in Moldova lags behind (International Telecommunication Union, 2008). The evolution of fixed and mobile telephony in Moldova in 2000-2007 is shown in fig.1.

The fraction of the information and communication technologies sector in the GDP has increased from 6.3% in 2002, to 10.1% in 2007 (Statistical Yearbook of the Republic of Moldova, 2008). In 2007, the value of the ICT service market reached nearly 5.40 billion Moldovan lei ($519 million), of which 4.19% consists of Internet access services (Statistical Yearbook of the Republic of Moldova, 2008, ANRTI, 2007).

In 2005, the number of users of Internet access services by dial-up showed an increase of 26.9% Y/Y. This increase was due, mainly, to the extension of point of presence of alternative operators in county centers, and to the reduction in the fees for the use of this service from 42 bani per minute in 2004 to 8.4 bani per minute in 2005 (ANRTI,

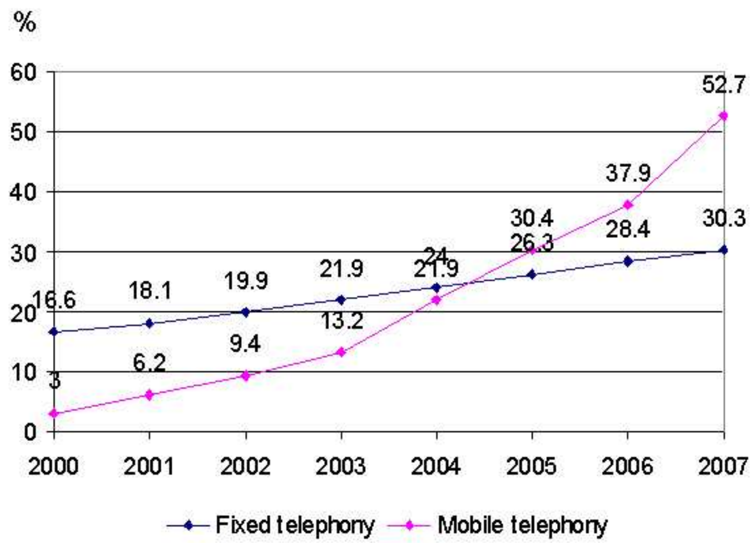

Figure 1. Evolution of fixed and mobile phone penetration

2005). In 2006 this rate was down to 7 bani per minute everywhere in Moldova. In 2007, the number of subscribers to services of Internet Access Providers increased 33.2% over the level of 2006.

The changing number of Internet users per 100 inhabitants is shown in fig. 2; it is, practically, linearly increasing. Internet use penetration in Moldova in 2007 (18.45%) was less than in Ukraine (21.64%), Russia (21.05%), Bulgaria (24.94%), Romania (52.24%) and Belarus (61.93%) (Statistical Yearbook of the Republic of Moldova, 2008; ITU 2008). The evolution of computer usage in 2001-2007 years is reflected in fig. 3 (Statistical Yearbook of the Republic of Moldova, 2008). In 2004 there were 3.2 PCs per 100 inhabitants in Moldova, more than in Ukraine (2.8 PC), but less than in Bulgaria (5.9 PC), Romania (11.3 PC), and Russia (13.2 PC) (ITU, 2004).

By 31 December 2000, 14 ATMs and nearly 200 Point Of Sale terminals had been installed. Payment by electronic cards in 2000 constituted only 0.6% of total consumer sales. Although this was small,

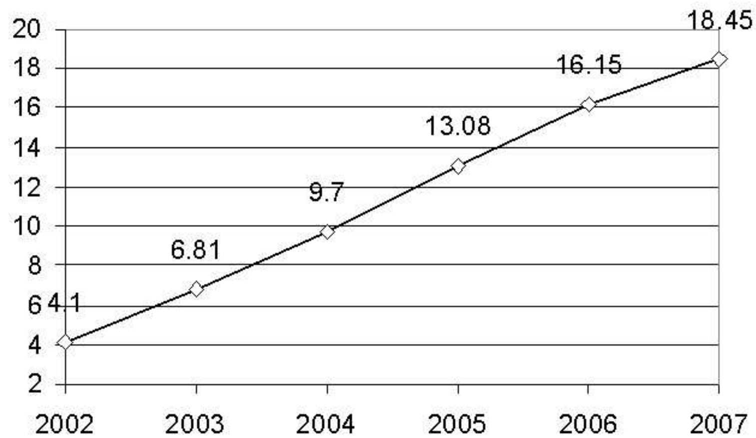

Figure 2. Number of Internet users per 100 inhabitants

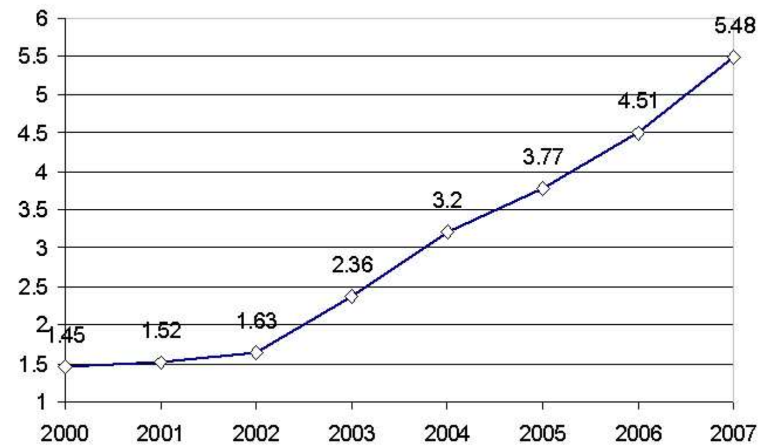

Figure 3. Number of PCs per 100 inhabitants

analysis of payment services using electronic cards shows that this sector is developing at a high rate. The number of cards per 100 inhabitants was 1.5 in 2001, 4.4 in 2002, 10 in 2004, 12.7 in 2005, and 19.0 in September 2008 (Banca Nationala a Moldovei, 2008). Of the 16 commercial banks in Moldova, 12 banks offer electronic cards. It should be noted that Moldova has done a generation skip in this technology. Very few Moldovans have used checks, but they are now beginning to use bankcards.

The indicators are significantly lower than in developed countries, but comparable with other countries in the FSU. As mentioned above, the fraction of the GDP that is generated by Information Technology industries has seen robust growth. The mobile phone and cable Internet markets continue to grow. Even in Moldova's complicated economy, ICT has grown and should continue to do so. Although this growth is not fueled by e-government needs, it will serve them.

Adoption and content of strategic documents

Moldova continues to be a country in which central government decrees are necessary preconditions for action and change. The government has adopted a wide range of documents related to strategies and action plans for e-government development. Notably, the President of Moldova issued, on 19 March 2004, a decree "Regarding the edification of Moldova's information society" nr. 1743-III. After this decree a series of Parliamentary laws and Governmental decisions in this domain has followed, including a law "Regarding the electronic document and digital signature" nr. 264-XV from 15 July 2004; a law "Regarding electronic commerce" nr. 284-XV from 22 July 2004; the Government decision "Regarding the National strategy for the edification of the information society - Electronic Moldova" nr. 255 from 9 march 2005; and the Government decision "Regarding the Conception of electronic government" nr. 733 from 28 June 2006.

The conception of electronic government foresees, for example:

- Preparation of the Design of an integrated system for the circulation of electronic documents (by 1 August 2006);
- Preparation of the Design of a governmental portal (by 1 October

2006), and creation of the model of this portal to provide public services online (by 20 October 2006);

- Preparation of proposals regarding the creation of the Center for electronic government (by 15 September 2006);
- The adoption of Methodological norms for instruction and certification of public functionaries in information and communication technologies (by 1 September 2006);
- Preparation of an annual plan of actions for the implementation of electronic government.

Analysis of the content of these strategic documents shows that the Moldovan Government has, generally, taken into account best practices accumulated in industrialized countries in the field of information society and e-government development (Relyea & Shuler, 2001; Wong, 1996; Kraemer & King, 1996). These documents are designed to encourage the development of different aspects of the information society, and of e-government. At present, it is too early to assess the effectiveness of the projects envisioned by these documents.

Financing the development of e-government

We have already discussed some investments in e-government. But these government investments were not enough to support modern requirements. As mentioned above, external support helped, although corruption lessened the desired effects (Lambsdorff 2001). Financial conditions have been better, with a more stable budget, since the organisation of an independent Ministry of Information Development in 2004. Prior to this, Informatics had not existed as a separate Ministerial level division. It had either been a lower level division, or had been subsumed in the Ministry of Transport and Communication and had been plagued by erratic financing.

Social Issues

Trust and security issues

It is well known that computer information systems are more vulnerable than traditional paper-based information systems. Research shows that concerns about security can be a major impediment to the development of e-government (Teicher & Dow, 2002). In our opinion, trust is a very important factor not only for the different structural parts of the government, but especially for relations between citizens and e-government.

Since most aspects of this problem of trust and security are common to all countries, including Moldova, we will mention only some things specific to Moldova. The level of trust in online environments is rather low in Moldova. Recent radical changes in the country have lead to many losses. For example, most people lost all the money they had in the banking system, and corruption during the transitional phase after the break up of the Soviet system caused many problems. This has had a negative impact on the measure of trust of citizens in government in general and in modern institutions (including e-government), in particular. The situation becomes worse in the context of additional factors such as insufficient ICT skills and security.

In 2005, three elections for the mayor of Chisinau were invalidated because the turnout was below the legal minimum of 33% of the registered voters. Some scholars have argued that online/internet voting can be used to increase voter participation. Despite this, widespread distrust among the general public would make it extremely difficult to use e-voting in Moldova. In fact, voting in Moldova is still done using paper ballots.

Indeed, we can see a lack of trust in the field of e-commerce, e-cards, and so on even among sophisticated academics. Even in technical departments, the implementation of e-cards for the receipt of academic salaries was not welcomed voluntarily, but rather was imposed by administrative fiat.

Digital divide issues

One of the well-documented (Teicher & Dow, 2002; Relyea & Shuler, 2001) factors that impede the spread of e-government is the digital divide, the disparity in access to computers and networks among different groups of society. Some reasons for poor access include "lack of financial resources, living in remote areas, disabilities, and lack of education and/or language skills" (Teicher & Dow, 2002).

Moldova is a compact country with a high density of population, and thus, has no real problems with remote areas, although the penetration of technology into the villages is much less than into the cities. Despite a population containing many ethnic groups, there are also no essential problems with language skills. The two languages known by almost all people in the country (Romanian and Russian) and English as an international language are used in most government websites. The main problem is the lack of sufficient financial resources. Nevertheless, many steps have been taken to make computers and the Internet accessible to a larger part of the people.

Knowledge and skills issues

As mentioned above, the use of ICT services can be significantly limited by an insufficient level of related skills. Even in the most advanced countries special attention has been paid to this. In Moldova, there have been a number of positive efforts in this area, chief among these are:

1) Efforts of the Ministry of Education and the Ministry of Labor and Social Protection designed to improve instruction in the ICT fields for students and other citizens.

2) The adoption of Methodological norms for instruction and certification of public functionaries in information and communication technologies (by 1 September 2006).

Of special importance in the support of instruction in informatics for employees is the role of the private sector. Managers of companies (in most cases members of the younger generation, with new education and vision) understand the strategic importance of using modern information technologies as a support for daily activities, and support instruction of personnel. Moreover, they encourage the development of informatics knowledge in potential employees, leading to improved instruction in informatics for graduate students. In practice, it is now impossible to find a good job in a modern company or organization without skills in ICT.

Moldova has a relatively good system of education. It is significant that specialists with higher education (i.e. with university qualification) make up 11.2% of the population in Moldova compared with, for example, 16% in Germany (University Degrees, 2003).

Motivation and barriers to implementing e-government

The general motivation for implementing e-government is the same as in other countries: e.g. increased convenience for citizens, better informed citizens, increased citizen participation in many aspects of government.

Thus, we will address only some specific aspects. Access to government information and the providing of more advanced e-government services do not depend only on the level of ICT penetration. To a great extent it depends on the readiness of the society to reach a certain level of access to information (government openness). Although, theoretically, Moldova's society has chosen the strategic way towards democracy, the actual state is still far from ideal. It has proved to be the case that it is not enough to declare a democratic society, it is necessary to build it step by step, and it is a long path, depending not only on the government but on the citizens, too. They are not accustomed to pushing for their rights, an activity which might well have been fatal in the USSR. This "culture" is changing in the new generation, albeit slowly.

In such a situation, even a high level of ICT development in the country won't lead to a full use of the opportunities offered by e-government. E-government in these conditions might be merely a façade to cover internal traditional bureaucracy, supporting the functioning of bureaucratic structures, not citizens. On the other hand, we should note that e-government makes it both easier and anonymous to obtain government information. This is one major way that e-government can help foster the rights of the people in developing countries.

Motivation and barriers to the development of integrated services

Again, the motivation for the development of integrated services is similar to that in other countries. Specific obstacles are similar to those to implementing (the simpler) e-government we have just discussed; however, given the existing level of democracy and the traditional bureaucratic culture, implementing and developing integrated e-government services, which require much more cooperation, would face even more difficulty at present. This is a challenge for the future.

4.2 Impact

We do not know of any prior systematic research on the impact of Moldova's e-government. Certainly the major positive effects of e-government, including improved, more convenient and faster citizen services, should hold in Moldova.

We do wish to note that e-government has a special role to play in a developing country like Moldova. The development of ICT and e-government can lead to greater involvement of the people in governing, and hence to a strengthening of democracy (Bolun & Lupan, 2004). The active interconnection between citizens and different levels of government, based on on-going feedback, can be an efficient engine for solving complex problems.

However, we need to emphasize that these positive effects will be totally realized only after reaching the next levels of e-government development. These levels cannot be reached without better integration of e-government services, orienting them to life-events.

A survey of university students' and teachers' perspectives on e-government in Moldova

In order to assess some aspects of the impact of e-government on the citizens of Moldova a survey was undertaken in the Academy of Economic Studies of Moldova (ASEM), one of the largest universities in Moldova. The aim of the survey was to assess to what extent students and teachers are accessing e-government services. A questionnaire was mailed to 600 people. Responses were received from 293 students and 157 members of the teaching staff (faculty), from all departments of ASEM.

In the four IT-specialized Departments 40% of student respondents and 67% of teacher respondents use Internet access time to visit different government sites. They are mainly looking for information: laws, decisions, government documents, newspapers and information about political parties, etc. Recall that our census revealed that this is really all that can be done on government sites at this time.

In non-IT departments the results look very different. First of all, while there was a 100% response rate among students, the participation of teachers in the survey was much less (only a 47% response rate). After the fact, nonrespondents explained that they don't use the Internet to access e-government. Taking this fact into consideration in overall statistics, we see that teachers access the Internet, generally and more specifically for e-government work, more rarely (16%) than the students of these non-IT departments (35%).

We asked for suggestions for the improvement of e-government services. The proposals we received were formulated by students from across the campus and by IT professors, but none came from teachers of non-IT departments.

As expected, professors in the IT-specialized departments use websites in research and preparation. Outside of those departments, the students seemed to be more engaged with e-government and more ready to use e-government services. Note that ASEM is an institute of Economics and Business, and all the faculty at ASEM, including those in non-IT departments, have access to and some knowledge of technology.

In Moldova, University students are almost all of "traditional" age - between 18 and 23. Today, the average age of professors is higher than in the past, because few young scholars are choosing the profession, which is poorly paid in Moldova. Our work also confirms that, at least within our specialized survey population, people are accessing the e-government sites. This population is characterized by both familiarity with and access to technology. Among professors, the more technically literate groups clearly access e-government sites more often. While this is also true among the students, the difference is smaller, and a significant percentage of both IT and non-IT students report using e-government. None of this is surprising and this research clearly needs follow-up work, but it does emphasize the need to take special measures in order to increase IT availability and literacy in all parts of the society. It also seems to confirm our hypothesis that the acceleration of the development of information society and e-government services is possible mostly because of the younger generation.

5 Possible future directions of e-government development

We will now examine existing obstacles impeding the development of e-government and factors contributing to its development, and will formulate some suggestions on future directions. Analyzing the actual situation in e-government development in Moldova, we can see a range of negative and positive factors that could impede or contribute to the further development of e-government.

5.1 Positive and Negative factors influencing e-government potential

Positive factors

1. The growing worldwide information society environment, which facilitates transfer of experience to developing countries.

2. A generally strong program of Government initiatives.

3. Doing informatization and re-engineering of public institutions in parallel enables informatization to be done once, in accordance with the best practices accumulated in the field.

4. The small size of the country could be a positive factor in building its information society, as suggested by the work of some authors, including Pawłowska (2004).

5. Moldova has a good system of education, and quite a high level of literacy and human potential in the IT field.

Negative factors, existing obstacles to e-government development

1. A relatively weak information infrastructure in rural areas.
2. Complications linked with transitional processes in Moldova, when the processes of informatization and re-engineering of public institutions are taking place in parallel. While this could be an advantage, as we suggested above, Pawłowska (2004) argues that such conditions increase the risk of unsuccessful informatization.
3. There is currently a dilemma in designing Management Information Systems for the transitional period. It is widely accepted among Moldovan ICT professionals that centralized planning, coordination and control are proper only for Soviet regimes, and, once we build a new society, based on democratic principles and a market economy, we don't need centralized command and control at all. However, it seems impossible to build the integrated information space of an information society, including e-government, without well-organized information management. In the most developed countries with the highest level of information society and e-government development, we see a lot of examples of the positive role of government initiatives, strategies, and coordination of the processes of informatization. In fact, the best results in this field were obtained in those countries where there are centralized permanent organizations and coordination of the process of informatization. We will return to this important social dilemma immediately in our proposals for the future.

5.2 Proposals for future directions in e-government development

The dilemma discussed at the end of the last section lies near the heart of the process of democratization in Moldova. While wishing to avoid centralized control associated with their Soviet and czarist pasts, Moldovans are often attracted to the efficiency, and to the familiarity of the strong center.

There are good arguments for central government involvement in the process of developing e-government. Analysis of the experience of leading e-government countries shows that successful e-governments do not come about accidentally. On the contrary, they are the result of purposeful and well-organized government activities, of a well-determined strategy, laws, government decisions and specialized organizational structures. The government has played such a guiding role in e-government in, for example, France (Acaud & Lakel, 2003), the USA (Janssen et. al., 2004; Kramer & King, 1996), Hong-Kong (Poon & Huang, 2002), and the EU (Lee, Tan & Trimi, 2005).

In the USA (Relyea & Shuler, 2001; Kramer & King, 1996), we can see a very well-defined strategy on developing national information infrastructure and an active governmental position in promoting coordinated information management efforts, designed to minimize unnecessary duplication of effort, capitalize on successes, and "promote the development of innovative technologies, standards, and practices among the agencies, state and local governments, and the private sector."

Similar initiatives have been undertaken in other leading e-government countries, and we'd like to emphasize the importance of coordinating efforts in e-government development.

In France, the Agency for Information Technology in the Administration played a supervisory role of providing expertise and guidance to various administrations (Acaud & Lakel, 2003). The government in Hong Kong organized an e-Government Coordination Office, which was established to plan and oversee the e-government policies and initiatives, including the ESDLife project mentioned above (Poon & Huang, 2002).

The experiences of Poland (Pawłowska, 2004), which is very similar to Moldova, confirm the necessity to involve the government in the coordination of ICT projects. Two departments with activities directly related to information society development were established in Poland. One was the Department of Information Society, which is mostly a coordinator of projects dealing with information society and e-government.

In this context, we note that, although the Government of Moldova has undertaken a range of useful e-government initiatives, following the best practices of advanced countries, it would seem to be necessary to recommend an increase in the role of the central government in promoting and organizing the further development of e-government and extending it at all levels of local governance.

Indeed, given the current political environment in Moldova, it would seem essentially impossible to accomplish anything without such an increase.

However, Moldovan scholars are likely to understand this recommendation differently than we mean. It is vital to notice that we have described the roles played by central agencies as "guiding", "providing expertise", as a "coordinator," etc. Even these words might be misunderstood, so let us be very clear. We are recommending a stronger role as facilitator, as coordinator, but not as dictator, or as "the" deciding agency. In contrast to what is often assumed in Moldova, the existence of a central agency does not require that it be absolutely superior to everything else.

This difference between the western view and an all-too-common post-Soviet view leads us directly to another recommendation. The creation of the information society and its components requires very large ICT projects with significant risks of failure. To ensure the success of such large-scale projects, it is necessary to organise them very well and to coordinate them at all levels of the hierarchy (both vertically and horizontally). We recommend that the government organisations with concrete responsibilities involve experts, both researchers and practitioners, from academia and industry in the discussions of major e-government and information society projects. We also note that it is necessary to consult representatives of the citizenry that is to be served. In other words, we strongly suggest that governments listen to people from outside.

Let us illustrate this with a more concrete example. A next possible stage of Moldovan e-government development is to increase the informatization of the county (raional) level. There is a significant volume of work and cost involved, hence it is necessary to find ways to accomplish this in the shortest possible time while economizing severely limited financial resources. Practically all governments in Moldova at the same level of hierarchy (county level, city, village level) have very similar structures and functions and offer similar services to citizens. In these conditions, the most economical way to solve the problem of effectiveness of informatization is the design and implementation of prototypical, standardized projects for all governments at each level. This would minimize duplication and expenditures. But we do not mean that the design has to be done by the national government, which might be the immediate traditional Moldovan reaction. It could be done by a few counties with the expectation that the best practices would then be shared. It could even be done by an NGO. However it is done, care must be taken to assure that prototyping does not restrict creativity too much.

In recommending a more inclusive, more democratically-inclined, yet still central organization, we are aware that there will be difficulties. However, this is a strategy that can be used within the current Moldova. In addition to this recommendation, we have a several more suggestions which we will discuss briefly.

Continue a National strategy for making ICT accessible to all citizens. The Clinton administration, on 9 December 1999, issued a presidential memorandum (Relyea & Shuler, 2001) designed to foster computer and Internet use among all Americans. Moldova needs e-government efforts like this initiative, designed to make computers and networks available to all citizens. It is worth noting that Moldova does have an innovative phone-based Internet service that requires no registration and is relatively inexpensive (about $0.32/hour).

Restructure, don't merely automate. It is necessary to avoid the easy way of creating e-government, based merely on traditional bureaucratic structures and their operations. The idea that e-government should be oriented to support needs of citizens, not the functioning of bureaucracies, is commonly promoted by researchers (Janssen et. al., 2004; Kunstelj & Vintar, 2004; Lee et. al., 2005; Poon & Huang, 2002; Teicher & Dow, 2002). To really enhance public services, e-government should be based on the integration of government processes, external (online) and internal information services and systems.

Use Integrated Quality Management in e-government services. The initial deployment of e-government at the local and national government levels will be only the start of an ongoing process of evolving e-government. Active online citizens should develop new interests in and requirements for government services. This, in turn, should accelerate the development of e-government. This can be facilitated by the use of Quality Management of Information Services. This is a relatively new subfield of Information Management. It provides methods to evaluate systems on the basis of on-going feedback that measures user satisfaction with the offered services (e.g. usefulness of services, access to services, quality of information, timeliness). Our recent research suggests that it can be an efficient subsystem of integrated information management (Costas, 2005). The collected information can be used for generalisation and further improvement of the system and services. This feedback loop facilitates democracy in action, enabling organizations to better monitor customer satisfaction and to more rapidly incorporate customer suggestions for improvements into future e-government activities.

Integrated record management. As mentioned by Mittal et.al. (2004), record management is one of the most essential and widely used services in the e-governance framework. It is necessary to "provide a record management system to electronically capture, preserve, manage, protect, and ultimately dispose of records" (Mittal et.al., 2004). Our research in the field of information management (Costas, 2003; Costas, 2004) has argued for the necessity of an integrated document management system, including record management. This is one of the e-government functions that can be totally unified and realized as a typical project and application to be disseminated to all local e-governments.

6 Conclusions

In Moldova, the best situation in e-government has been reached at the national level, and very little has been done at local levels of government. With better organization and coordination of informatization, involving independent experts in the discussion of ICT projects, Moldova could have reached a much better level of e-government development even in the difficult conditions the country has faced during the last 17 years. Parallel development of e-government and total re-engineering of government structures has certainly complicated the development of e-government, but at the same time, this could have been a good opportunity. This situation could have been used to develop new structures and government processes (back-office and front-office) to offer integrated e-government services. Unfortunately, this opportunity was lost in the conditions of transitional chaos.

Achievements are more modest than potential and conditions admit. But we believe that Moldova has a good chance to overcome its transitional problems. Certainly, ICT and e-government cannot wait until the process of democratization is finished. It is necessary to continue the process of ICT spread, because it synergistically accelerates the development of other components of the information society, including democratic changes. For example, the Australian federal government addresses these innovations, claiming that "ICTs have the potential to act as a catalyst for greater social interaction and community participation" (Teicher & Dow, 2002).

Even if the actual state of e-government is not yet as developed as is desirable, e-government in Moldova can make some contributions that e-government cannot make in more developed countries, where democratic traditions are more firmly established. It can accelerate the democratization of the society by promoting government openness, by making information more accessible, and by making government more accountable to its citizens. It can allow citizen-government interactions to be more anonymous, helping to encourage citizens scarred by years of totalitarian rule.

Although ICT infrastructure in Moldova is still not developed enough, the penetration and adoption rate of ICT is comparable with the rates of other transitional countries, sometimes even higher.

There are other positive factors, which can contribute to further essential development. The most important of these are a good literate human potential and a good system of education, particularly good in the ICT field; a favorable geographical location (almost in the central part of Europe), with a relatively favorable environment, consisting of countries with a relatively high interest in e-government (Ukraine, Romania, etc.). In these conditions, with support from developed countries, Moldova has a good chance to make essential movement towards new levels of e-government development.

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e-Services in Moldova: State of the Art

Ion Coşuleanu

Abstract

The article provides an overview of the first steps in public e-services development in Moldova and a view of Moldova position from international perspective. The article identifies the constraints, weakness and strengths for e-services and ICT development in Moldova.

1 Introduction

In the modern era knowledge is recognized as an essential driver of development and information is seen as a valuable resource as much as other natural resources - metals, oil and others. Now information has become a critical resource that assists in ensuring the accountability of government, enables governments to manage their operations, and allows the public to participate in the governance of their country. As the result, the role and the size of the governments is changing. As it is mentioned in The *United Nations e-Government Survey 2008 "From e-Government to Connected Governance"* [1], if in the early 1990s governments had a full ownership of all infrastructure, banking and natural resources, now they become e-Governments, owning only a part of the infrastructure, some mining and all citizens services offered over the Internet. The traditional government models with traditional service today are replaced by *e-Governments* with e-government services. The government sizes are diminishing drastically. Future governments in the forthcoming 10-15 years are expected to own a bit of infrastructure, a little mining and few services for citizens and e-Governments will be transformed in *Connected Governments* with high value of services for citizens, increasing the effectiveness to the maximum possible

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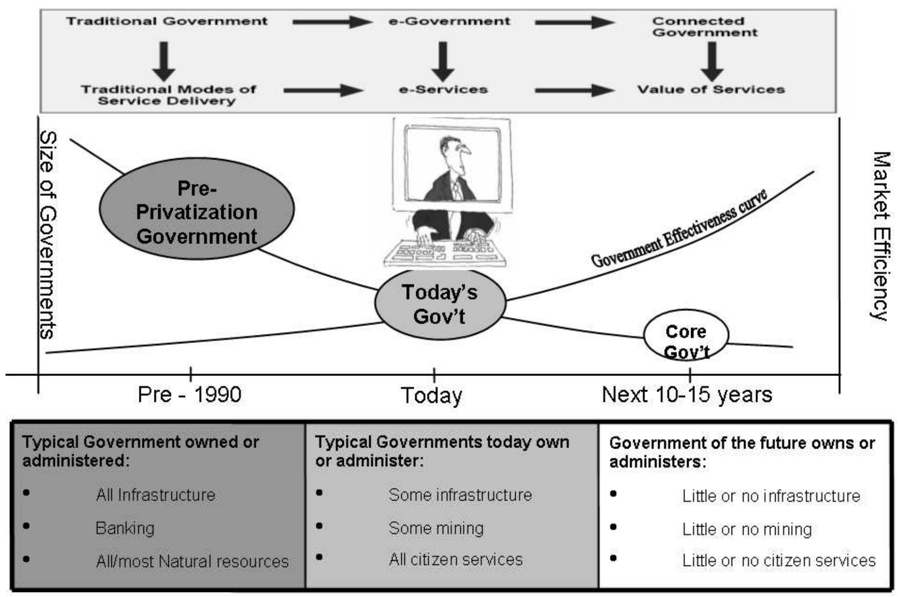

Sources: compiled from Andi Dervishi presentation by IFC. June 25, 2008. Some functions of today's Government, UN eGov Report 2008

Figure 1. The changing role of Governments

extent [2] (see Figure 1).

The connected government should keep and respect a key principle, namely, that: *the end-goal of all e-government and connected governance efforts must remain better public service* delivery. Improvements in the quality of governance and the responsiveness and effectiveness of government should still serve to empower the citizen. In that sense, citizens must be given the chance to play a role in influencing these e-government solutions [1].

An effective connected government is about a 'bigger and better' front-end with a 'smaller and smarter' back-end [3].

The National Strategy "e-Moldova" provides: "The Government of Moldova *will take* a leading role in developing appropriate conditions that favour emergence of a national information society fully integrated into European Information Society." [4]

One of the important activities in this respect is the emerging project "Building e-Governance in Moldova" - a logical continuation of the UNDP-Government of Moldova project that started in 2003 and assisted national authorities to develop E-Moldova Strategy and Action Plan. The main objective of the project is to assist the Government to improve its new role in enhancing public service delivery, while improving the efficiency and productivity of government processes and systems, to develop and promote on-line electronic public services, to increase the level of ICT knowledge among public servants and to promote democratic values through application of ICTs.[5] The project developed e-Governance Concept, approved in 2006, e-Governance Portal Concept approved in 2007, and other important documents, establishing the key elements for building e-Governance. Figure 2 is explaining the interaction of different actors within e-Governance environment.

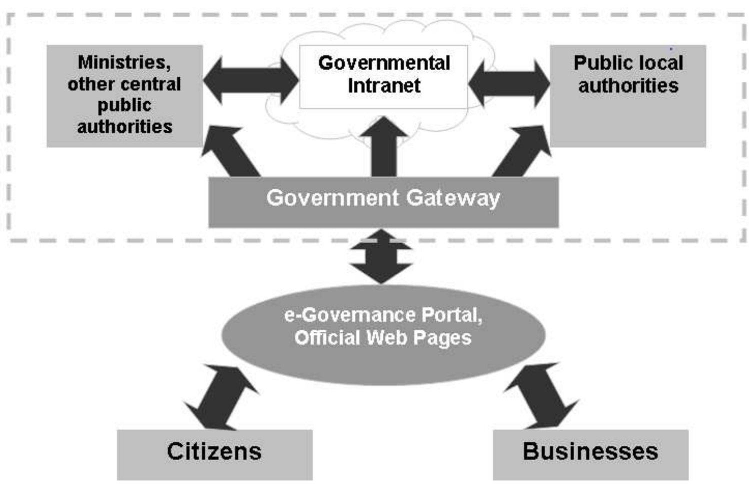

Figure 2. The interaction between different stakeholders
Source: E-Governance Concept.

E-Governance Concept [6] is not about making wrong processes more efficient through usage of technology, it is more about providing the solutions for abandoning inappropriate processes and rules being an important pillar for Public Administration Reform.
E-Governance concept enables government to deal with more challenging tasks in increasingly complex environments with less direct governing power and control.

2 E- Services in Moldova – view from inside

The e-Governance Concept provides recommendations for development of necessary elements for offering of 20 basic electronic services as they are defined by the e-Europe (12 for citizens and 8 for business). Based on approved e-Governance Concept some practical steps are being taken by the Government of Moldova supported by UNDP for their implementation.
Accessibility of services depends primarily on the level of public institutions preparedness to provide these services. It is widely recognised, that improving service delivery is not only about ICT, but mainly about cultural change, re-organisation, cooperation and interoperability between government bodies.

In the process of creating services there are some fundamental technical requirements for provision of services electronically. These conditions are related to the infrastructure of telecommunications, information resources, standards for interoperability, technology, security and data protection, the electronic identity of the person and company. For full functionality of e-services the digital signature mechanism also is necessary. The situation in Moldova with these components is shown in the Table 1.

The telecommunication infrastructure is a basic condition for access of citizen to e-services. In the last 4 years the access infrastructure growth was fast achieving the level of penetration (2007) in fixed telecommunications 32%, mobile telecommunications -61%, and Internet users 23.4%. The mobile telecommunications growth was faster than fixed telecommunications (Figure 3).

Table 1. The conditions for e-services

Necessary conditions	**Situation**	**Suggestions**
Telecommunication infrastructure	Fixed 32%, Mobile-61%, Internet 23,4%, Broadband-2.2%, Government Intranet	Needs improvement
Main information resources	4 main state information registers: -*population register* -*companies register* -*transport means register* -*drivers register*	Develop e-services based on existing resources
Interoperability standards	In progress of establishment	
Local content for services	Lack of local content	Needs
Technology	available	Further implementation
Security of data, confidentiality	Government Intranet	Expansion in rural areas
Privacy (personal data protection),	Law on Data protection, Data protection body not established yet Law on cyber crime drafted	To establish a Data Protection agency (provision of the law)
AND		
Electronic identity	Electronic identity-*implemented only in banking system.* It is expected soon the new identity card to be implemented	Implement the identity card with embedded digital signature
Interoperable electronic registration of companies	Interoperable electronic registration of companies- *partially implemented with limited access*	Implement full operable service
Interoperable digital signature	Interoperable digital signature- *incipient phase of implementation The access to wide use of digital signature is limited by: a) procedures, b) cost*	Expand PKI certification infrastructure Find cheaper and easier solution for citizens

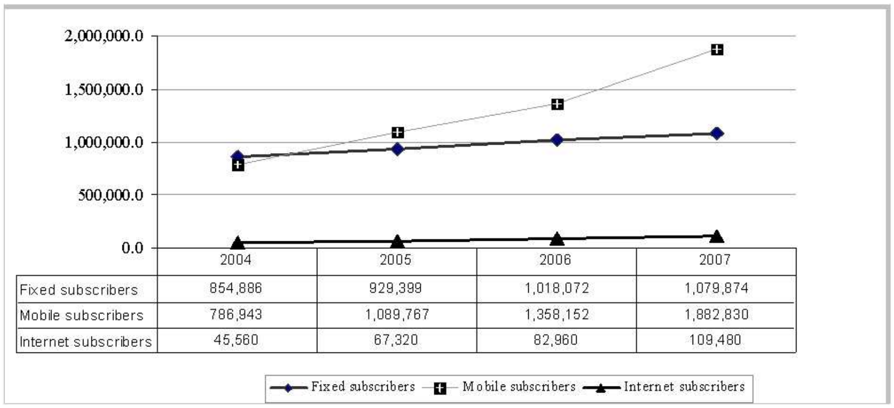

Figure 3. Dynamics of access infrastructure development
Source: ANRCETI

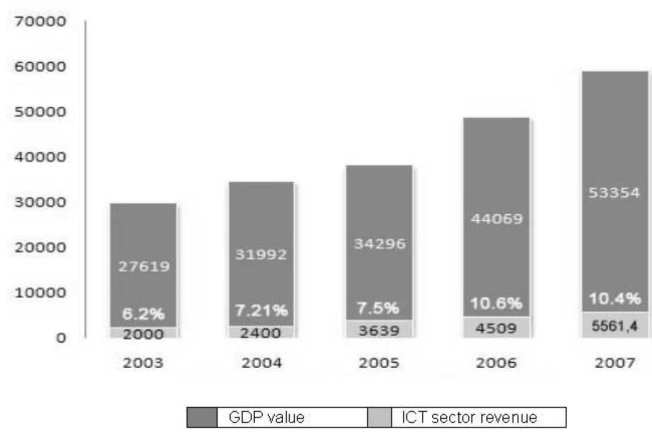

Figure 4. ICT sector value revenue share dynamics in GDP
Source: National Bureau of Statistics, Ministry of Information Development *http://www.mdi.gov.md/stat105_md/*

Due to the accelerated access infrastructure development, the ICT sector revenue was growing faster than GDP (Figure 4).

Moldova is taking the first steps in creation of e-services back office infrastructure. The following e-systems are in progress of implementation:

- E-Registration of companies (the personal signature and payment for service is necessary to pick up documents),
- E-Tax declarations system (is being implemented for businesses),
- E-statistics (The Concept of IT system is developed and being approved). UNDP is assisting in development of Statistics e-Reporting system,
- E-voting (The Concept approved by the Parliament, Register of Voters in the process of implementation),
- E-Procurement (incipient phase).

Based on e-Tax Declarations system development and implementation the following main Issues were met:

- The biggest problem – there are too many *"For the first time"*. For the first time for the country were developed, tested and implemented:
  - E-tax Declarations- first country-wide system of interaction of Government and society with the use of digital signature and user connection procedures,
  - *A* "time stamp" mechanism,
  - The dispute resolution procedure,
- Lack of experience in the society and in public authorities in use of digital signature
- Lack of public awareness in use of electronic services.

E-Services Offer-Demand Gap

A survey on e-services needs, undertaken in 2006 by e-Governance Project [8] to understand what citizens and companies of Moldova are expecting to have, showed, that the majority of the population is requiring services, related to health and to personal documents (Figure 5) and a substantial gap exists between the offer and the demand. For example, none of medical institutions were offering the on-line services. Only job search services are offered on-line by labour offices (as well as by private advertising companies).

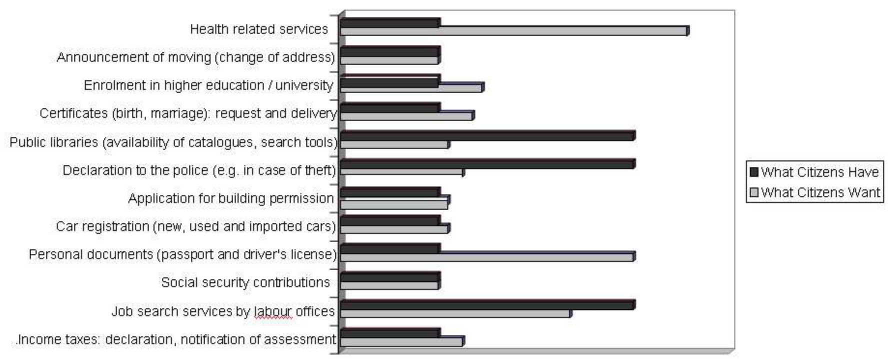

Figure 5. The e-services demand perception by citizen and the reality
Source: Report "Population priorities needs assessment in public services " Business Intelligent Service, for UNDP Moldova Project "Building e-Governance in Moldova", Chisinau, 2007, Author's estimation

Almost the same is the situation with e-services for companies. Companies need the on-line services related to tax returns, public procurement, social contribution for employees and submission of data to Statistical offices. Currently only the e-Tax Declarations system has started offering online services (Figure 6) for companies.

Based on above mentioned data, Moldova should be positioned in e-services implementation at low level, having only one service fully on-line (Figure 7).

Conclusions:

- Moldova is mainly at the 2nd stage of e-Government services

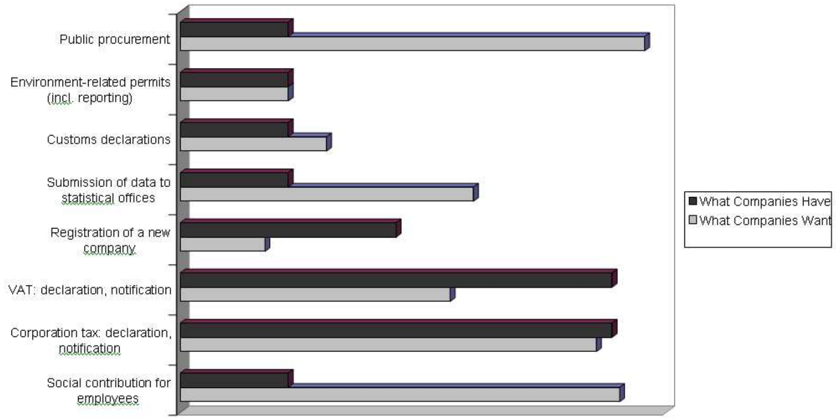

Figure 6. The e-services demand perception by companies and the reality

Source: Report "Population priorities needs assessment in public services " Business Intelligent Service, for UNDP Moldova Project "Building e-Governance in Moldova", Chisinau, 2007

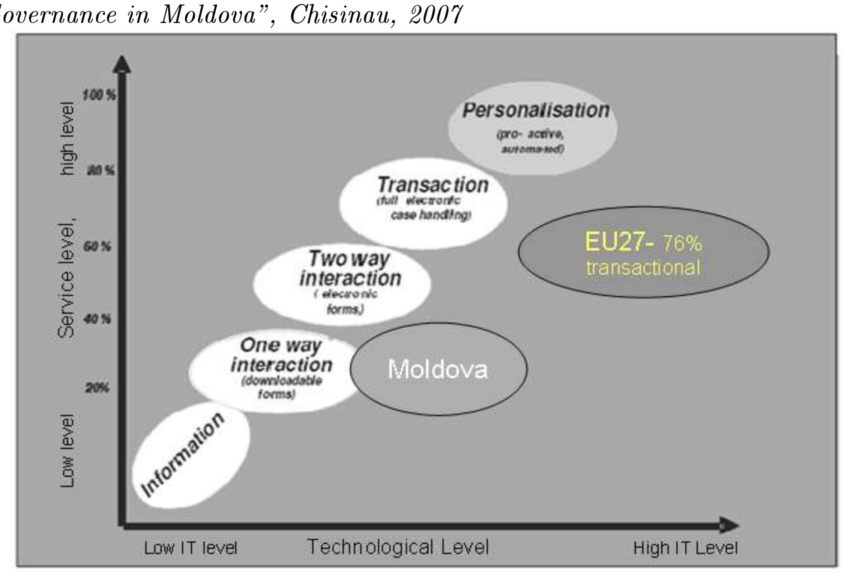

Figure 7. The level of e-services in Moldova and in EU27

Sources: Adapted from: 2007 European Commission Directorate General for Information Society and Media The User Challenge, Benchmarking the supply of online public services, 2007, author estimation.

implementation.

- Actions have to be taken to accelerate e-Services implementation.
- The implementation has to be prioritized according citizen and business expectations.

While substantial efforts are taken by the Government with donors assistance and the positive trends in development, there are at least three issues standing on the way of speeding up the implementation of electronic public services, namely: (a) the pace and the manner of implementing the Law on electronic documents and digital signature, (b) the weakness of IT departments within the ministries and other public authorities, caused by the level of wages of IT professionals, (c) lack of initiative and political will of public authorities to implement ICT in daily work. Probably, the last issue and strategic planning represent the most vulnerable weak point of the public institutions in general.

While the law on electronic document and digital signature was implemented technologically entirely, in terms of creating the necessary infrastructure for PKI protected transactions, the situation remains still unsatisfactory. Digital signature is used mainly for public servants (at quite limited extent) because of complicated procedures and the high cost. This may become an obstacle for the citizen in accessing the public electronic services. Strong political will is necessary to find a relevant solution to this problem, giving citizens the opportunity to benefit from on-line public services, using digital signature.

Practice of other countries shows that government electronic services offered to citizens should not require additional costs for accessing these services; in other words, citizens should not be forced to pay for digital signatures.

Some government institutions are technologically able to offer public services online, but the chain of procedures can not be finished without the physical presence of citizens because of digital signature and on-line payment procedure weakness.

For example, the registration of Companies through Internet (one of the 12 basic services for citizens), might be available 2 years ago from the State Registration Chamber, but unfortunately, to complete the process, the presentation of the applicants to the desk is required for signing documents and for presentation of payment voucher.

The e-Tax Declaration system, which is being implementing started to be used only by companies. The high cost of digital signature (100 USD when average wage is 250 USD) and the procedure of its obtaining make unattractive (or impracticable) the use by citizen of on-line tax declaration services.

However, despite of barriers and constrains, one could conclude that the development of e-Services and ICT in Moldova is fast and the situation is encouraging, if do not take into consideration the position of Moldova in Figure 6. Moldova position in e-services stage suggests, that the situation should be evaluated in comparison with other countries, analysing how Moldova is seen in international context.

3 E-Services and ICT development in Moldova: view from outside. International context.

E-services and ICT development level is an indicator of the overall development of the country, depending on many factors. A range of international reports on ICT and/or knowledge based economy development were recently published, ranking the countries by different indicators internationally agreed. It is important to see the position of the country among others in this respect and to compare the development dynamics; to analyse the gaps in specific indicators for better understanding what kind of actions have to be taken to diminish the existing gap.

We will refer to the following indicators:

- The Networked Readiness Index
- The Knowledge Economy Index
- The Digital Opportunities Index
- The ICT Opportunities Index

- The Prosperity Index.

The Networked Readiness Index (NRI)
Moldova rank is 96 of 127 countries [10].
The NRI assesses:

- the presence of an ICT-friendly and conducive environment;
- the level of ICT readiness and preparation to use; and
- the actual use of ICT.

The NRI (Figure 8) is measured based on a mixture of hard data collected by International Telecommunication Union (ITU), the World Bank, and the United Nations, and survey data from the Executive Opinion Survey, conducted annually by the World Economic Forum in each of the economies included in The Global Information Technology Reports [10].

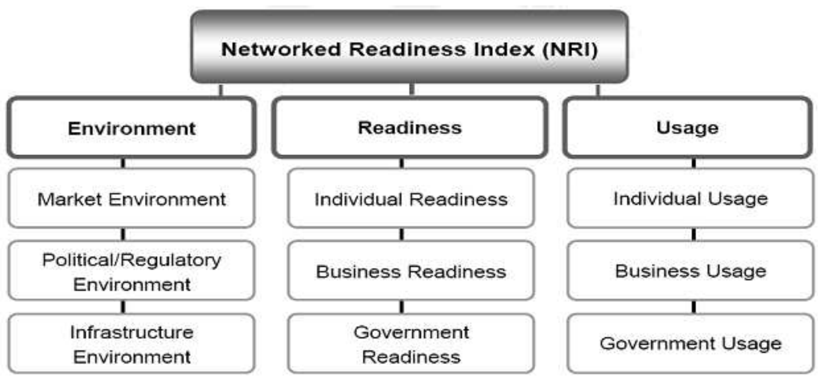

Figure 8. NRI Composition.
Source: Irene Miam. Senior Economist Global Competitiveness Network, World Economic Forum "Insights from the Global Information Technology Report 2005-2006"

The recently published Global Information Technology Report 2007-2008 is analysing 127 economies NRI. The level of NRI for

Moldova (3.21) is lower than the world average (3.96). Moreover, the growth of the NRI in 2006-2007 for Moldova by 0.08 is lower then world average (0.09) and is almost three times lower then NRI growth for Georgia -0.22 (Figure 9).

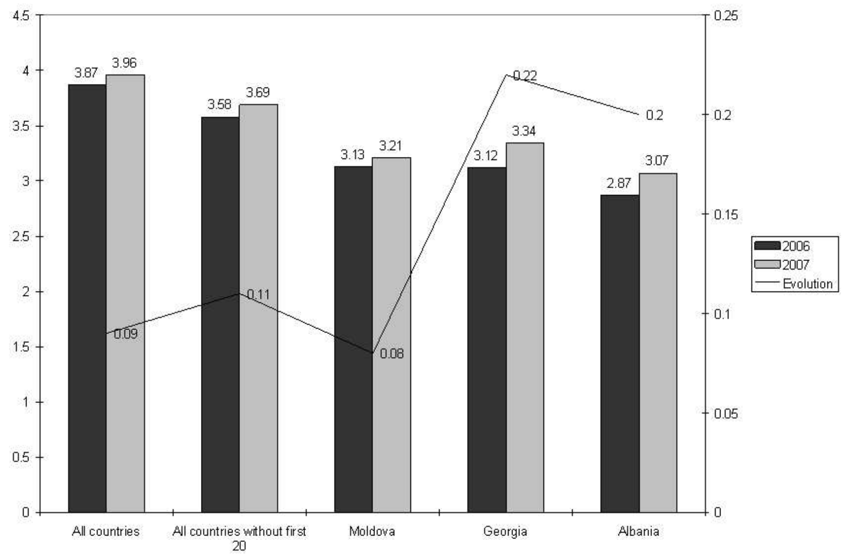

Figure 9. NRI dynamics 2006-2007.

Source: adapted from [10]

The government is the most important contributor to Moldova's overall well being and ICT promotion in particular. Looking at the value of and rank by components of NRI for Moldova one could ask, while government recognizes that ICT is an important factor for development it performs very poorly procuring first of all equipment without having information systems and does not invest enough in human capital to use it. It does not procure advanced technology (Table 2). Also, while the effectiveness of Law making bodies is satisfactory, implementation and judicial independence needs serious affirmative actions. The last period actions (Guillotine Law) cut the excessive regulations and special tax facilities for IT companies were introduced.

Conclusion: Keeping the existing conditions and pace of NRI dy-

Table 2. The strengths and weaknesses of Moldova based on NRI components

Strengths (Economy's 10 best ranks)		**Weaknesses (Economy's 10 worst ranks)**	
Variable Name	**Rank**	**Variable Name**	**Rank**
Number of procedures to enforce a contract	26	State of cluster development	126
Time to enforce a contract	26	Availability of latest technologies	125
Presence of ICT in government offices	47	Gov't procurement of advanced tech products	121
Time required to start a business	48	Prevalence of foreign technology licensing	120
Number of telephone lines	53	Extent of business Internet use	119
Tertiary enrollment	56	Cost of mobile telephone call	119
Number of procedures required to start a business	58	Freedom of the press	118
Total tax rate	59	Residential telephone connection charge	117
Quality of math and science education	59	Local supplier quantity	116
Education expenditure	60	Local supplier quality	116

Source: The Global Information Technology Report 2007-2008 © 2008 World Economic Forum

namics of the country could worse the situation and increases the gap with developed countries.

The Knowledge Economy Index (KEI)

Moldova rank is 74 of 140 countries [12]

The Knowledge Economy Index is calculated based on Knowledge Assessment Methodology (KAM) [10] with the unique strength of cross-sectoral approach, allowing the user to take a holistic view of a wide range of relevant factors rather than just focusing on one area. The variables serve as proxies for the 4 ***pillars of the Knowledge Econ-***

omy framework [12]:

- *An economic and institutional regime to provide incentives for the efficient use of existing and new knowledge and the flourishing of entrepreneurship (EIR);*
- *An educated and skilled population to create, share, and use knowledge well;*
- *An efficient innovation system of firms, research centers, universities, consultants and other organizations to tap into the growing stock of global knowledge, assimilate and adapt it to local needs, and create new technology;*
- *Information and communication technology to facilitate the effective creation, dissemination, and processing of information (ICT).*

Since 1995 to 2008, Moldova has improved its KEI ranking by 15 positions to 74. Moldova's EIR pillar increased by 23 positions to rank at 76 in 2008 due to significant reduction in trade barriers and strengthening in regulatory quality. Similarly, ICT also made significant improvements in computer and Internet penetration leading the pillar to jump by 15 positions to rank 81. However, in adult literacy rate, secondary and tertiary education enrolment rates decrease led to the education pillar decreasing 9 positions to be ranked at 52.

The position of Moldova (4.59) seems better than of Albania only (3.62) in this group of the countries and its KEI is lower than of the world average (6) and of the European and Central Asia group average (6.35). The dynamic of the Moldova KEI is positive (0.07), but is much lower than of European and Central Asia group (0.3) and of Czech Republic (0.37) for example (Figure 10).

Conclusion: The existing gap in KEI level between Moldova and developed countries is increasing!

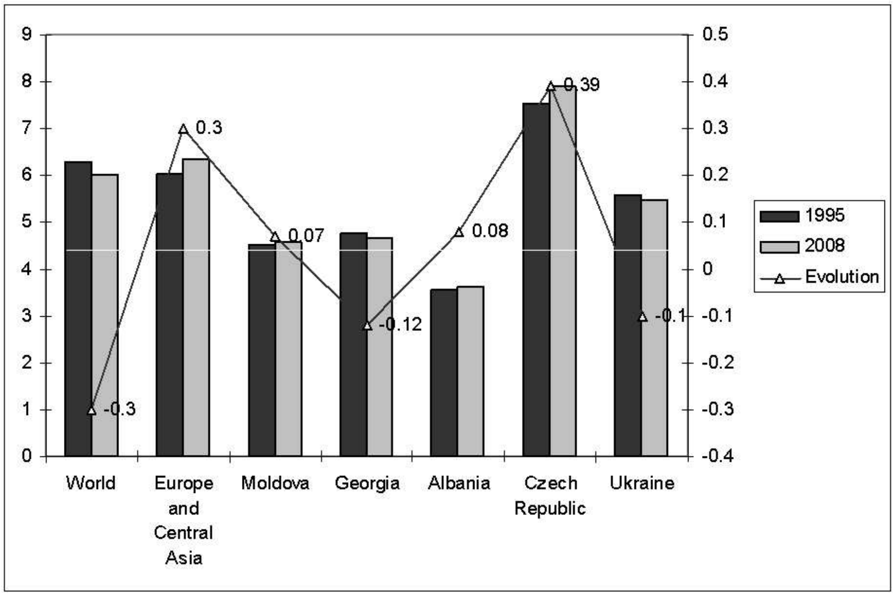

Figure 10. Knowledge Economy Index Dynamics 1995-2008.
Source: based on information from
http://info.worldbank.org/etools/kam2/KAM_page5.asp

<u>The e-Government Readiness Index</u>

<u>Rank 93 of 191 countries [1]</u>

The e-government readiness index is a composite index comprising the web measure index, the telecommunication infrastructure index and the human capital index.

The 5 stages of e-government readiness are identified:

<u>Stage I - Emerging</u>
<u>Stage II – Enhance</u>
<u>Stage III – Interactive</u>
<u>Stage IV - Transactional</u>
<u>Stage V – Connected</u>

The Web Measure Index

The web measure were based on a questionnaire, which allocated a *binary value to the indicator based on the presence/absence of specific electronic facilities/services available.*

Telecommunication Infrastructure Index

The telecommunication infrastructure index 2008 is a composite index of five primary indices::

1. *Internet Users /100 persons*
2. *PCs /100 persons*
3. *Main Telephones Lines /100 persons*
4. *Cellular telephones /100 persons*
5. *Broad banding /100 persons*

Each index represents 20 per cent of the overall telecommunication infrastructure index. Source of data: International Telecommunication Union (ITU).

Human Capital Index

The human capital index is a composite of the adult literacy rate and the combined primary, secondary and tertiary gross enrolment ratio, with two thirds weight given to the adult literacy rate and one third to the gross enrolment ratio. Sources of data: United Nations. Educational, Scientific and Cultural Organization (UNESCO), UNDP Human Development Report.

The 2008 Survey [1] represents an improvement from positions of 2005 on the e-government, when Moldova was the 37^{th} of 42 European countries and the 109^{th} – of 179 countries, and in 2008 gained 16 positions. While this shows some progress, Moldova having the higher growth of e-Government Index (0.1051) than the world average (0.0247) and the region one (0.0133), it remains on the last position in the region with the lowest e-Government Readiness level (0.451) among the countries of the Eastern Europe Region (Czech Republic, Hungary, Poland, Slovakia, Ukraine, Bulgaria, Romania, Belarus, and Russian Federation)-0.5689 (see Figure 11).

Conclusion: the pace of e-Governance development should be accelerated or at least kept at the same level for lowering the gap with developed countries in ensuring that its citizens are connected and have easy access to online government services.

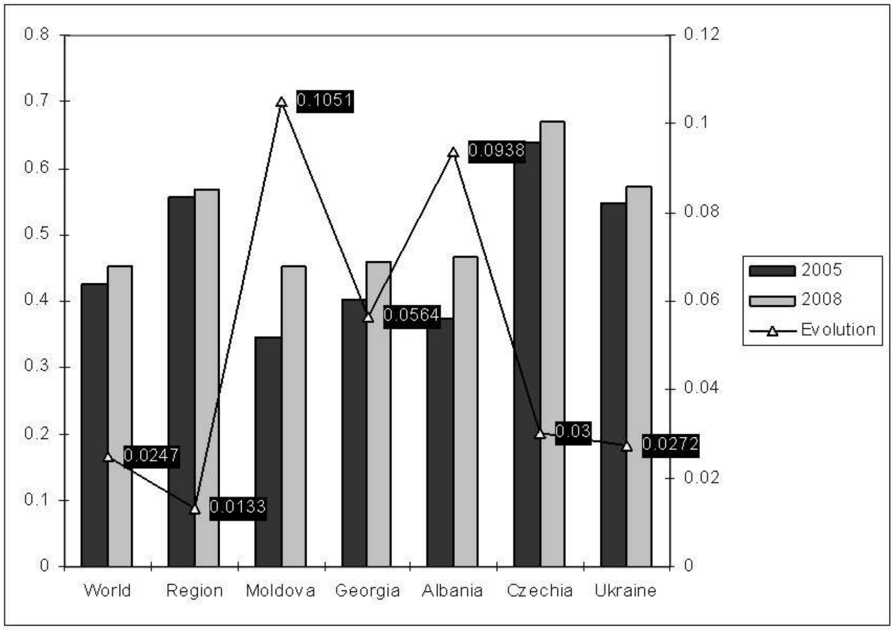

Figure 11. E-Government Readiness Dynamics 2005-2008
Source: based on United Nations e-Government Survey 2008 From e-Government to Connected Governance data

The Digital Opportunities Index (DOI)

Rank 111 of 181 countries [13]

The Digital Opportunity Index (DOI) is a composite index that *measures access to telecommunications and digital opportunity in 181 economies worldwide and considers the policy implications for the further evolution of the Information Society, and is one of the two indices endorsed in the WSIS Tunis Agenda.*

The Digital Opportunity Index is based on 11 ICT indicators, grouped in 3 clusters: opportunity, infrastructure and utilization (Figure 12).

According to the [13] Moldova DOI (0.35) was lower than the average DOI score worldwide in 2005/2006 (0.40), up from 0.37 a year

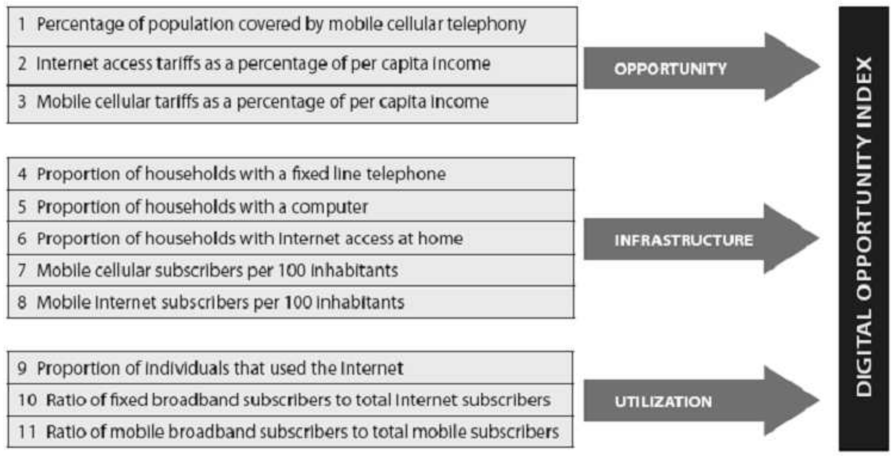

Figure 12. Composition of Digital Opportunities Index
Source: World Information Society Report 2007: Beyond WSIS, 2nd Edition, 2007 www.itu.int/wisr

earlier (an increase by 0.03). (Figure 13). This indicator is ranking Moldova among lower middle income countries (average DOI -0.38).

Conclusion: As soon as the speed of DOI growth of Moldova is the same as of the World and of Europe, the disparity with European Union countries will be preserved and Moldova will remain at the lowest level in Europe if additional actions will not be taken.

The ICT Opportunities Index (ICT-OI)

Rank 83 of 183 countries [13]

ICT Opportunity Index measures access to and usage of ICT by individuals and households in its inclusive sense interpreting the notion of ICT access and usage within the context of a global Information Society, thus recognizing ICT opportunities as an important part of social development [13]. See below Figures 14,15.

Moldova ICT-OI evolved from 59.59 in 2001 to 102.19 in 2005, by 42.6, achieving the reference country level of 2001 (Figure 16). In spite of the fact that Moldova had higher pace of development than other

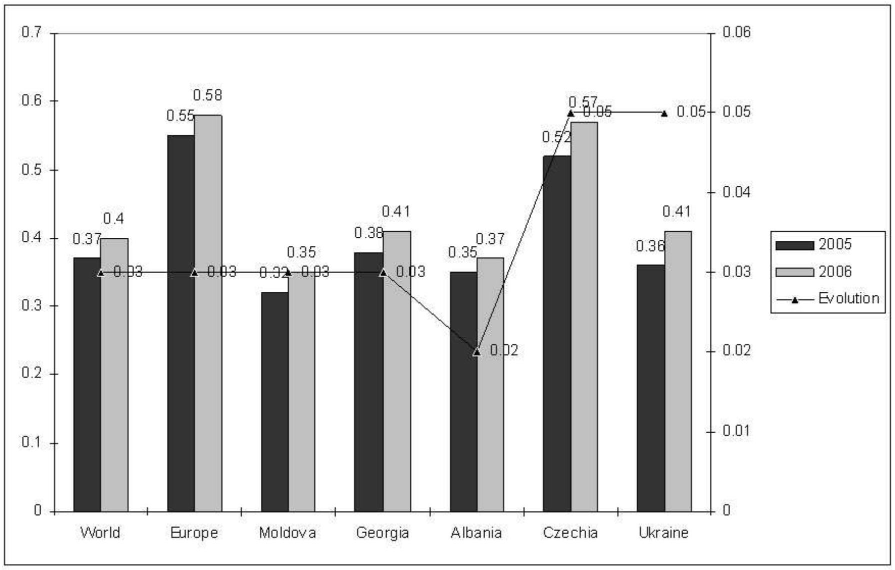

Figure 13. Digital Opportunities Index Dynamics
Source: World Information Society Report 2007

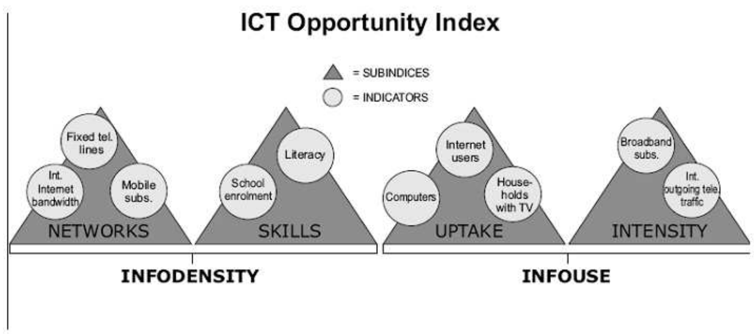

Figure 14. ICT Opportunities Index composition
Source: World Information Society Report 2007: Beyond WSIS, 2nd Edition, 2007 www.itu.int/wisr

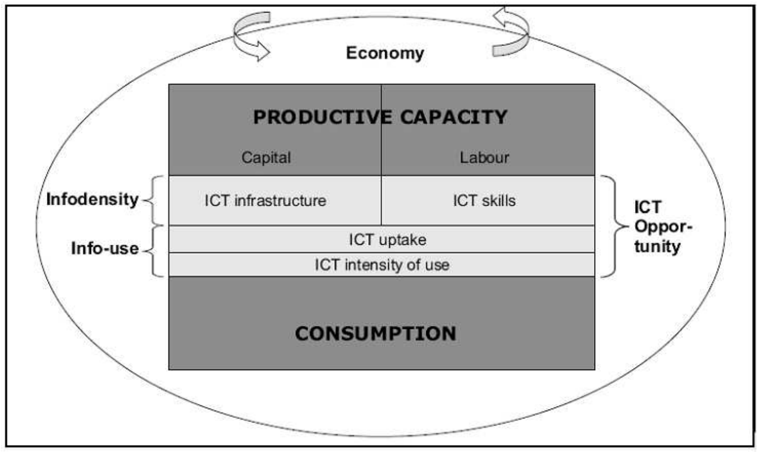

Figure 15. Conceptual ICT –OI framework
Source: World Information Society Report 2007: Beyond WSIS, 2nd Edition, 2007 *www.itu.int/wisr*

countries from the region as Georgia, Albania, and Ukraine, the average world growth of ICT-OI (the reference country ICT-OI 2005= 147.56) in 2001-2005 was higher than of Moldova.

For comparison, the EU countries, for example, Czech Republic ICT-OI growth (67.53) was substantially higher than of the Republic of Moldova (42.6).

Let us see were Moldova is lagging the most in the ICT-OI components. The networks, intensity , uptake and infouse indexes are the most lagging in comparison with the first ranked country- Sweden and the greatest gap with the reference country is in intensity index (Figure 17)

As mentioned above, the advanced infrastructure is a mandatory condition for the development of public e-services. Therefore, the stage of e-services implementation depends primarily on the ICT level of development, as well as on overall economic development, taking into consideration the high cost of necessary back office infrastructure.

As could be observed from Figure 18, the main ICT indicators of

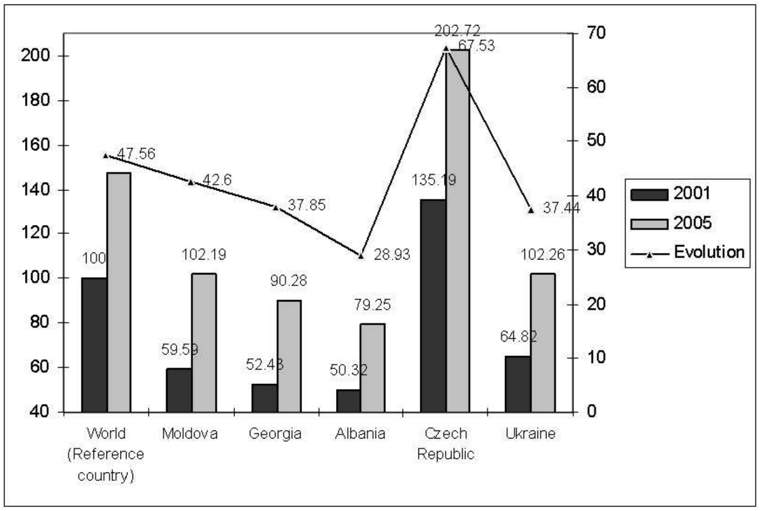

Figure 16. ICT-Opportunities Index Dynamics 2001-2005
Sources: The Global Information Technology Report 2007-2008, ITU & UNCTAD World Information Society Report 2007

Moldova even in 2007 are far behind the UE average level, excepting the ICT share in GDP.

Conclusion: Keeping the existing level of ICT-OI for Moldova the gap with developed countries will increase.

Annual sector revenues have reached close to 10 percent of GDP in 2007 and have averaged over 7 per cent over the past years. This is notably higher than other countries in the region, and significantly higher than the international average of about 3 to 4 per cent of GDP (5.3% in EU). According to [17] *"the overweight contribution of the sector to the economy might indicate either a weakness or an opportunity. The sector might be larger relative to the economy because the country is in transition and the official statistics might not reflect every economic activity. Alternatively, this could indicate that Moldavians are seeing the value of, and are using telecommunications services. The rapid expansion of the telecommunications sector relative to the national economy*

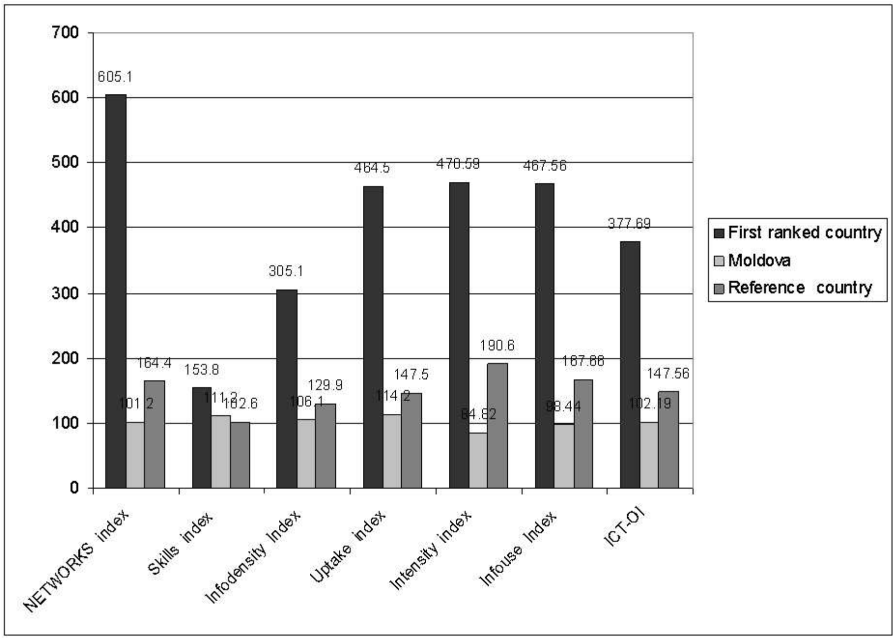

Figure 17. ICT-Opportunities Index components
Source: based on data from The Global Information Technology Report 2007-2008, ITU & UNCTAD World Information Society Report 2007

also supports such an interpretation".

The reality is somewere in the middle of such interpretation. It is rather a weakness of the country overall economic situation with \$1299 per capita (2007)[1] and \$1,218,230 (1/3 of GDP) of remittances [18,19] than an opportunity. According [24] Moldova is ranked 176 of 229 countries with \$2300 per capita (PPP) in 2007.

It is important to understand why Moldova is ranked so low in these reports. As mentioned above, e-governance- a basement for e-services- is rather a matter of cultural change, re-organisation, cooperation and interoperability between government bodies than a matter

[1] 2300 (PPP) according https://www.cia.gov/library/publications/the-world-factbook/rankorder/2004rank.html

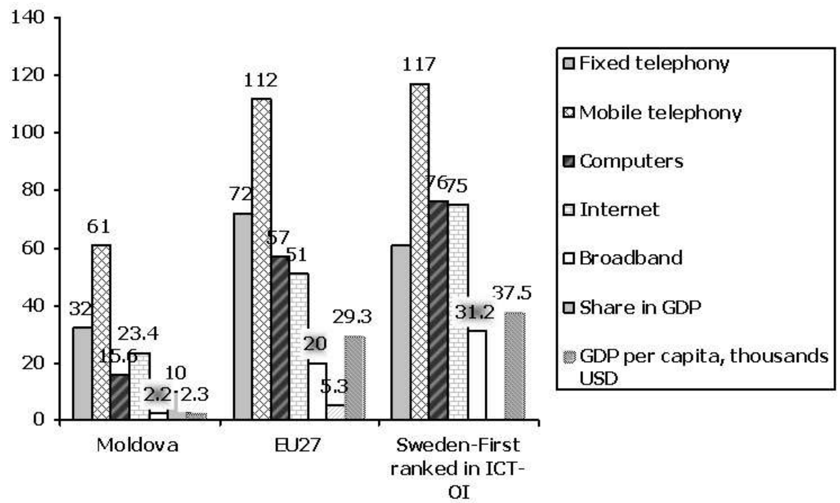

Figure 18. Main ICT indicators Moldova- EU average, Sweden, 2007
Sources: The Global Information Technology Report 2007-2008, ITU & UNCTAD World Information Society Report 2007,UN Global E-Government Survey 2008, ANRCETI Moldova, Eurobarometer nr.293, CIVIS 2007 Business & Houshold Survey for eGovernance Project, http://en.wikipedia.org/wiki/Economy_of_the_European_Union

of technology only. However as figure 15 shows there is a strong correlation between ICT indicators and GDP per capita value, especially for broadband.

While the electronic services for the citizens are seen for the one hand as a factor of good governance and for the other hand as a commodity, satisfaction of people, wellbeing and prosperity, let us see a newly indicator, analysed in another recent research -*The 2008 Legatum Prosperity Index results "An Inquiry into Global Wealth and Wellbeing"* [22].

The Prosperity Index (PI)

Rank 83 of 104 countries [20]

Again, the rank of Moldova is quite low. PI is the result of an investigation into the various factors that drive prosperity in different countries [21]. Recent research advances have made it possible to compare not only the material wealth but also the life satisfaction of people in countries worldwide. Accordingly, the PI defined national prosperity as the well-rounded combination of both of these factors. The 2008 Prosperity Index combines more than 70 variables into 20 key indicators in order to rank countries, based on the degree to which the actions of their people and governments drive or restrain the creation of well-rounded prosperity.

The Prosperity Index:

Takes a holistic view of prosperity, encompassing both material wealth and life satisfaction.

Assesses the drivers and causes of prosperity, rather than measuring outcomes.

Includes factors that relate to government policy and to individual citizens.

The PI clearly indicates that both individuals and governments have a role to play. The 2008 Prosperity Index includes both factors driven by individual choice and factors driven by the choices of policymakers.

The results of the 2008 Prosperity Index research highlight a number of general principles relevant to the promotion of national prosperity.

- Freedom of choice is crucial.
- Holistic prosperity cannot be created or maintained without effective and accountable governance.
- For poorer countries, raising incomes is a top priority.
- For richer countries, wellbeing means more than just money. Entrepreneurship is a path to material wealth.
- Growth in invested capital is crucial to long-term economic growth.

- Economic openness can help poorer countries catch up faster.
- Climate and the environment impact happiness.
- Charitable giving is associated with wellbeing.
- Geography is no longer destiny.
- Governments and citizens each have a role in building the prosperity of nations.

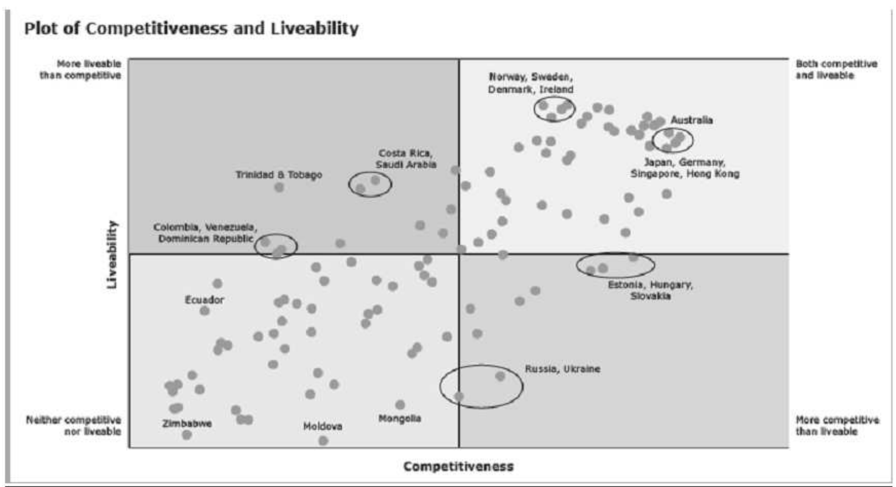

Figure 19. How Moldova is seen: Neither Competitive Nor liveable
Source: The 2008 Legatum Prosperity Index results "An Inquiry into Global Wealth and Wellbeing"

According to the [22] Moldova's Comparative Liveability indicators are as poor as its Economic Competitiveness indicators, including a low average income. Despite the existence of low unemployment (4.7%), citizens do not enjoy income levels that match their aspirations. This results in an outward migration rate of 34%, the highest in the region. Therefore Moldova is placed in the lower left quadrant of Figure 19 along with other countries which have a low rank owing especially poor liveability and poor competitiveness scores (like Zimbabwe).

The low rank of Moldova in this research demonstrates that neither government nor people make enough efforts in accelerating the development.

Below one could see that in competitiveness indicator all components (excepting secondary education) are below the world average. The most essential deviation is observed in Government Effectiveness (Figure 20).

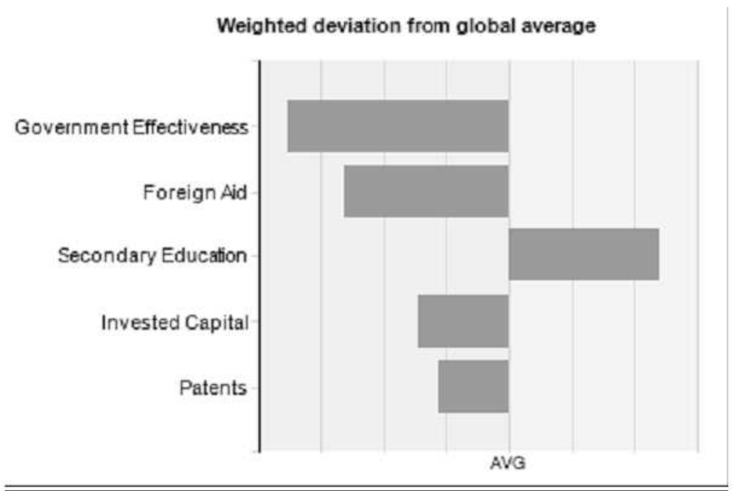

Figure 20. Moldova economic competitiveness indicators deviation from world average
Source: http://www.prosperity.com/country.aspx?id=MD

As Figure 21 shows, all the components of liveability are lower than world average. The major deviation is in the income per person.

4 Conclusions and recommendations

Moldova is ranked in all cited reports almost lower than the world average, excepting *e-government readiness index.* Moreover, the speed

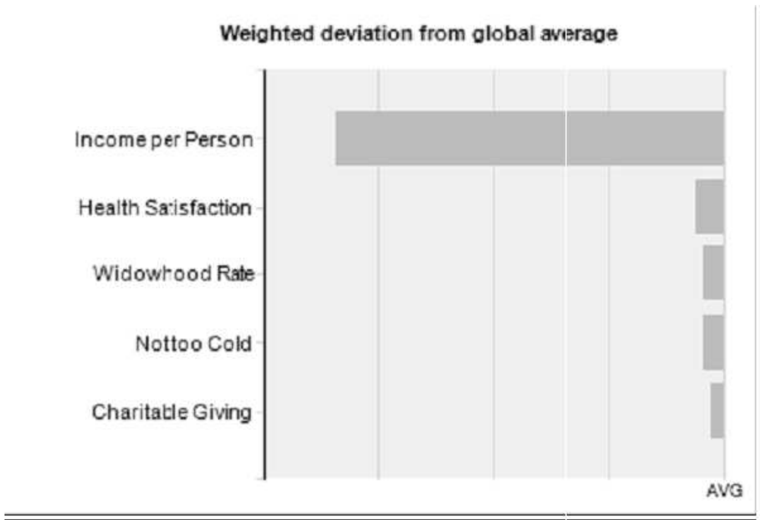

Figure 21. Moldova comparative liveability deviation from world average
Source: http://www.prosperity.com/country.aspx?id=MD

of development is lower than of many countries from the region, which should especially aware the responsible government bodies.

The weakest positions according to the Reports Moldova holds in:

- Income per person (very low)
- State of cluster development
- Availability of latest technologies
- Government's procurement of advanced technologies products
- Prevalence of foreign technology licensing
- Extent of business Internet use
- Cost of mobile telephone call

- Freedom of the press
- Residential telephone connection charge
- Local supplier quantity
- Insufficient competitiveness.

The following constraints for ICT and e-services development were identified by sector representatives within roundtables held by "e-Governance Project":

- The Centre for Electronic Governance is not yet created
- Private public keys Certification centres are not yet established
- Digital signature procedures applications are complicated and costly
- Lack of financial resources
- Lack of motivation of IT specialists to work in public institutions (low wages)
- Not yet mechanism for interoperability in place.
- Lack of law enforcement
- Lack of independence of Justice
- Lack of fair competition in fixed telecommunications market
- Protectionism from the government over incumbent operator
- Lack of access to the local loop and associated facilities for alternative operators
- Lack of funding and the high costs of investment in the creation and development of optical infrastructure FTTx / GPON that limits the broadband penetration

- High wages in developed countries create conditions for the exodus of qualified personnel
- Offer-Demand gap in ICT specialists' education
- Insufficient use by the Government outsourcing and expertise of local private IT companies
- Inconsistencies in the reforms and declared hostile business climate (according to the study "Doing business in Moldova" 2008 Moldova is ranked 92 of 172 economies);
- Insufficient use of OS/OSS limits local companies' development, and, creates the risk that the public authorities could lose access to their documents and archives.

The responsible bodies shall use the main strengths of the country for development mentioned in the Reports, such as:

- Acceptable number of procedures and time to enforce a contract
- Presence of ICT in government offices
- Number of procedures and time required to start a business
- Satisfactory number of telephone lines
- Tertiary enrolment, Education expenditure, quality of mathematic and science education
- Total tax rate

Use of existing strengths will create new opportunities to decrease the gap.

All the analysed in this paper World reports are showing that the level of ICT development in Moldova is below the world average. Moreover, the pace of development is lower than of majority of countries of the region. This low rate of development creates the risk of increasing the existing gap between Moldova and European Union countries. To

avoid such a scenario, Moldova needs not simply a fast development in the area of the Information Society but an accelerated development in the areas were new technologies will have the greatest impact to improve the situation in the next few years.
A Strategic Paper for ICT sector development 2009-2012 is being formulated by the Ministry of Information Development with UNDP assistance. All listed constrains have to be addressed in the paper and in the action plan to create relevant environment for the development.

However, creation of necessary conditions for the implementation of electronic services is a long term task for Moldova that needs political will, favourable legal and regulatory environment, changed paradigm of public servants, financial resources, trained personal and time. The high ranked countries in all the reports could serve as a best practice to be followed.

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Confidentiality and Security in Medical Information Systems

Victor Papanaga

Abstract

Behind the technologies Medical System contains different types of information including patient information also. The patient data is classified as confidential and is one of the patient rights based on World Health Organization declaration. There are several compromises in solutions selection based on hardware and software requirements, performance, usability, portability. This article presents the investigation results and proposes the secured solution principles for the medical system that deal with patient data.

Keywords: Medical Information System, World Health Organization, Patient Rights, Patient Confidentiality, Patient Data, Medical Investigations Data, Medical Security, Symmetric/Asymmetric Encryption, Encryption Key

1 Introduction

Medical Information Systems (MIS) in majority cases deal with patient information. According to World Health Organization (WHO), coordinating authority for public health, patient rights include:

- The right to receive information from physicians and to discuss the benefits, risks, and costs of appropriate treatment alternatives;
- The right to make decisions regarding the health care that is recommended by the physician;

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- The right to courtesy, respect, dignity, responsiveness, and timely attention to health needs;
- The right to confidentiality;
- The right to continuity of health care;
- The basic right to have adequate health care.

In most of the cases physicians assure that patient rights are not compromised but with implementation of the MIS the responsibility to assure the confidentiality of the patient information becomes the actual problem for Software Engineers that create and implement MIS. In this situation the accuracy in selecting the solution that assures the patient confidentiality becomes vital for the project implementation. This article presents the investigation results for Secure MIS implementation based on the most common hardware and software constraints.

2 Patient information in Medical Information Systems

Behind the pure medical information like Medical Knowledge Databases, present medical systems usually deal with additional information like user database, patient database, medical investigation database, other. In situation when the database contains patient information, the system must assure the confidential access to such information.

Minimal patient information usually available in Medical Systems is:

a. **Patient data.** Minimal data to identify the patient are: **Name**, **Surname** but there are some constraints that require additional information as following:

   - Using **Name** and **Surname** there are some risks to identify ununiquely the patient. For unique identification we will require additional information like **Unique Identification Number** (IDNP) allocated for each citizen on birth. **IDNP** is required also in registration of the patient visits to physicians provided by Healthcare Institution to Medical Assurance Institutions.

- To differentiate physiological aspects for male and female anatomy the systems can add additional description for patient like **Sex**.
- Differences based on patient age can be monitored using additional descriptor like **birth date**.

b. **Patient investigation data.** Patient investigation data include the following:

- Information about fact of addressing for medical help;
- Present health situation;
- Diagnostic Disease;
- Other medical data collected after additional investigation.

Sometimes when the medical data are collected using specialized devices the patient information is included in digital collected information (example of ultrasound investigations as images, movies contains patient information), that's why the confidentiality of the patient data requires additional attention to digital resources also.

Based on World Health Organization, all the **Patient investigation data** are classified as confidential and require attention in elaboration and implementation of MIS.

There are several international standards elaborated by specialized institution that describe and offer the recommendations in implementation of the security components in MIS like:

- ISO TC 215 (ISO)
- HL7 (Health Level Seven)

- CIHI (Canada)
- GEHR
- ICPC (EUPHID)
- SONOMED
- Current Procedural Terminology
- OPCS-4, UT
- DICOM
- XML (W3C Consortum)

Chosen solution requires conformance with existing standards and deep attention in application implementation.

3 Constraints in provided solutions

There are a lot of existing solutions that assure the encryption of the textual, digital and media, stream information. Several solutions are pure hardware, other are software or combined. Based on application type the solutions can be: free or priced, open source or custom, platform independent or based on specific platform. In our case there are several constraints that require attention in solution providing/implementation as following:

- **Easy system integration** – solution requires to be integrated or part of the main system;
- **Short processing time** – solution that performs the action fast;
- **Independent solution** – solution that does not require human interaction directly;
- **Platform Independent** – solution that easy can be used and/or adjusted for each type of OS platform;

- **With low implementation cost** – solution that does not require additional costs for hardware upgrades, software license, other.
- **With minimal resources usage** – solution that supports minimal hardware configuration for the system and does not affect the performance.
- **Solution that allows textual and digital information encryption / decryption**.
- **Solution that allows integration with third part components** (like database readers/writers).
- **Solution with easy further maintenance.**

4 Available software to satisfy the selection criteria

In order to find the possible third part solution to satisfy our application the set of the Internet Available applications were investigated and the results of investigation are presented in Table 1.

General comments related to preliminary investigation:

- There is no free or non free software that satisfies all mentioned criteria for Medical Systems;
- All the existing solution are classified as final products and can't be integrated using API or other technologies to our system;
- All the identified solutions are platform dependent and based on Windows OS predominantly;
- There is no free (only shareware) solution to satisfy our necessity;
- All the identified solutions are easy to maintain based on specialized interface but not assure the integration with third part components.
- We can obtain some progress in this direction only if we will develop our custom solution.

Table 1. Encryption/Decryption software acceptance criteria conformance

Application	Easy system integration	The short processing time	Independent solution	Platform Independent	With low implementation cost	With minimal resources usage	Solution that allow textual and digital information encryption /decryption	Solution that allow integration with third part components	Solution with easy further maintenance.
File Encryption XP 1.5.123		■				■	■		■
Best Folder Encryptor 15.72		■					■		■
BestCrypt Volume Encryption 2.11.01		■					■		■
Max File Encryption 1.8		■					■		■
SecureIT Encryption Software 3.1.8		■				■	■		■
Encryption And Decryption Pro 1.2		■					■		■
Strong File Encryption Decryption 1.0		■					■		■
Folder Crypto Password 2.0		■					■		■
Dekart Secrets Keeper 3.11		■				■	■		■

Legend:

	The selected criteria are not satisfied
■	The selected criteria are satisfied

5 Cryptographic solution investigation for MIS

To satisfy all our requirements it is proposed to develop custom solution for our application. The investigation process includes two steps:

- **Step 1** – identification of the encryption methodologies (symmetric, asymmetric);
- **Step 2** – identification of the encryption algorithm

There are some constraints related to asymmetric key encryption related to public authority that must be involved in the process of key emission like VeriSign, other. Asymmetric key also has it own life cycle and needs to be re-emitted periodically (once a year). That's why it compromises one of our criteria: **Solution with easy further maintenance.** At the same time we can't decide to exclude it from our performance investigation that's why the most representative asymmetric algorithm was selected for performance compare.

Step 1 – the investigations main purpose was the identification of encryption methodologies.

Exercise was done using IntelliJ Idea 7.3 on Windows Vista with 2 GB RAM and 100 GB hard disk for 10 times and average results are presented in Table 2.

Note: This test includes run-time symmetric key generation, encryption of plain text into cipher text and decryption of cipher text back into plain text.

Summary of observations:

- Regardless of the algorithm (and therefore the perceived complexity of the algorithm) the amount of time it takes for encryption and decryption remains more or less the same.
- The size of the input text does not make any difference.

Note: This test includes run-time symmetric key generation, encryption of plain text into cipher text and decryption of cipher text back into plain text. Asymmetric key algorithm performance is presented in Table 3.

Summary of observations:

- The time taken is certainly more than what symmetric key encryption algorithms require.

Table 2. Symmetric algorithm encryption/decryption performance

Algorithm used	Original text	Length of original text (kb)	Key size (bit)	Execution time (ms)
Blowfish	Small plain text for encryption	32	128	40.0159098
	Medium plain text for encryption	85	128	41.2025411
	Big plain text for encryption	203	128	40.0815467
3DES	Small plain text for encryption	32	160	41.5861513
	Medium plain text for encryption	85	160	40.5570750
	Big plain text for encryption	203	160	41.3329487
AES	Small plain text for encryption	32	256	42.7647718
	Medium plain text for encryption	85	160	42.3849121
	Big plain text for encryption	203	160	42.4304486

- The size of the input text is important. The more the plain text, the higher is the time taken for encryption and decryption.

Conclusion:

- To assure performance in Medical Information Systems the Symmetric Key Algorithms are preferable and provide the best performance as Cryptography methodology;

- Not all symmetric cryptographic solutions are approved by international standards (from our list only 3DES and AES satisfy this criteria).

Table 3. Asymmetric algorithm encryption/decryption performance

Algorithm used	**Original text**	**Length of original text (kb)**	**Key size (bit)**	**Execution time (ms)**
RSA	Small plain text for encryption	32	1024	86.1157563
	Medium plain text for encryption	85		88.0750277
	Big plain text for encryption	104		93.0025534

Step 2 – the investigations main purpose was the identification of encryption algorithm and minimal hardware environment for Encryption process.

Exercise was done using JDK 1.4.2 and JDK 1.5.13 on Pentium IV 2.4 GHz with 256 Mb RAM, running Windows XP Professional edition. Exercise was performed for 10 times and average results in **milliseconds** are presented in the Table 4.

Summary of observations:

- Observation was performed using minimal hardware environment for Pentium IV technologies;
- Observation includes encrypt and decrypt process;
- For low and medium block size, AES algorithm performs encryption and decryption much faster;
- For increased block size the algorithms have the temptations to provide the same results;
- AES algorithm is acceptable as Encryption Algorithm by international standards;
- AES algorithm has the public implementation and can be developed in platform independent mode using Java technologies.

Table 4. 3Des and AES encryption/decryption performance

Block Size		**Java SDK**			
		Java 1.4.2		**Java 1.5.13**	
		Encrypt	**decrypt**	**encrypt**	**Decrypt**
1Kb	**3DES**	0	0	0	0
	AES	0	0	0	0
10Kb	**3DES**	0	0	0	0
	AES	0	0	0	0
100Kb	**3DES**	31	31	15	31
	AES	0	0	0	16
1Mb	**3DES**	328	360	343	375
	AES	94	109	109	141
10Mb	**3DES**	2860	2891	2859	2958
	AES	500	547	500	562
50Mb	**3DES**	16063	14948	14703	16373
	AES	4641	4250	2328	4875
100Mb	**3DES**	116687	124969	78765	110828
	AES	98094	103813	66531	74375

6 Conformance with initial selection criteria

Utilization of the Java based solution for our algorithm satisfies the initial selection criteria as following:

- **Easy system integration** – utilization of the universal solution (Example: based on Java);
- **The short processing time** – the processing time is shorter than other concurrent solutions;
- **Independent solution** – solution can be developed in such a way that to exclude human interventions;
- **Platform Independent** – utilization of Java Based solutions for algorithm;
- **With low implementation cost** – utilization of existing open source solutions for algorithm;

- **With minimal resources usage** – algorithm is faster than several concurrent solutions. Additional constraint was added in order to satisfy WHO and international standards requirements;
- **Solution that allows textual and digital information encryption / decryption** – was demonstrated under test process;
- **Solution that allows integration with third part components** – as custom solution there are possibilities to create the Public API in order to integrate to third part components.
- **Solution with easy further maintenance** – there are no any constraints in symmetric key lifecycle and the maintenance of the key can be properly organized and/or documented.

7 Conclusions

- There is no public free solution to satisfy Medical Information System requirements in cryptography implementation;
- Symmetric algorithms are faster than asymmetric algorithms and can be used in simple Medical Information System with low resource limitation;
- AES is faster than 3DES algorithm for small and medium block size. For block size more than 100Mb the performance is comparable with 3DES algorithm;
- AES algorithm satisfies the initial selection criteria and can be used as possible solution for Medical Information Systems.

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