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Nash equilibrium set function in dyadic mixed-strategy games

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Abstract

Dyadic two-person mixed strategy games form the case for which Nash equilibrium sets can be determined simply. In this paper the set of Nash equilibria in a particular game is determined as an intersection of graphs of optimal reaction mappings of the first and the second players. In contrast to other games, it is obtained not only an algorithm, but a multi-valued Nash equilibrium set function that gives directly as its values the Nash equilibrium sets corresponding to the values of payoff matrix instances. To give an expedient form to such a function definition, it is used a code written in the Wolfram language. Additionally, it is also applied a Wolfram language code to prove the main theoretic result.

Keywords: Dyadic mixed-strategy game, Nash equilibrium, Nash equilibrium set function, NES function, graph of best response mapping.

MSC 2010: 91A05, 91A10, 91A43, 91A44.

1 Introduction

The problem of Nash equilibrium [2] computing in noncooperative games [1] remains important in game theory at least for last decades. Moreover, the problem of the entire Nash equilibrium set computing becomes important too because of non equivalence of Nash equilibrium instances.

We studied the problem of Nash equilibrium sets computing in a series of works [5]–[8], [10], [11], [15], [16]. The main idea of those works

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is based on the concept of a graph of best response mapping and the representation of a Nash equilibrium set as the intersection of best-response mapping graphs. The idea was extended to Pareto-Nash-Stackelberg games too [9], [12]–[14], [17]–[21].

In this paper, we present a formula of the Nash equilibrium set function $NES(A, B)$ (NES function). It is a multi-valued function that has as its domain the Cartesian product $\mathbb{R}^{2\times 2} \times \mathbb{R}^{2\times 2}$ of two real spaces of two 2×2 matrices and as a NES function image all possible sets of Nash equilibria in dyadic bimatrix mixed-strategy games. These types of games where studied earlier in a series of works, e.g. [3]–[5].

2 Game statement and its simplification

The dyadic two-player mixed strategy game is defined by a tuple

$$\Gamma' = \langle N, X, Y, f_1(\mathbf{x}, \mathbf{y}), f_2(\mathbf{x}, \mathbf{y})\rangle,$$

where

$N = \{1, 2\}$ is a set of players,

$X = \{\mathbf{x} \in \mathbb{R}^2 : x_1 + x_2 = 1, x_1 \geq 0, x_2 \geq 0\}$ is a set of strategies of the first player,

$Y = \{\mathbf{y} \in \mathbb{R}^2 : y_1 + y_2 = 1, y_1 \geq 0, y_2 \geq 0\}$ is a set of strategies of the second player,

$f_i : X \times Y \to R$ is a player's $i \in N$ payoff function,

$$f_1(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T A\mathbf{y}, \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad A \in \mathbb{R}^{2\times 2},$$

$$f_2(\mathbf{x}, \mathbf{y}) = \mathbf{x}^{\mathbf{T}} B\mathbf{y}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad B \in \mathbb{R}^{2\times 2}.$$

We suppose that every player maximizes the value of his payoff function. It is a requirement that is dictated by a need to be consistent with the Wolfram language code.

Remark, the strategy set is a segment for each player, i.e. it is a hypotenuse of the correspondent unit simplex (right triangle) in $\mathbb{R}^2$.

We reduce the game Γ' to a simpler game Γ by substitutions:

$$x_1 = x,\ x_2 = 1 - x,\ 0 \le x \le 1,$$

$$y_1 = y,\ y_2 = 1 - y,\ 0 \le y \le 1.$$

Thereby, in the game Γ both the players have as their strategy sets the segment $[0, 1]$ and as their payoff functions:

$$f_1(x, y) = (\alpha y - \alpha_0)\, x + (a_{21} - a_{22})\, y + a_{22},$$

$$f_2(x, y) = (\beta x - \beta_0)\, y + (b_{21} - b_{22})\, x + b_{22},$$

where

$$\alpha = a_{11} - a_{21} + a_{22} - a_{12},$$

$$\alpha_0 = a_{22} - a_{12},$$

$$\beta = b_{11} - b_{12} + b_{22} - b_{21},$$

$$\beta_0 = b_{22} - b_{21}.$$

Remark 2.1. *To compute their optimal strategies, the players can omit the last two members of their payoff functions because the first player chooses a value for the variable x and the second chooses a value for the variable y. Thus, they solve the simplified game:*

$$\Gamma = \langle [0, 1], [0, 1]\,;\ \tilde{f}_1(x, y) = (\alpha y - \alpha_0)\, x,\ \tilde{f}_2(x, y) = (\beta x - \beta_0)\, y \rangle.$$

Proposition 2.1. *From the strategy point of view, the games Γ' and Γ are equivalent.*

Proof. The truth of the statement follows from the reduction of the game Γ' to the game Γ provided above. □

Further, we will use both the game Γ' and the game Γ to construct the sets of Nash equilibria.

3 Optimal Value Functions and Best Response Mappings

The game Γ represents a game on a unit square. The payoff functions are bilinear, i.e. for a fixed value of one variable, the functions become linear functions in relation to the other variable. To choose his best strategy, every player must solve a parametric linear programming problem on a unit segment.

For any player, we can define the optimal value function and the best response mapping:

$\varphi_1(y) = \max\limits_{x\in[0,1]} f_1(x,y)$ — the optimal value function of the first player,

$\gamma_1(y) = \operatorname{Arg}\max\limits_{x\in[0,1]} f_1(x,y) = \operatorname{Arg}\max\limits_{x\in[0,1]} \tilde{f}_1(x,y)$ — the best response mapping of the first player,

$\varphi_2(x) = \max\limits_{y\in[0,1]} f_1(x,y)$ — the optimal value function of the second player,

$\gamma_2(x) = \operatorname{Arg}\max\limits_{y\in[0,1]} f_2(x,y) = \operatorname{Arg}\max\limits_{y\in[0,1]} \tilde{f}_2(x,y)$ — the best response mapping of the second player.

To determine Nash equilibrium sets for particular instances of matrices A and B, we need the graphs of best response mappings:

$$Gr_1(A,B) = \left\{ (x,y) \in [0,1]\times[0,1] : \begin{array}{l} y \in [0,1] \\ x \in \operatorname{Arg}\max_{x\in[0,1]} \tilde{f}_1(x,y) \end{array} \right\},$$

$$Gr_2(A,B) = \left\{ (x,y) \in [0,1]\times[0,1] : \begin{array}{l} x \in [0,1] \\ y \in \operatorname{Arg}\max_{y\in[0,1]} \tilde{f}_2(x,y) \end{array} \right\}.$$

As these graphs are functions of payoff matrices A and B, it is evidently that the Nash equilibrium set is a function of the payoff matrices A and B too:

$$NES(A,B) = Gr_1(A,B) \bigcap Gr_2(A,B).$$

Remark 3.1. *It is important to observe once again that the graphs of best responses $Gr_1(A,B)$ and $Gr_2(A,B)$ are multi-valued (set-valued) functions of the matrices A and B. As the Nash equilibrium set in a particular game is determined as the intersection of these graphs, the Nash equilibrium set function (NES function) is a function of the matrices A and B. Such a function must have eight arguments corresponding to eight elements of the matrices A and B:*

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad A \in \mathbb{R}^{2\times 2},$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad B \in \mathbb{R}^{2\times 2}.$$

Because of the initial game Γ' simplification to the game Γ, we define both the graphs of best response mappings and the NES function as the functions of the four arguments: α, α_0, and β, β_0, the formulas for which are defined above as the functions of the eight matrix elements.

Now, according to the above remark, let us consider the problem of a Nash equilibrium set computing as a problem of defining a Nash equilibrium set function with the payoff matrices A and B (or α, α_0, and β, β_0) as their arguments, i.e. let us consider analytically the problem of a NES function definition and its computing. In the same context, let us consider the graphs of best response mappings as the functions of α, α_0, and β, β_0.

$$\mathrm{g1}(\alpha, \alpha_0) = \left\{ (x,y) \in [0,1] \times [0,1] : \begin{array}{l} y \in [0,1] \\ x \in \operatorname{Arg} \max\limits_{x \in [0,1]} (\alpha y - \alpha_0)\, x \end{array} \right\},$$

$$\mathrm{g2}(\beta, \beta_0) = \left\{ (x,y) \in [0,1] \times [0,1] : \begin{array}{l} x \in [0,1] \\ y \in \operatorname{Arg} \max\limits_{y \in [0,1]} (\beta x - \beta_0)\, y \end{array} \right\}.$$

Optimal solutions of the optimization problems in the expressions for $\mathrm{g1}(\alpha, \alpha_0)$ and $\mathrm{g2}(\beta, \beta_0)$ are attained either on one of the extremities, or on the whole segment $[0,1]$. As the results depend of both the values

of $\mathrm{g1}(\alpha,\alpha_0)$ and $\mathrm{g2}(\beta,\beta_0)$, we can establish exactly expressions for these graphs as piecewise functions.

The graph of best response mapping of the first player is

$$\mathrm{g1}(\alpha,\alpha_0)=\begin{cases} [0,1]\times[0,1] & \text{if } \alpha=0\&\alpha_0=0,\\ [[1,0],[1,1]] & \text{if } (\alpha\geq 0\&\alpha 0>0)\| \\ & \quad (\alpha<0\&\alpha+\alpha 0>0),\\ [[0,0],[0,1]] & \text{if } (\alpha\leq 0\&\alpha 0<0)\| \\ & \quad (\alpha>0\&\alpha+\alpha 0<0),\\ [[0,0],[1,0]]\bigcup[[1,0],[1,1]] & \text{if } \alpha>0\&\alpha 0=0,\\ \left[[0,0],\left[0,-\frac{\alpha 0}{\alpha}\right]\right]\bigcup & \text{if } \alpha>0\&\alpha 0<0\& \\ \left[\left[0,-\frac{\alpha 0}{\alpha}\right],\left[1,-\frac{\alpha 0}{\alpha}\right]\right]\bigcup & \quad \alpha+\alpha 0>0,\\ \left[\left[1,-\frac{\alpha 0}{\alpha}\right],[1,1]\right] & \\ [[0,0],[0,1]]\bigcup[[0,1],[1,1]] & \text{if } \alpha>0\&\alpha+\alpha 0=0,\\ [[1,0],[0,0]]\bigcup[[0,0],[0,1]] & \text{if } \alpha<0\&\alpha 0=0,\\ \left[[1,0],\left[1,-\frac{\alpha 0}{\alpha}\right]\right]\bigcup & \text{if } \alpha<0\&\alpha 0>0\& \\ \left[\left[1,-\frac{\alpha 0}{\alpha}\right],\left[0,-\frac{\alpha 0}{\alpha}\right]\right]\bigcup & \quad \alpha+\alpha 0<0,\\ \left[\left[0,-\frac{\alpha 0}{\alpha}\right],[0,1]\right] & \\ [[1,0],[1,1]]\bigcup[[1,1],[0,1]] & \text{if } \alpha<0\&\alpha+\alpha 0=0. \end{cases}$$

Remark 3.2. *Above, we denoted the unit square by* $[0,1]\times[0,1]$ *and the segment which connects two points, e.g.* $[0,0]$ *and* $[1,0]$*, by an expression of the type* $[[0,0],[1,0]]$.

Remark 3.3. *From the expression for the graph* $\mathrm{g1}(\alpha,\alpha_0)$*, we can conclude that the set of its values or the image of the function* $\mathrm{g1}(\alpha,\alpha_0)$ *is formed by a union of the following four alternatives:*

a unit square,

a unit segment,

a union of two connected segments on the boundary of the unit square (one horizontal and one vertical),

a union of three connected segments (two vertical, on the opposite vertical sides of the unit square, connected by the third interior horizontal segment, from one lateral side to the other).

It is important to observe that the condition specified by the concrete values of α and $\alpha 0$ corresponds to an entire class of matrices $A \in \mathbb{R}^{2\times 2}$. More the more, accordingly to the formulas that define α and α_0, the function $\mathrm{g1}(\alpha, \alpha_0)$ is defined on entire space $\mathbb{R}^{2\times 2}$, i.e. it is defined for any numeric dyadic matrix A.

The graph of best response mapping of the second player is

$$\mathrm{g2}(\beta, \beta_0) = \begin{cases} [0,1] \times [0,1] & \text{if } \beta = 0 \& \beta_0 = 0, \\ [[0,1],[1,1]] & \text{if } (\beta \geq 0 \& \beta 0 > 0) \| \\ & \quad (\beta < 0 \& \beta + \beta 0 > 0), \\ [[0,0],[1,0]] & \text{if } (\beta \leq 0 \& \beta 0 < 0) \| \\ & \quad (\beta > 0 \& \beta + \beta 0 < 0), \\ [[0,0],[0,1]] \bigcup [[0,1],[1,1]] & \text{if } \beta > 0 \& \beta 0 = 0, \\ \left[[0,0], \left[-\frac{\beta 0}{\beta}, 0\right]\right] \bigcup & \text{if } \beta > 0 \& \beta 0 < 0 \& \\ \left[\left[-\frac{\beta 0}{\beta}, 0\right], \left[-\frac{\beta 0}{\beta}, 1\right]\right] \bigcup & \quad \beta + \beta 0 > 0, \\ \left[\left[-\frac{\beta 0}{\beta}, 1\right], [1,1]\right] & \\ [[0,0],[1,0]] \bigcup [[1,0],[1,1]] & \text{if } \beta > 0 \& \beta + \beta 0 = 0, \\ [[0,1],[0,0]] \bigcup [[0,0],[1,0]] & \text{if } \beta < 0 \& \beta 0 = 0, \\ \left[[0,1], \left[-\frac{\beta 0}{\beta}, 1\right]\right] \bigcup & \text{if } \beta < 0 \& \beta 0 > 0 \& \\ \left[\left[-\frac{\beta 0}{\beta}, 1\right], \left[-\frac{\beta 0}{\beta}, 0\right]\right] \bigcup & \quad \beta + \beta 0 < 0, \\ \left[\left[-\frac{\beta 0}{\beta}, 0\right], [1,0]\right] & \\ [[0,1],[1,1]] \bigcup [[1,1],[1,0]] & \text{if } \beta < 0 \& \beta + \beta 0 = 0. \end{cases}$$

For the graph $\mathrm{g2}(\beta, \beta_0)$ analogical conclusions are valid as for the graph $\mathrm{g1}(\alpha, \alpha_0)$.

4 Nash equilibria and NES function

As every graph of best response mappings has 9 different possible forms for particular instances of the payoff matrices A and B, their intersection abstractly may generate 81 possible instances/cases of Nash equilibrium sets. Some of them may coincide.

Next theorem has to highlight distinct cases in the form of the NES function definition on the matrices A and B. Remark once again that such a function is a set-valued function.

Theorem 4.1. *In the dyadic mixed strategy game, the Nash equilibrium set function* $nes[\alpha, \alpha 0, \beta, \beta 0]$ *may be defined as a piecewise function formed by 36 distinct pieces.*

Proof. The proof is constructive. It enumerates distinct possible cases in the form of the Wolfram language code.

First, the best response mapping graphs are defined as the Wolfram language functions.

Second, the NES function is defined as the Wolfram language function too.

In the end, we present a Wolfram language code to manipulate all the elements together and to highlight results for different initial data. The Wolfram language primitives, such as Rectangle, Line, Point, have clear meanings and are used as a simple language means to expose both the proof, and a Mathematica 10.4.1 program.

```
g1[α_, α0_]:=Piecewise[{
{Rectangle[{{0,0}, {1,1}}], α == 0&&α0==0},
{Line[{{1,0},{1,1}}],(α >= 0&&α0 > 0)||(α < 0&&α + α0 > 0)},
{Line[{{0,0},{0,1}}],(α <= 0&&α0 < 0)||(α > 0&&α + α0 < 0)},
{Line[{{0,0},{1,0},{1,1}}], α > 0&&α0 == 0},
{Line[{{0,0}, {0,-α0/α}, {1,-α0/α}, {1,1}}], α > 0&&α0 < 0&&α + α0 > 0},
{Line[{{0,0}, {0,1}, {1,1}}], α > 0&&α + α0 == 0},
{Line[{{1,0}, {0,0}, {0,1}}], α < 0&&α0 == 0},
{Line[{{1,0},{1,-α0/α}, {0, -α0/α}, {0, 1}}], α < 0&&α0 > 0&&α + α0 < 0},
{Line[{{1,0},{1,1},{0,1}}], α < 0&&α + α0 == 0}
}]
```

```
g2[β_, β0_]:= Piecewise[{
{Rectangle[{{0,0},{1,1}}], β == 0&&β0==0},
{Line[{{0,1},{1,1}}], (β >= 0&&β0 > 0)||(β < 0&&β + β0 > 0)},
{Line[{{0,0},{1,0}}], (β <= 0&&β0 < 0)||(β > 0&&β + β0 < 0)},
{Line[{{0,0},{0,1},{1,1}}], β > 0&&β0 == 0},
{Line[{{0,0}, {-β0/β, 0}, {-β0/β,1},{1,1}}], β > 0&&β0 < 0&&β + β0 > 0},
{Line[{{0,0},{1,0},{1,1}}], β > 0&&β + β0 == 0},
{Line[{{0,1},{0,0},{1,0}}], β < 0&&β0 == 0},
{Line[{{0,1},{-β0/β, 1}, {-β0/β, 0}, {1, 0}}], β < 0&&β0 > 0&&β + β0 < 0},
{Line[{{0,1},{1,1},{1,0}}], β < 0&&β + β0 == 0}
}]

nes[α_, α0_, β_, β0_]:=Piecewise[{

(*1*) { {Point[{{0,0},{1,0},{1,1},{0,1}}],Rectangle[{0,0},{1,1}]},
    (α == 0&&α0 == 0)&&(β == 0&&β0==0) },

(*2*) { {Point[{{0,1},{1,1}}],Line[{{0,1},{1,1}}]},
    (α == 0&&α0 == 0)&&((β ≥ 0&&β0 > 0)||
(β < 0&&β + β0 > 0)))||((α > 0&&α + α0 == 0)&&((β ≥ 0&&β0 > 0)||
(β < 0&&β + β0 > 0)))||
((α > 0&&α + α0 == 0)&&(β < 0&&β + β0 == 0))||
((α < 0&&α +  α0 == 0)&&(β > 0&&β0 == 0))||
((α < 0&&α + α0 == 0)&&((β ≥ 0&&β0 > 0)||
(β < 0&&β + β0 > 0))) },

(*3*) { {Point[{{0,0},{1,0}}],Line[{{0,0},{1,0}}]},
    ((α == 0&&α0 == 0)&&((β ≤ 0&&β0 < 0)||
(β > 0&&β + β0 < 0)))||
((α > 0&&α0 == 0)&&((β ≤ 0&&β0 < 0)||(β > 0&&β + β0 < 0)))||
((α > 0&&α0 == 0)&&(β < 0&&β0 == 0))||
((α < 0&&α0 == 0)&&(β > 0&&β + β0 == 0))||
((α < 0&&α0 == 0)&&((β ≤ 0&&β0 < 0)||(β > 0&&β + β0 < 0))) },

(*4*) { {Point[{{0,0},{0,1},{1,1}}],Line[{{0,0},{0,1},{1,1}}]},
    ((α == 0&&α0 == 0)&&(β > 0&&β0 == 0))||
((α > 0&&α + α0 == 0)&&(β == 0&&β0 == 0))||
((α > 0&&α + α0 == 0)&&(β > 0&&β0 == 0)) },

(*5*) { {Point[g2[β, β0][[1]]], g2[β, β0]},
```

$((\alpha == 0\&\&\alpha 0 == 0)\&\&(\beta > 0\&\&\beta 0 < 0\&\&\beta + \beta 0 > 0))\|$
$((\alpha == 0\&\&\alpha 0 == 0)\&\&(\beta < 0\&\&\beta 0 > 0\&\&\beta + \beta 0 < 0))$ },

(*6*) { {Point[{{0,0},{1,0},{1,1}}],Line[{{0,0},{1,0},{1,1}}]},
$((\alpha == 0\&\&\alpha 0 == 0)\&\&(\beta > 0\&\&\beta + \beta 0 == 0))\|$
$((\alpha > 0\&\&\alpha 0 == 0)\&\&(\beta == 0\&\&\beta 0 == 0))\|$
$((\alpha > 0\&\&\alpha 0 == 0)\&\&(\beta > 0\&\&\beta + \beta 0 == 0))$ },

(*7*) { {Point[{{0,1},{0,0},{1,0}}],Line[{{0,1},{0,0},{1,0}}]},
$((\alpha == 0\&\&\alpha 0 == 0)\&\&(\beta < 0\&\&\beta 0 == 0))\|$
$((\alpha < 0\&\&\alpha 0 == 0)\&\&(\beta == 0\&\&\beta 0 == 0))\|$
$((\alpha < 0\&\&\alpha 0 == 0)\&\&(\beta < 0\&\&\beta 0 == 0))$ },

(*8*) { {Point[{{0,1},{1,1},{1,0}}],Line[{{0,1},{1,1},{1,0}}]},
$((\alpha == 0\&\&\alpha 0 == 0)\&\&(\beta < 0\&\&\beta + \beta 0 == 0))\|$
$((\alpha < 0\&\&\alpha + \alpha 0 == 0)\&\&(\beta == 0\&\&\beta 0 == 0))\|$
$((\alpha < 0\&\&\alpha + \alpha 0 == 0)\&\&(\beta < 0\&\&\beta + \beta 0 == 0))$ },

(*9*) { {Point[{{1,0},{1,1}}],Line[{{1,0},{1,1}}]},
$(((\alpha \geq 0\&\&\alpha 0 > 0)\|$
$(\alpha < 0\&\&\alpha + \alpha 0 > 0))\&\&(\beta == 0\&\&\beta 0 == 0))\|$
$(((\alpha \geq 0\&\&\alpha 0 > 0)\|(\alpha < 0\&\&\alpha + \alpha 0 > 0))\&\&(\beta > 0\&\&\beta + \beta 0 == 0))\|$
$(((\alpha \geq 0\&\&\alpha 0 > 0)\|(\alpha < 0\&\&\alpha + \alpha 0 > 0))\&\&(\beta < 0\&\&\beta + \beta 0 == 0))\|$
$((\alpha > 0\&\&\alpha 0 == 0)\&\&(\beta < 0\&\&\beta + \beta 0 == 0))\|$
$((\alpha < 0\&\&\alpha + \alpha 0 == 0)\&\&(\beta > 0\&\&\beta + \beta 0 == 0))$ },

(*10*) { {Point[{1,1}]},
$(((\alpha \geq 0\&\&\alpha 0 > 0)\|(\alpha < 0\&\&\alpha + \alpha 0 > 0))\&\&((\beta \geq 0\&\&\beta 0 > 0)\|$
$(\beta < 0\&\&\beta + \beta 0 > 0)))\|(((\alpha \geq 0\&\&\alpha 0 > 0)\|$
$(\alpha < 0\&\&\alpha + \alpha 0 > 0))\&\&(\beta > 0\&\&\beta 0 == 0))\|(((\alpha \geq 0\&\&\alpha 0 > 0)\|$
$(\alpha < 0\&\&\alpha + \alpha 0 > 0))\&\&(\beta > 0\&\&\beta 0 < 0\&\&\beta + \beta 0 > 0))\|$
$((\alpha > 0\&\&\alpha 0 == 0)\&\&((\beta \geq 0\&\&\beta 0 > 0)\|(\beta < 0\&\&\beta + \beta 0 > 0)))\|$
$((\alpha > 0\&\&\alpha 0 < 0\&\&\alpha + \alpha 0 > 0)\&\&((\beta \geq 0\&\&\beta 0 > 0)\|$
$(\beta < 0\&\&\beta + \beta 0 > 0)))$ },

(*11*) { {Point[{1,0}]},
$(((\alpha \geq 0\&\&\alpha 0 > 0)\|(\alpha < 0\&\&\alpha + \alpha 0 > 0))\&\&((\beta \leq 0\&\&\beta 0 < 0)\|$
$(\beta > 0\&\&\beta + \beta 0 < 0)))\|(((\alpha \geq 0\&\&\alpha 0 > 0)\|$
$(\alpha < 0\&\&\alpha + \alpha 0 > 0))\&\&(\beta < 0\&\&\beta 0 == 0))\|(((\alpha \geq 0\&\&\alpha 0 > 0)\|$
$(\alpha < 0\&\&\alpha + \alpha 0 > 0))\&\&(\beta < 0\&\&\beta 0 > 0\&\&\beta + \beta 0 < 0))\|$
$((\alpha < 0\&\&\alpha + \alpha 0 == 0)\&\&((\beta \leq 0\&\&\beta 0 < 0)\|(\beta > 0\&\&\beta + \beta 0 < 0)))\|$

$((\alpha < 0\&\&\alpha 0 > 0\&\&\alpha + \alpha 0 < 0)\&\&((\beta \leq 0\&\&\beta 0 < 0)\|$
$(\beta > 0\&\&\beta + \beta 0 < 0)))$ },

(*12*) { {Point[{{0,0},{0,1}}],Line[{{0,0},{0,1}}]},
$(((\alpha \leq 0\&\&\alpha 0 < 0)\|$
$(\alpha > 0\&\&\alpha + \alpha 0 < 0))\&\&(\beta == 0\&\&\beta 0 == 0))\|$
$(((\alpha \leq 0\&\&\alpha 0 < 0)\|(\alpha > 0\&\&\alpha + \alpha 0 < 0))\&\&(\beta > 0\&\&\beta 0 == 0))\|$
$(((\alpha \leq 0\&\&\alpha 0 < 0)\|(\alpha > 0\&\&\alpha + \alpha 0 < 0))\&\&(\beta < 0\&\&\beta 0 == 0))\|$
$((\alpha > 0\&\&\alpha + \alpha 0 == 0)\&\&(\beta < 0\&\&\beta 0 == 0))\|$
$((\alpha < 0\&\&\alpha 0 == 0)\&\&(\beta > 0\&\&\beta 0 == 0))$ },

(*13*) { {Point[{0,1}]},
$(((\alpha \leq 0\&\&\alpha 0 < 0)\|(\alpha > 0\&\&\alpha + \alpha 0 < 0))\&\&((\beta \geq 0\&\&\beta 0 > 0)\|$
$(\beta < 0\&\&\beta + \beta 0 > 0)))\|(((\alpha \leq 0\&\&\alpha 0 < 0)\|$
$(\alpha > 0\&\&\alpha + \alpha 0 < 0))\&\&(\beta < 0\&\&\beta 0 > 0\&\&\beta + \beta 0 < 0))\|$
$(((\alpha \leq 0\&\&\alpha 0 < 0)\|(\alpha > 0\&\&\alpha + \alpha 0 < 0))\&\&(\beta < 0\&\&\beta + \beta 0 == 0))\|$
$((\alpha < 0\&\&\alpha 0 == 0)\&\&((\beta \geq 0\&\&\beta 0 > 0)\|(\beta < 0\&\&\beta + \beta 0 > 0)))\|$
$((\alpha < 0\&\&\alpha 0 > 0\&\&\alpha + \alpha 0 < 0)\&\&((\beta \geq 0\&\&\beta 0 > 0)\|$
$(\beta < 0\&\&\beta + \beta 0 > 0)))$ },

(*14*) { {Point[{0,0}]},
$(((\alpha \leq 0\&\&\alpha 0 < 0)\|$
$(\alpha > 0\&\&\alpha + \alpha 0 < 0))\&\&(\beta > 0\&\&\beta + \beta 0 == 0))\|$
$(((\alpha \leq 0\&\&\alpha 0 < 0)\|(\alpha > 0\&\&\alpha + \alpha 0 < 0))\&\&((\beta \leq 0\&\&\beta 0 < 0)\|$
$(\beta > 0\&\&\beta + \beta 0 < 0)))\|(((\alpha \leq 0\&\&\alpha 0 < 0)\|$
$(\alpha > 0\&\&\alpha + \alpha 0 < 0))\&\&(\beta > 0\&\&\beta 0 < 0\&\&\beta + \beta 0 > 0))\|$
$((\alpha > 0\&\&\alpha 0 < 0\&\&\alpha + \alpha 0 > 0)\&\&((\beta \leq 0\&\&\beta 0 < 0)\|$
$(\beta > 0\&\&\beta + \beta 0 < 0)))\|((\alpha > 0\&\&\alpha + \alpha 0 == 0)\&\&((\beta \leq 0\&\&\beta 0 < 0)\|$
$(\beta > 0\&\&\beta + \beta 0 < 0)))$ },

(*15*) { {Point[{{0,0},{1,1}}]},
$((\alpha > 0\&\&\alpha 0 == 0)\&\&(\beta > 0\&\&\beta 0 == 0))\|$
$((\alpha > 0\&\&\alpha + \alpha 0 == 0)\&\&(\beta > 0\&\&\beta + \beta 0 == 0))$ },

(*16*) { {Point[{{0,0},$\{-\frac{\beta 0}{\beta}, 0\}, \{1, 1\}\}$], $Line[\{\{0, 0\}, \{-\frac{\beta 0}{\beta}, 0\}\}]$},
$(\alpha > 0\&\&\alpha 0 == 0)\&\&(\beta > 0\&\&\beta 0 < 0\&\&\beta + \beta 0 > 0)$ },

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(*18*) { {Point[g1[$\alpha, \alpha 0$][[1]]], g1[$\alpha, \alpha 0$]},

$((\alpha > 0\&\&\alpha 0 < 0\&\&\alpha + \alpha 0 > 0)\&\&(\beta == 0\&\&\beta 0 == 0))\|$
$((\alpha < 0\&\&\alpha 0 > 0\&\&\alpha + \alpha 0 < 0)\&\&(\beta == 0\&\&\beta 0 == 0))$ },

(*19*) { {Point[{{0,0},{0,$-\frac{\alpha 0}{\alpha}$},{1,1}}], Line[{{0,0},{0,$-\frac{\alpha 0}{\alpha}$}}]},
$(\alpha > 0\&\&\alpha 0 < 0\&\&\alpha + \alpha 0 > 0)\&\&(\beta > 0\&\&\beta 0 == 0)$ },

(*20*) { {Point[{{0,0},{$-\frac{\beta 0}{\beta}$, $-\frac{\alpha 0}{\alpha}$},{1,1}}]},
$(\alpha > 0\&\&\alpha 0 < 0\&\&\alpha + \alpha 0 > 0)\&\&$
$(\beta > 0\&\&\beta 0 < 0\&\&\beta + \beta 0 > 0)$ },

(*21*) { {Point[{{0,0},{1,$-\frac{\alpha 0}{\alpha}$},{1,1}}], Line[{{1,$-\frac{\alpha 0}{\alpha}$},{1,1}}]},
$(\alpha > 0\&\&\alpha 0 < 0\&\&\alpha + \alpha 0 > 0)\&\&(\beta > 0\&\&\beta + \beta 0 == 0)$ },

(*22*) { {Point[{{0,0},{0,$-\frac{\alpha 0}{\alpha}$}}], Line[{{0,0},{0,$-\frac{\alpha 0}{\alpha}$}}]},
$(\alpha > 0\&\&\alpha 0 < 0\&\&\alpha + \alpha 0 > 0)\&\&(\beta < 0\&\&\beta 0 == 0)$ },

(*23*) { {Point[{{$-\frac{\beta 0}{\beta}$, $-\frac{\alpha 0}{\alpha}$}}]},
$((\alpha > 0\&\&\alpha 0 < 0\&\&\alpha + \alpha 0 > 0)\&\&$
$(\beta < 0\&\&\beta 0 > 0\&\&\beta + \beta 0 < 0))\|$
$((\alpha < 0\&\&\alpha 0 > 0\&\&\alpha + \alpha 0 < 0)\&\&$
$(\beta > 0\&\&\beta 0 < 0\&\&\beta + \beta 0 > 0))$ },

(*24*) { {Point[{{1,-$\frac{\alpha 0}{\alpha}$},{1,1}}], Line[{{1,$-\frac{\alpha 0}{\alpha}$},{1,1}}]},
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(*25*) { {Point[{{0,0},{$-\frac{\beta 0}{\beta}$,1},{1,1}}], Line[{{$-\frac{\beta 0}{\beta}$,1},{1,1}}]},
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(*26*) { {Point[{{0,1},{$-\frac{\beta 0}{\beta}$,1}}], Line[{{0,1},{$-\frac{\beta 0}{\beta}$,1}}]},
$(\alpha > 0\&\&\alpha + \alpha 0 == 0)\&\&(\beta < 0\&\&\beta 0 > 0\&\&\beta + \beta 0 < 0)$ },

(*27*) { {Point[{{0,0},{$-\frac{\beta 0}{\beta}$,0}}], Line[{{0,0},{$-\frac{\beta 0}{\beta}$,0}}]},
$(\alpha < 0\&\&\alpha 0 == 0)\&\&(\beta > 0\&\&\beta 0 < 0\&\&\beta + \beta 0 > 0)$ },

(*28*) { {Point[{{$-\frac{\beta 0}{\beta}$,0}, {1,0},{0,1}}], Line[{{$-\frac{\beta 0}{\beta}$,0}, {1,0}}]},
$(\alpha < 0\&\&\alpha 0 == 0)\&\&(\beta < 0\&\&\beta 0 > 0\&\&\beta + \beta 0 < 0)$ },

(*29*) { {Point[{{1,0},{0,1}}]},
$((\alpha < 0\&\&\alpha 0 == 0)\&\&(\beta < 0\&\&\beta + \beta 0 == 0))\|$
$((\alpha < 0\&\&\alpha + \alpha 0 == 0)\&\&(\beta < 0\&\&\beta 0 == 0))$ },

(*30*) { {Point[{{0,$-\frac{\alpha 0}{\alpha}$},{0,1}}], Line[{{0,$-\frac{\alpha 0}{\alpha}$},{0,1}}]},

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(*31*) { {Point[{{1,−α0/α},{1,0}}], Line[{{1,−α0/α},{1,0}}]},
    (α < 0&&α0 > 0&&α + α0 < 0)&&(β > 0&&β + β0 == 0) },

(*32*) { {Point[{{0,−α0/α},{0,1},{1,0} }], Line[{{0,−α0/α},{0,1}}]},
    (α < 0&&α0 > 0&&α + α0 < 0)&&(β < 0&&β0 == 0) },

(*33*) { {Point[{{0,1},{−β0/β, −α0/α},{1,0}}]},
    (α < 0&&α0 > 0&&α + α0 < 0)&&
(β < 0&&β0 > 0&&β + β0 < 0) },

(*34*) { {Point[{{1,−α0/α},{1,0},{0,1}}], Line[{{1,−α0/α},{1,0}}]},
    (α < 0&&α0 > 0&&α + α0 < 0)&&(β < 0&&β + β0 == 0) },

(*35*) { {Point[{{−β0/β,1},{1,1}}], Line[{{−β0/β,1},{1,1}}]},
    (α < 0&&α + α0 == 0)&&(β > 0&&β0 < 0&&β + β0 > 0) },

(*36*) { {Point[{{0,1},{−β0/β,1},{1,0}}], Line[{{0,1},{−β0/β,1}}]},
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TableHeadings → {{"1","2"},{" 1"," 2"}}, TableSpacing →{2,2}]],
SaveDefinitions → True]
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As the enumerated 36 cases include all the 81 abstractly possible cases of the results of graph intersections, the proof is complete. □

Even though we establish the 36 different forms/cases needed to define exhaustively the Nash equilibrium set function, we can summarize them and can conclude additionally that a Nash equilibrium set may be:

1. **a border point** as one of the vertices of the square (cases: 10, 11, 13, 14),

2. **an interior point** of the square (cases: 5, 23),

3. **two border points** as two opposite vertices of the square (cases: 15, 29),

4. **a unit border segment** as one of the sides of the square (cases: 2, 3, 9, 12),

5. **two unit border segments** as two connected sides of the square (one vertical and one horizontal) (cases: 4, 6, 7, 8),

6. **a union of one point and one non-unit segment** as one vertex of the square and one non-unit segment on opposite side of the square (cases: 16, 19, 21, 25, 28, 32, 34, 36),

7. **one non-unit segment** as a segment on one of the sides of the square (cases: 17, 22, 24, 26, 27, 30, 31, 35),

8. **a graph** of one of the players as a union of three connected segments (case 18),

9. **three distinct points** as two corner opposite vertices of the square and one interior point (cases: 20, 33),

10. **a unit square** (case 1).

Corollary 4.1. *Nash equilibrium set in dyadic mixed strategy games may be formed by*

1. *a point,*

2. *two points,*

3. *three points,*

4. *a segment,*

5. *two connected segments,*

6. *three connected segments,*

7. *union of non-connected one point and one segment,*

8. *the unit square.*

5 Conclusions

For the dyadic mixed strategy games we developed an analytic method for Nash equilibrium set computing as the value of the NES function. The function is defined by a formula in the proof of theorem 4.1 applying a Wolfram language function definition. The corollary summarizes all the results in a simple and useful statement.

A preceding version of the proposed approach/algorithm was realized in the Wolfram language too. It was published in the Wolfram Demonstrations Project [15]. That code may be freely viewed, verified and downloaded from the address [15].

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Performance Evaluation of TDMA Medium Access Control Protocol in Cognitive Wireless Networks*

Muhammed Enes Bayrakdar, Ali Calhan

Abstract

Cognitive radio paradigm has been revealed as a new communication technology that shares channels in wireless networks. Channel assignment is a crucial issue in the field of cognitive wireless networks because of the spectrum scarcity. In this work, we have evaluated the performance of TDMA medium access control protocol. In our simulation scenarios, primary users and secondary users utilize TDMA as a medium access control protocol. We have designed a network environment in Riverbed simulation software that consists of primary users, secondary users, and base stations. In our system model, secondary users sense the spectrum and inform the base station about empty channels. Then, the base station decides accordingly which secondary user may utilize the empty channel. Energy detection technique is employed as a spectrum sensing technique because it is the best when information about signal of primary user is acquired. Besides, different number of users is selected in simulation scenarios in order to obtain accurate delay and throughput results. Comparing analytical model with simulation results, we have shown that performance analysis of our system model is consistent and accurate.

Keywords: Cognitive radio, delay analysis, TDMA, wireless networks.

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1 Introduction

In recent years, the usage of wireless communication technologies has been growing rapidly with the increase of wireless devices and services [1]. Due to the increasing number of users and fixed spectrum allocation strategies, spectrum scarcity problem has revealed [2]. Cognitive radio is a new wireless communication technology that aims to utilize the spectrum in a more dynamic and effective way [3]. In cognitive radio networks, secondary users exploit spectrum holes temporarily without causing any interference to the primary users. Secondary users use various signal detection techniques and algorithms in order to sense the unutilized portions of the spectrum [4]. Among spectrum sensing techniques, energy detection is widely preferred due to not requiring any prior knowledge about signal of the primary user [5].

Several medium access control protocols have been described in the literature to regulate access [6]. They mainly are grouped into two categories, namely distributed and centralized. In the distributed approach, each node decides on its own when is the best time for the transmission process [7]. If multiple users transmit, then these protocols must provide mechanisms to resolve the collision [8]. In the centralized approach, a base station is responsible for deciding who can access the channel [9]. The base station allocates a time slot for every user in turn.

In this work, performance analysis of TDMA (Time Division Multiple Access) based wireless cognitive radio network is examined. The proposed network structure is composed of two different groups of users including primary and secondary users. Both groups of users employ TDMA as a medium access control technique. All of the users share time slotted based common communication channel. Primary and secondary users communicate through primary base station and secondary base station correspondingly.

Because primary users are licensed users, they have a right to use the spectrum any time. Secondary users only access to the spectrum when action of primary users is not sensed in the spectrum. Simulation scenarios have been developed using Riverbed simulation software. In

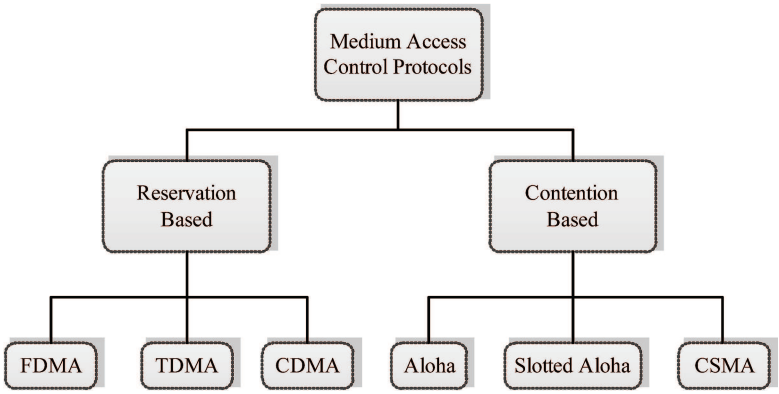

Figure 1. Classification of medium access control protocols

order to evaluate performance of our network model; end to end delay analysis, throughput analysis and capacity analysis are carried out. Besides, equations for throughput and delay characteristics of TDMA protocol are derived.

2 Medium access control protocols in cognitive radio networks

Multiple access techniques allow an access to the limited bandwidth of the spectrum for multiple users at the same time effectively [10]. Because available spectrum is limited for wireless communications, spectrum needs to be allocated efficiently [11]. Medium access control protocols are employed to utilize the limited bandwidth effectively by means of reservation based and contention based techniques [12].

In Figure 1, classification of medium access control protocols in wireless cognitive radio networks is shown. FDMA (Frequency Division Multiple Access), TDMA and CDMA (Code Division Multiple Access) are reservation based protocols while Aloha, Slotted Aloha and CSMA (Carrier Sense Multiple Access) are contention based protocols.

2.1 Related works

In recent years, a lot of researches about cognitive radio and effective spectrum access are made. Li et al. proposed a new Slotted Aloha based distributed access cognitive radio network [11]. In their study, secondary users select a random set of channels to sense, and then perform the transmission process after detecting an idle channel. They have also confirmed the obtained analytical results with some important simulation models. Choe proposed a Slotted Aloha based multichannel cognitive radio network [4]. In his work, delay analysis of the network model has been carried out. While calculating end to end delay, spectrum sensing time is not taken into account in his study.

Lien et al. presented a new CSMA based protocol in cognitive radio networks [12]. In their study, secondary users employ CSMA based medium access technique that is different from conventional network systems. For the proposed network model, delay analysis is also calculated. When compared to the conventional CSMA based network systems, throughput performance of the proposed system is significantly increased.

Yang examined design of medium access protocols for primary users and secondary users in cognitive radio networks [13]. In his work, primary users employ Slotted Aloha technique while secondary users utilize Slotted Aloha and CSMA techniques as a medium access control protocol in order to exploit idle time slots of primary users. In addition, the average delay performance and throughput performance of the network model are examined. All analytical results obtained are verified with the simulation results in his work.

Choe and Park presented a Slotted Aloha based new protocol for multichannel cognitive radio networks [5]. In addition, the idle channel selection based protocols are confirmed to have better performance than the random channel selection protocols in their study. Moreover, Slotted Aloha technique has shown to give better results than Aloha technique. However, spectrum sensing time is ignored in their delay and throughput equations.

Different than studies in the literature, spectrum sensing time and

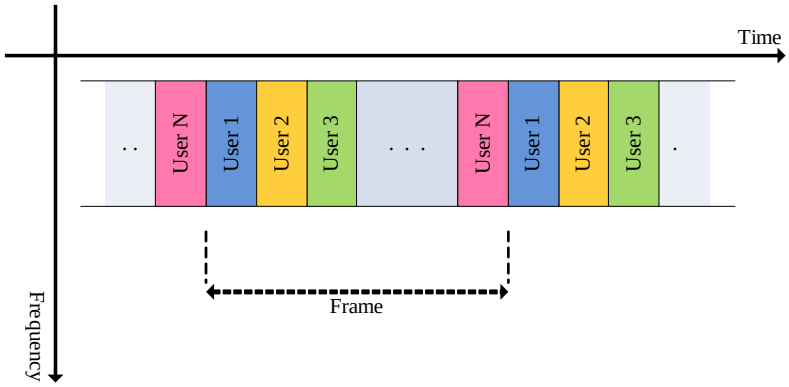

Figure 2. TDMA frame structure in wireless cognitive radio networks

probability of missed detection parameters are taken into consideration in our network models. Spectrum sensing time is described as a duration secondary users spend time for finding an idle spectrum. Probability of missed detection is expressed as detecting a supposed idle channel when it actually is busy. Opposite to the traditional networks, probability of missed detection decreases the throughput of the cognitive radio networks.

2.2 Reservation based medium access control protocols

One of the best advantages of reservation based medium access control protocols is that they prevent collision by allocating bandwidths to users [14]. Reservation based medium access techniques are mainly examined under three different sections, namely FDMA, TDMA and CDMA [15].

In Figure 2, TDMA frame structure in cognitive radio networks is shown. In TDMA systems, the radio spectrum is divided into time slots and each user sends or receives packets in their own time slots. If this system is applied to certain time duration, multiple users on the same spectrum and in different time slots may perform the transmission operation independently.

In TDMA structure, each resource in the spectrum is sampled in order to obtain data series in the environment [16]. Sampling frequency is the frequency of the user which is changing frequency band most rapidly and selected according to the sampling theorem [13]. It also has features similar to all the information sources used in the same sampling frequency [17].

In TDMA medium access control protocol, if resources with different characteristics of the spectrum are available, these resources are joined together to have better performance [13]. The data rate is directly related to the bandwidth of the spectrum [18]. If the bandwidth of the transmission medium is high, then it is possible to achieve very high data rates [3].

3 System model of the network

The Riverbed (OPNET) Modeler simulation software includes several tools such as designing, simulation, and data aggregation [19]. In Riverbed software, performance of a simulation model is evaluated by means of discrete event simulations [19].

Configuration of the network model is carried out in three stages: namely network stage, node stage, and process stage. In the network stage, topology of the network is built. The node stage defines the characteristics of the node. The process stage is characterized by state machines that are created with states and transitions between states. The source code of Riverbed Modeler is written on C programming language [19].

Our network model composes of secondary users and the secondary base station in the same communication environment with the primary users and the primary base station. TDMA is employed as medium access control protocol by primary and secondary users [20]. The procedure of a secondary user in the network model is as follows. In the beginning of each time slot, every secondary user senses the spectrum in order to learn status of the time slots. If the time slot sensed is used by a primary user, then the secondary user waits for the availability of following time slots. If the time slot sensed is empty, then the

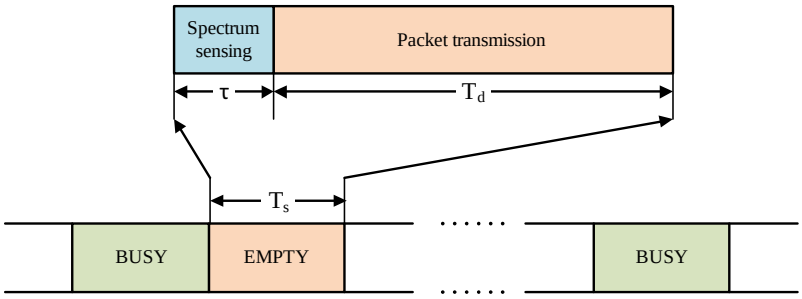

Figure 3. TDMA time slot structure used in the network model

secondary user transmits its packet [21].

Total number of channels that is supported simultaneously in a TDMA system is expressed with the following equation;

$$NC = \frac{m(B_t - 2B_g)}{B_c}, \tag{1}$$

where m is maximum number of users that can be supported by each frequency channel [3], [6], [18]. In (1), B_t denotes total spectrum allocation, B_g denotes guard band and B_c denotes channel bandwidth.

Throughput delay characteristics of TDMA protocol with different number of users is expressed as follows;

$$D = 1 + \frac{M}{2(1 - S)}, \tag{2}$$

where D, M and S represents delay, number of users and throughput correspondingly [1], [6], [10]. By means of (2), various throughput delay characteristics of TDMA may be obtained using different values. Expected end to end delay for TDMA scheme is calculated as dividing sum of delay in a time slot by number of slots in a frame [6].

In the network model, time slot structure shown in Figure 3 is used. Because the primary users employ TDMA technique as a medium access control protocol, the time axis is divided into fixed length of time slots. As shown in the figure, some of the time slots are used by primary users while some of them are empty. Time slots utilized by primary

users are called busy. By means of cognitive radio, secondary users may exploit empty time slots of primary network opportunistically. Busy and empty time slots of users are directly related to offered load of primary users. The rise of offered load reduces empty time slots while increasing the number of busy time slots.

The time slots of primary users are composed of two parts, namely spectrum sensing and packet transmission. In spectrum sensing stage, secondary users sense the signal of primary users without attempting to transmit. If the absence of primary users is detected, secondary users may send their packets in packet transmission stage. If the idle time slot is not found, secondary users continue to sense the next time slots.

3.1 System model of the primary network

In the communication area of the network model, there are a variable number of primary users and a primary base station. Primary users communicate via the base station using TDMA scheme. In the proposed primary network model, number of time slots in a frame is assumed to be equal to the number of primary users. Therefore, a unique time slot is allocated to each primary user for transmitting their packets.

Figure 4 shows simulation scenario designed for primary network on Riverbed software where primary users and the primary base station are illustrated. Characteristics of all the primary users in the network scenario are identical. Code of each designed user and the base station are written in Riverbed simulation software using C programming language.

Moreover, each primary user is presumed to have unlimited buffer. Packet length of primary users is equal to the time slot in a frame. Besides, each primary user is an independent Poisson source where λ_p represents the average number of packets produced per time slot. The total load offered to the network, G_p, is obtained by using the following expression:

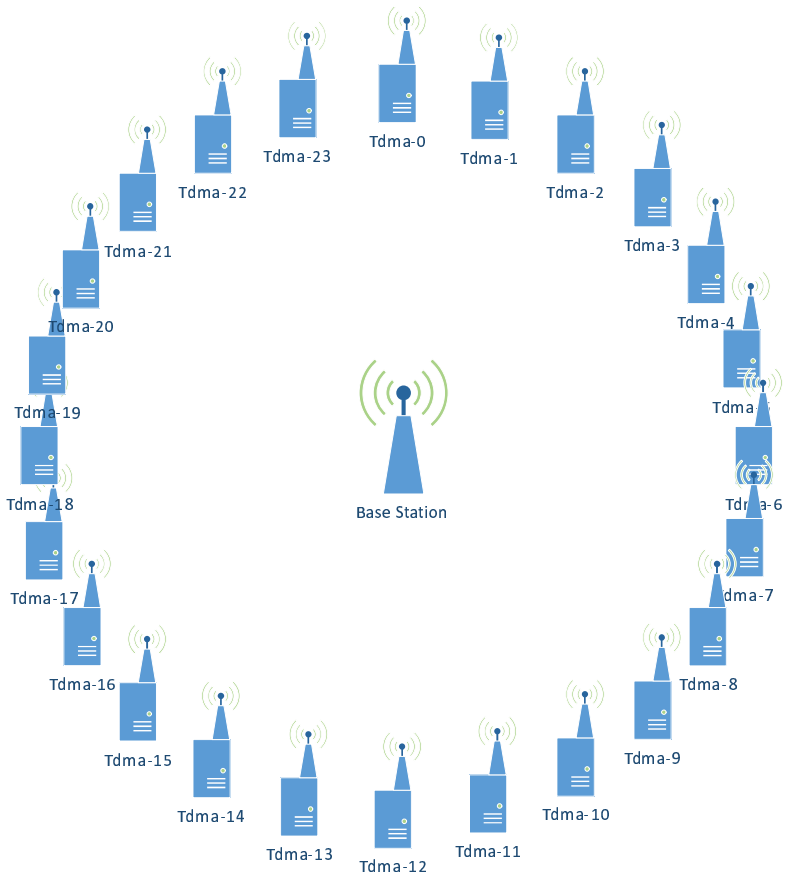

Figure 4. Primary network model in Riverbed simulation software

$$G_p = \lambda_p N_p, \tag{3}$$

where N_p is the number of primary users [2]. Since throughput of the network is one of the performance metric used in this work, throughput of the analytical model and simulation model is evaluated. Throughput is defined as the average number of packets successfully received by the base station in a given time. The number of packets sent through the network in a given time is called offered load of the network [2].

Since TDMA medium access scheme is not a contention-based protocol, TDMA packets do not collide with each other [8]. Therefore, when offered load of the network is less than 1, throughput equals to offered load in traditional networks. In the case where the load is equal to 1 or greater than 1, throughput is equal to 1.

However, the packets of the secondary users may collide with the packets of primary users because of the missed detection probability in cognitive radio networks [17]. Missed detection slightly decreases the throughput of the network. Taking missed detection into account, throughput of TDMA scheme for primary users in wireless cognitive radio networks is expressed as in (4).

$$S_p = \begin{cases} G_p(1 - P_m), G_p < 1 \\ (1 - P_m), G_p \geq 1 \end{cases} . \tag{4}$$

In (4), P_m indicates the probability of missed detection and it is obtained by subtracting detection probability from 1 [16]. S_p represents throughput of TDMA based primary network in wireless cognitive radio networks.

3.2 System model of the secondary network

In the communication area of the secondary network model, there are ten secondary users and a secondary base station. Secondary users communicate through the secondary base station using TDMA medium access control protocol.

In our secondary network model, it is assumed that each secondary user produces packets according to the Poisson process. Furthermore,

Table 1. Simulation parameters utilized for the secondary network

Parameter	**Value**
Time slot length of the primary users	100 ms
Number of secondary users	10
Receiving frequency	2.4 GHz
Transmitting frequency	1.85 GHz
Bandwidth	2 MHz
Modulation scheme	BPSK
Transmit power	0.1 W
Probability of detection	0.90
Probability of false alarm	0.10
Data rate	1 Mbps
Packet length of secondary users	10 Kb

the packet length of secondary users is shorter than that of primary users due to the effect of spectrum sensing time. Simulation parameters of the secondary users used in Riverbed software are given in Table 1.

Taking missed detection and false alarm into consideration, throughput of TDMA scheme for secondary users in wireless cognitive radio networks is computed as in (5);

$$S_s = \begin{cases} G_s(1 - P_m - P_{fa}), G_s < 1 \\ (1 - P_m - P_{fa}), G_s \geq 1 \end{cases}, \tag{5}$$

where P_{fa}, S_s and G_s denote probability of false alarm, throughput of secondary network and offered load of secondary users respectively. Different from throughput equation of primary network, throughput of secondary network is affected from probability of false alarm. In addition to probability of missed detection, probability of false alarm reduces throughput.

End to end delay characteristics of secondary users are expressed as follows taking spectrum sensing time into consideration;

$$D = 1 + \frac{M}{2(1-S)} + T_s, \tag{6}$$

where D, M and S denote delay, number of users and throughput respectively [1], [16], [20]. In (6), T_s denotes spectrum sensing time which reduces packet length of secondary users [21].

4 Analytical and simulation results

For our network simulation scenarios, we have evaluated throughput, capacity and end to end delay characteristics of TDMA scheme. All analytical results obtained from equations precisely match to the simulation results obtained from Riverbed simulation software [19].

4.1 Throughput characteristics of the network

In traditional TDMA scheme, throughput is proportional to the offered load. In cognitive radio networks, different parameters like missed detection may decrease throughput.

In Figure 5, comparison of TDMA scheme under different network conditions is made. In cognitive radio networks, throughput is lower due to the missed detection. Missed detection is defined as sensing the spectrum idle when it actually is utilized by primary users. When offered load is more than one, throughput remains about 0.9 in cognitive radio networks.

In Figure 6, network throughput values for primary network and secondary network are illustrated. Secondary network has lower throughput than primary network because of the false alarm parameter. False alarm is described as sensing the spectrum busy, when it actually is not used by primary users.

In Figure 7, channel capacity of TDMA medium access scheme is examined. In the analytical model of channel capacity, each frame is assumed to be divided into ten time slots. Accordingly, number of users supported by TDMA is acquired. Each frequency channel supports ten

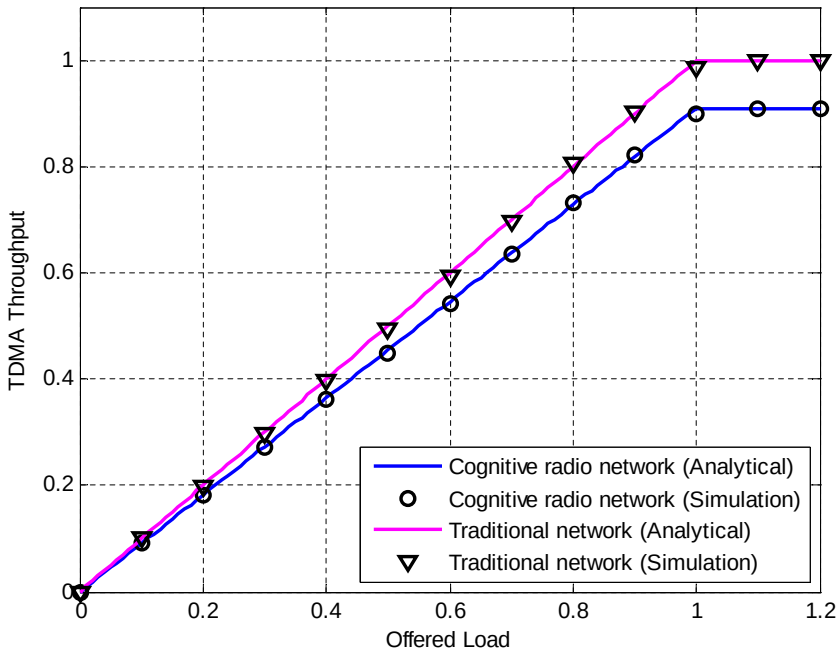

Figure 5. TDMA throughput analysis in traditional and cognitive radio networks

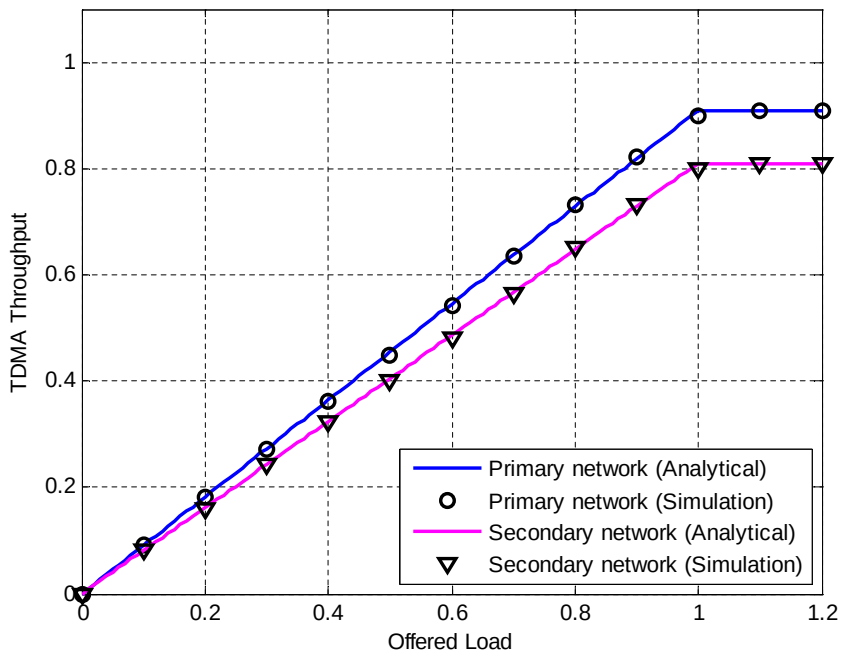

Figure 6. TDMA throughput comparison for primary and secondary network

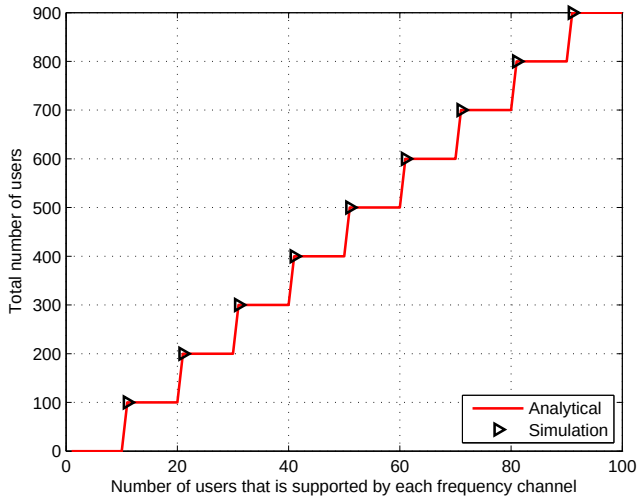

Figure 7. Number of users supported by TDMA assuming each frame is divided into ten time slots

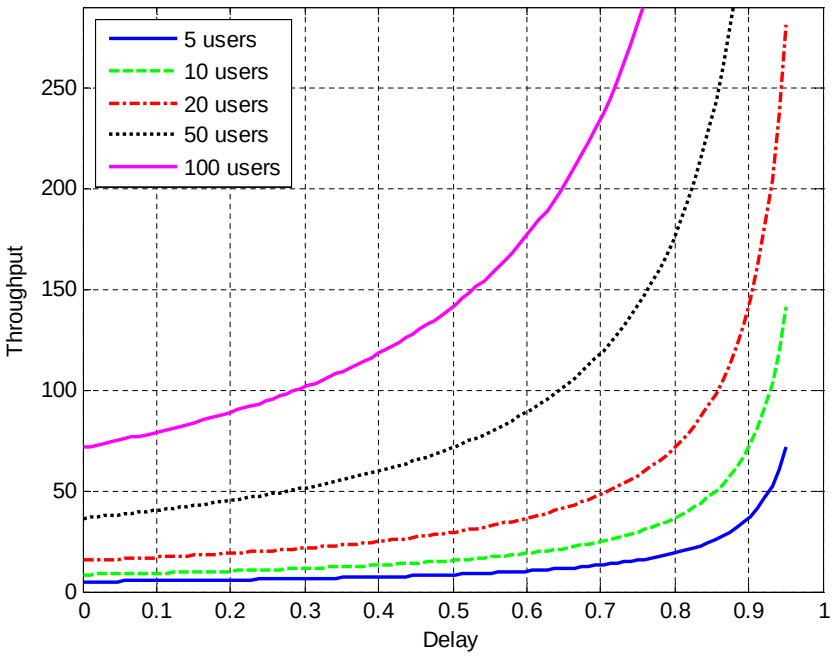

Figure 8. Throughput – delay characteristics of TDMA medium access control protocol

users in our model. However, because of the slot allocation, intermediate values do not affect the simulation results. For example, if forty five users are supported by each frequency channel, total number of users is still four hundred as is for forty users.

In Figure 8, throughput – delay characteristics of TDMA scheme is illustrated. As number of users is raised, exponential increase of delay and throughput are carried out. When maximum throughput is reached, delay becomes about one. Besides, maximum throughput changes according to the number of users.

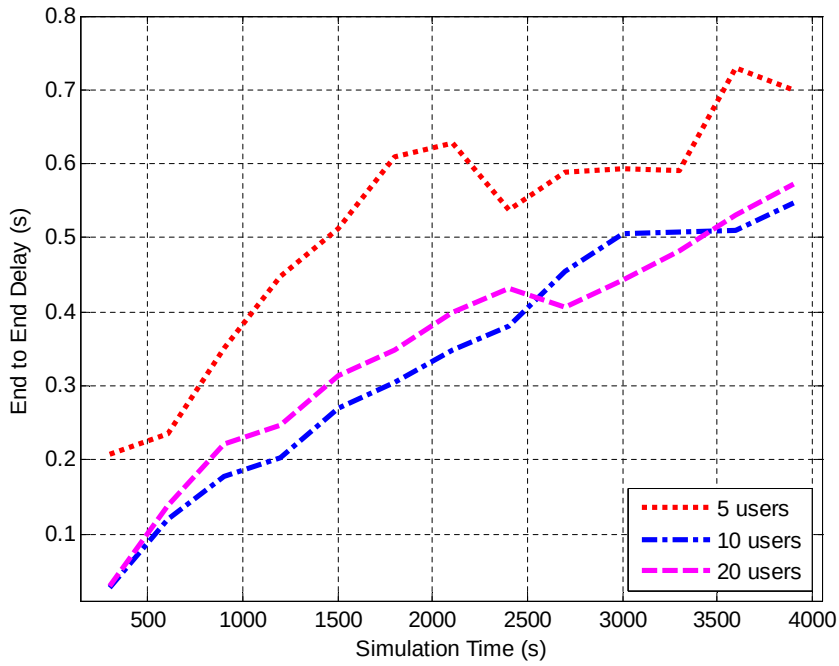

Figure 9. Delay analysis for primary network for less than 20 users

4.2 Delay results

Analytical end to end delay results of the network model are obtained using (2). And, simulation results obtained from Riverbed software are consistent with the analytical results.

In Figure 9, results of delay analysis for primary network are shown. For five users, end to end delay increases from 0.2 to 0.7. For ten and twenty users, end to end delay is almost the same. End to end delay for five users is higher because of the slot allocation. When ten time slots are assigned to five users, waiting time between slots gets higher so as to end to end delay.

In Figure 10, delay analysis of primary network with twenty five,

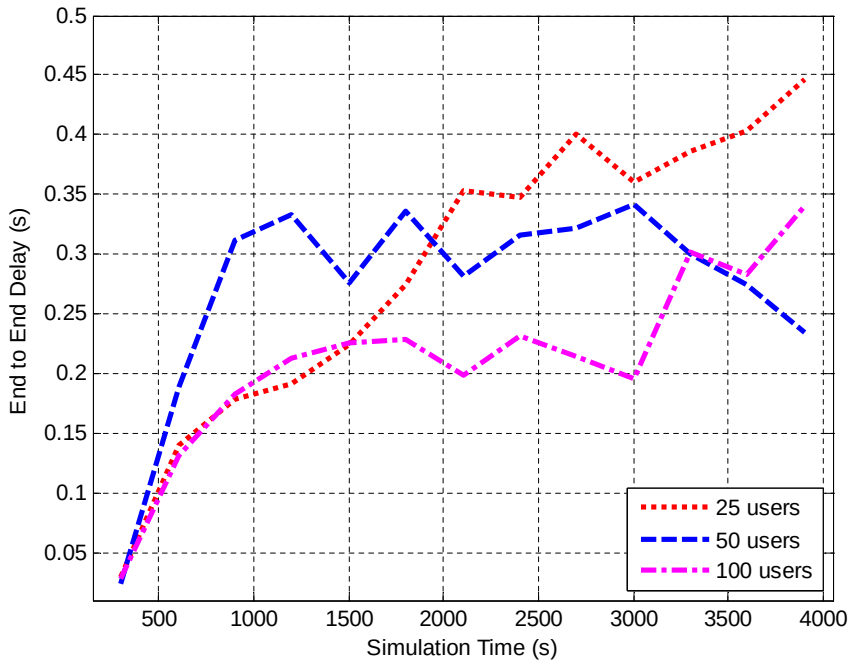

Figure 10. Delay analysis for primary network for more than 25 users

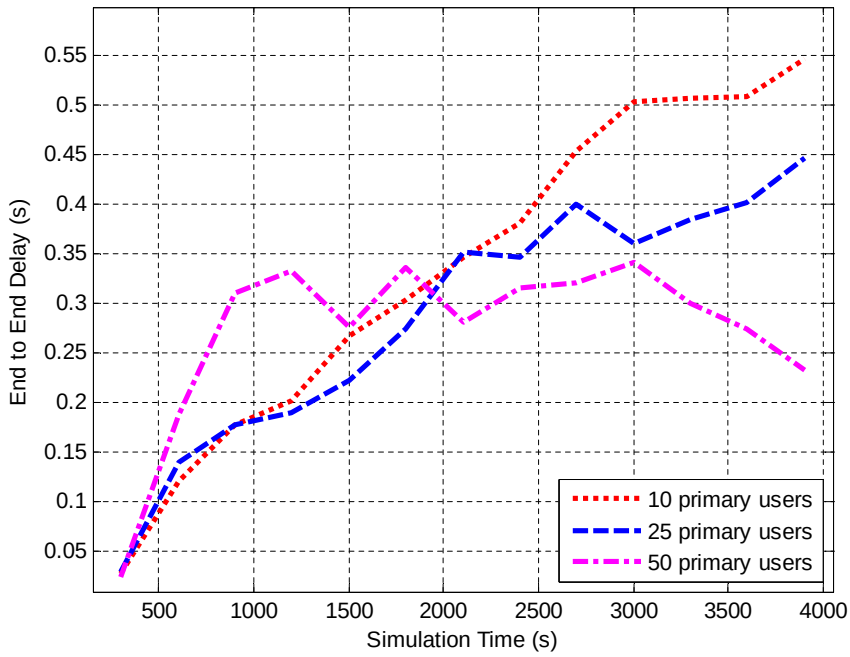

Figure 11. Delay analysis for primary network when number of secondary users is ten

fifty and hundred users are illustrated. The scenario which consists of twenty five users gives good results. For fifty and hundred users, some inconsistent results are obtained. Because the time slot of the users becomes smaller as number of users is increased.

In Figure 9 and Figure10, end to end delay results are obtained for primary networks without effect of secondary users. However, delay results are acquired when there are ten secondary users in the network scenario in Figure 11.

In Figure 11, end to end delay results of primary network that consists of ten secondary users are given. Because of the secondary users in the spectrum, end to end delay of the network increases. Owing

to the missed detection which causes collision with secondary users, packet transmission time in the network model arises.

5 Conclusions

In this study, we have evaluated the performance of reservation based TDMA medium access control technique. In the simulation scenarios, primary users and secondary users employ TDMA as a medium access control technique. We have designed a communication medium in Riverbed simulation software that consists of primary users, secondary users, and base stations. In the network model, secondary users sense the spectrum bands and inform the base station about idle channels. Then, the base station decides accordingly which secondary user may utilize the idle channel. Energy detection technique is used as a spectrum sensing technique because it is the best one when information about signal of primary user is known in advance. Moreover, different number of users is selected in simulation scenarios in order to achieve correct delay and throughput results. Comparing analytical model results with the simulation results, we have proved that performance analysis of our system model is consistent and accurate. In future works, the performance analysis of the secondary networks using various medium access control protocols may be studied.

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Performance of TDMA in Cognitive Wireless Networks

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Class Cohesion Metrics for Software Engineering: A Critical Review *

Habib Izadkhah, Maryam Hooshyar

Abstract

Class cohesion or degree of the relations of class members is considered as one of the crucial quality criteria. A class with a high cohesion improves understandability, maintainability and reusability. The class cohesion metrics can be measured quantitatively and therefore can be used as a base for assessing the quality of design. The main objective of this paper is to identify important research directions in the area of class cohesion metrics that require further attention in order to develop more effective and efficient class cohesion metrics for software engineering. In this paper, we discuss the class cohesion assessing metrics (thirty-two metrics) that have received the most attention in the research community and compare them from different aspects. We also present desirable properties of cohesion metrics to validate class cohesion metrics.

Keywords: Cohesion, Class cohesion metrics, Software engineering.

1 Introduction

Producing high quality software systems has always been one of the main goals of software designers and developers. Many software features have been identified that influence the quality of software, for example complexity, coupling and cohesion. Cohesion is an important software quality attribute and high cohesion is one of characteristics of well-structured software design. In overall, module cohesion indicates

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relatedness in the functionality of a software module [1]. It shows to which degree the elements within that module belong together or are related to each other. Modules with high cohesion are usually robust, reliable, reusable, and understandable while modules with low cohesion are allied with undesirable characteristics such as being difficult to understand, test, maintain, and reuse. Cohesion is usually expressed as high cohesion or low cohesion. There are different types of module cohesion; we can, therefore, create a nominal scale for cohesion measurement. Table 1 shows an ordinal scale for cohesion measurement [1].

Table 1. An ordinal scale for cohesion measurement

Rank	Cohesion type	Quality
6	Functional cohesion	Strongest (good)
5	Sequential cohesion	
4	Communication cohesion	
3	Procedural cohesion	
2	Temporal cohesion	
1	Logical cohesion	
0	Coincidental cohesion	Weakest (bad)

In Table 1, functional cohesion is judged to provide a strongest relationship because all of the elements contribute to a single task. Functional cohesion corresponds to single responsibility principle. Coincidental cohesion takes place when a module performs different and unrelated tasks. The serious problem with these methods is that it depends on subjective human assessment. It means that these metrics specify that the module is cohesive or not cohesive, and do not capture varying strengths of cohesion.

An object oriented program is made of many classes and each class consists of members called methods and attributes. In an object-oriented paradigm, a module can be a class, the data can be attributes, and the methods can be elements. We are mainly interested in the co-

hesion of object-oriented programs such as classes. Class cohesion can be defined as a measure of the degree to which members (i.e., methods and attributes) of a class belong together. Cohesion is usually measured on structural information extracted entirely from the source code (e.g., attribute references in methods and method calls).

In this paper, most class cohesion assessing metrics that have been presented in the literature are reviewed and categorized in both syntactic and semantic relationships. We also outline directions for further research in class cohesion assessing metrics, such as to define of better class cohesion assessing metrics or the improvement and evaluation of existing ones.

The structure of the paper is as follows. Section 2 presents three different applications of existing class cohesion metrics in order to motivate the need for assessing class cohesion. An overview of the state of the art for assessing class cohesion metrics is presented in Section 3. In section 4, the class cohesion metrics are evaluated from theoretical validation aspects. Section 5 introduces lack of discrimination anomaly problem, so that this problem is addressed on some class cohesion metrics. Section 6 presents some desirable properties of cohesion assessing metrics. Finally, Section 7 concludes the paper and open research challenges related to class cohesion are discussed in this section.

2 Class Cohesion Assessing Applications

Before we present the technical aspects of class cohesion metrics in Section 3, we describe three cases where class cohesion metrics can be used to solve important software engineering problems in the context of program understandability, software maintainability, and Reuse. It was argued that each module (class level) of well structured software should be focused on a single purpose. This means that it should have very few responsibilities. First, before addressing class cohesion metrics applications, we define a program's class design principle related with cohesion [2].

Principle: The Single Responsibility Principle (SRP) - A class should have one, and only one, reason to change.

The SRP indicates high cohesion for a class, in other words highly related, single responsibility, and strongly focused functionality for a class; where a responsibility [2] is defined as "a reason for change". For addressing class cohesion assessing applications, the following classes are considered: Class A performs three distinct operations (i.e., three responsibilities) while Class B performs only one operation (i.e., single responsibility); the cohesion of class B is higher than the class A.

```
Class A {
    1. Init disk() {...}
    2. Apply to college() {...}
    3. Circle Area Calculation () {...}
}
Class B {
    1. Method A part 1() {...}
    2. Method A part 2 () {...}
    3. Method A part 3 () {...}
}
```

2.1 Software Maintainability

In software engineering, maintainability is defined as the effort and the ease required to change in one module (e.g., class), to repair or to meet new requirements, maximize a product's useful life, maximize efficiency, reliability, and safety, cope with a changed environment, and etc. Since class B has only single responsibility, therefore, changing it is easier than changing class A; because changing a specific method in class A may affect other methods, due to the use of shared attributes. So, considering this side effect, it is a hard work to maintain a class. Maintainable class is a class that exhibits high cohesion. Higher cohesion and consequently higher maintainability means less time to make

a change. Class cohesion assessing metrics evaluate the cohesion of a class for determining the level of maintainability of a class.

2.2 Program Understandability

From Eighty-Twenty Rule, 20% of the time is spent creating, and 80% of the time is spent maintaining. Of the maintenance time, 20% is spent changing, while 80% of the time is spent just trying to understand the code. Therefore, understandability or program comprehension is one of the important characteristics of software quality, because it concerns the ways software engineers maintain the existing source code. In order to maintain the software, the programmers need to comprehend the source code. The understandability of the source code depends upon the cohesion, coupling and complexity of the program. A class with single responsibility is easier to understand compared to a class with multiple responsibilities. Class cohesion assessing metrics evaluate the cohesion of a class for determining the level of understandability of a class; a class with higher cohesion is more understandable than a class with low cohesion. For example, in our designed classes, since the class B only does "one thing," its interface usually has a small number of methods that are fairly self explanatory. It should also have a small number of member variables, thus it is more understandable than class A.

2.3 Software Reusability

One of the most powerful characteristics of object-oriented programming is code reuse. You can write a class and then reuse it many times. High cohesion with loose coupling, information hiding makes code more easily reusable, and decrease the cost and time of development. If a class has multiple responsibilities, and only one of those is needed to assemble new requirements in other programs, then the other redundant responsibilities hinder reusability. Single responsibility implies the reuse of a class anywhere without modification. In our designed classes, class B is more reusable than class A.

3 Class Cohesion Assessing Metrics

Using ordinal scale for cohesion measurement, comparing two cohesive or two non-cohesive classes, or to know whether a code modification increased or reduced the degree of cohesiveness is hard. In this paper, we review the metrics for determining whether a module (class) is cohesive or not cohesive and also the degree of its cohesiveness. Also, these metrics would allow us to judge which module is better designed and more cohesive. These metrics compute class cohesion using manipulations of class elements. The key elements of a class C are its attributes $A_1, ..., A_a$, methods $M_1, ..., M_m$, and the list of p parameter (or, argument) types of the methods $P_1, ..., P_m$. Using class cohesion assessing metrics for computing class cohesion, we need to show a class with corresponding graph. Let a, b, c, d denote attributes and m_1, m_2, m_3 show methods. Figure 1 shows a sample class with its corresponding graph.

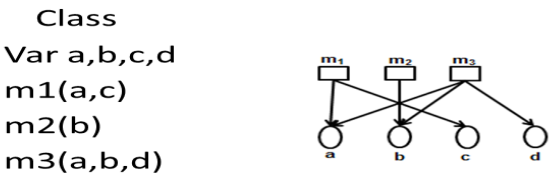

Figure 1. Class with corresponding graph

Class cohesion metrics can be broadly classified into two groups:
1. **Interface-based metrics** compute class cohesion from information in method signatures. Table 2 shows most interface-based metrics. Advantage of these metrics is that they can be calculated early in the design stage. Disadvantage of them can be concluded as from only design model, it is impossible to determine what exactly a method is doing (e.g., it may be using class attributes, or calling other methods on its class);
2. **Code-based metrics** also can be classified into two groups of conceptual metrics and structural metrics. Conceptual metrics use information retrieval methods for class cohesion measurement. Information

retrieval methods are based on the extraction the concepts of source code. In conceptual cohesion if methods of one class are conceptually related, then class is considered cohesive. Conceptual metrics of cohesion are introduced in the Tables 3 and 4. Many structural metrics have been proposed for cohesion measurement from research community. These metrics use the structural data that are extracted from the source code. The differences among structural metrics are based on the definition of the relationships between methods. Structural metrics compute class cohesion (method-method invocation) in terms of attribute accesses by methods (operations sharing attributes) and operations invocating other operations. The latter have the strongest cohesion compared to the former ones. We can further classify structural base cohesion metrics into four sub-types based on the methods of quantification of cohesion:

- Disjoint component-based metrics count the number of disjoint sets of methods or attributes in a given class. These metrics are addressed in Table 5.
- Pairwise connection-based metrics compute cohesion as a function of number of connected and disjoint method pairs. These metrics are addressed in Tables 6 and 7.
- Connection magnitude-based metrics count the accessing methods per attribute and indirectly find an attribute-sharing index in terms of the count (instead of computing direct attribute-sharing between methods). These metrics are addressed in Tables 8-11.
- Decomposition-based metrics compute cohesion in terms of recursive decompositions of a given class. The decompositions are generated by removal of pivotal elements that keep the class connected. These metrics are addressed in Table 12.

Table 2. Interface-based metrics

Class Cohesion Metrics	Definition/Formula
CAMC [3]	$CAMC = \frac{a}{kl}$, where, l is the number of distinct parameter types, k is the number of methods and a is the summation of the number of distinct parameter types of each method in the class.
NHD [4]	Let k and l have the same definitions as above for CAMC, and x_j be the number of methods that have a parameter of type j. Then, $NHD = 1 - \frac{2}{lk(k-1)} \sum_{j=1}^{l} x_j(k - x_j)$.
MMAC [5]	Let x_i be the number of methods that have a parameter or a return of type i. $MMAC(C) = \begin{cases} 0 & k = 0 \text{ or } l = 0 \\ 1 & k = 1 \\ \frac{\sum_{i=1}^{i=l} x_i(x_i-1)}{lk(k-1)} & \text{otherwise} \end{cases}$

Table 3. Conceptual metrics

Class Cohesion Metrics	Definition/Formula
LORM [6]	LORM=Total number of relations in the class / Total number of possible relations to the class, where, Total number of relations in the class = number of pairs of methods in the class for which one method contains conceptual relations forming external links out of the set of concepts that belong to the method to or from the set of concepts belonging to another method in the class. N= total number of member functions (methods) in the class. Total number of possible relations = $\frac{N(N-1)}{2}$.
C3 [7]	Let $M(C) = \{m_1, ..., m_n\}$ be the set of methods of class, V_{m_i} and V_{m_j} be the vectors corresponding to the $m_i, m_j \in M(C)$. Then $C3 = \begin{cases} ACSM, & \text{if } ACSM > 0 \\ 0 & \text{otherwise} \end{cases}$, where $CSM(m_i, m_j) = \frac{V_{m_i}^t . V_{m_j}}{\lvert V_{m_i} \rvert \times \lvert V_{m_j} \rvert}$ $ACSM = \frac{1}{N} \sum_{i=1}^{N} CSM(m_i, m_j)$ $N = C_n^2$
LCSM [7]	Let $M_i = \{m_j \lvert CSM(m_i, m_j) > ACSM(C), m_i \neq m_j\}$ and $P = (M_i, M_j) \lvert M_i \cap M_j = \emptyset$ and $Q = (M_i, M_j) \lvert M_i \cap M_j \neq \emptyset$ Then, LCSM(C)= $\begin{cases} \lvert P \rvert - \lvert Q \rvert, & \text{if } \lvert P \rvert > \lvert Q \rvert \\ 0 & \text{otherwise} \end{cases}$

Table 4. Conceptual metrics (continuation of Table 3)

Class Cohesion Metrics	Definition/Formula
MWE [8]	$MWE = max_{1<i<\|t\|}(O(t_i) \times D(t_i))$ where, $O(t_i) = \frac{\sum_{d=1}^{n} p_{t_i}^d}{n}$ $D(t_i) = \frac{\sum_{d=1}^{n} -q_{t_i}^d \times \log(q_{t_i}^d)}{\log n}$ $q_{t_i}^d = \frac{p_{t_i}^d}{\sum_{d=1}^{n} p_{t_i}^d}$ $p_{t_i}^d$=the probability of topic t_i in a method d $\|t\|$=number of topics n=number of methods

Table 5. Disjoint component-based metrics

Class Cohesion Metrics	Definition/Formula
LCOM1 [9]	Number of pairs of methods that do not share attributes.
TLCOM [10]	Number of method pairs that do not directly or transitively share a common attribute.
LC0M3 [11]	Number of connected components in the graph that represents each method as a node and the sharing of at least one attribute as an edge.
LCOM4 [12]	Similar to LCOM3, where graph G additionally has an edge between vertices representing methods m_i and m_j, if m_i invokes m_j or vice-versa.

Table 6. Pairwise connection-based metrics

Class Cohesion Metrics	Definition/Formula
LCOM2 [9]	Let P be the number of pairs of methods that do not share attributes and Q be the number of pairs of methods that share attributes. Then, $LCOM2(C) = \begin{cases} \lvert P\rvert - \lvert Q\rvert, & \text{if } \lvert P\rvert > \lvert Q\rvert \\ 0 & \text{otherwise} \end{cases}$
TCC [13]	Let NDC be the number of pairs of directly connected methods and NP be the maximum possible number of direct or indirect connection. Then, TCC is difined as: $TCC = \frac{NDC}{NP}$.
LCC [13]	Let NIC be the number of pairs of indirect connection in the class. Then, LCC=$\frac{NDC + NIC}{NP}$.
DC_D [14]	Fraction of directly connected pairs of methods, where two methods are directly connected if they satisfy the condition mentioned above for TCC or if the two methods directly or transitively invoke the same method.
DC_I [14]	Fraction of directly or transitively connected pairs of methods, where two methods are transitively connected if they satisfy the condition mentioned above for LCC or if the two methods directly or transitively invoke the same method.

Table 7. Pairwise connection-based metrics (continuation of Table 6)

Class Cohesion Metrics	Definition/Formula
CCM [15]	$CCM(C) = \frac{NC(C)}{NMP(C).NCC(C)}$, where NC(C) is the number of actual connections among the methods of class, NMP(C) is the number of the maximum possible connections among the methods of the class C, NCC(C) is the number of connected components of the connection graph G_c that represents each method as a node and two methods A and B are connected in the connection graph if A and B access one or more attributes in common or method A invokes method B or vice versa or methods A and B invoke one or more methods in common.

Table 8. Connection magnitude-based metrics

Class Cohesion Metrics	Definition/Formula
LCOM5 [16]	Let l be the number of attributes, k be the number of methods and a be the summing of the number of distance attributes that are accessed by each method in a class, then $LCOM5 = \frac{a - kl}{l - kl}$.
Coh [17]	Let a, k and l have the same definitions as above, then, $Coh = \frac{a}{kl}$.

Table 9. Connection magnitude-based metrics

Class Cohesion Metrics	Definition/Formula
OCC [18]	Consider a set of methods M=$\{m_1, ..., m_n\}$ and a set of attributes A within the class. Let $S = \{(m_i, m_j) \in M \times M \mid m_i \; invokes \; m_j\}$, $S^* = \{(m_i, m_j) \in M \times M \mid (m_i = m_j) \vee (\vee_{n \geq 1} m_i S^n m_j)\}$, $ac(m_i, a) \iff \exists m_j \in M \; s.t \; [(m_i S^* m_j) \wedge (m_j \, accesses \, a)]$ $R_w(m_i)$=set of methods which are reachable by $G_w(V, E)$ $G_w(V, E)$=weak-connection graph, where V=M(number of methods) and E is given by: $E = \{(m_i, m_j) \in M \times M \mid \exists a \in A \; s.t \; (ac(m_i, a) \wedge ac(m_j, a))\}$ Then, OCC(C)= $\begin{cases} max_{i=1,...,n}[\frac{\lvert R_w(m_i)\rvert}{n-1}] & n > 1 \\ 0 & \text{n=1} \end{cases}$
PCC [19]	$PCC(C) = \begin{cases} max_{i=1,...,n}[\frac{\lvert R_s(m_i)\rvert}{n-1}] & n > 1 \\ 0 & \text{n=1} \end{cases}$ where, $R_s(m_i)$=set of methods which are reachable by $G_s(V, E)$ $G_s(V, E)$=strong-connection graph, where V=M(number of methods) and E is given by: $E = \{(m_i, m_j) \in M \times M \mid \exists a \in A \; s.t \; (wr(m_i, a) \wedge re(m_j, a))\}$ $wr(m_i, a) \iff \exists m_j \in M \; s.t \; [(m_i S^* m_j) \wedge (m_j \, writes \; data \; on \; to \, a)]$, $re(m_i, a) \iff \exists m_j \in M \; s.t \; [(m_i S^* m_j) \wedge (m_j \, reads \; data \; from \; a)]$ and $S, S^*, M, A,$ have the same definitions above.

Table 10. Connection magnitude-based metrics

Class Cohesion Metrics	Definition/Formula
CC [19]	Let I_i and I_j be the sets of attributes that are referenced by methods m_i and m_j, respectively. CC is the ratio of the summation of the similarities between all pairs of methods to the total number of pairs of methods. The similarity between methods m_i and m_j is defined as: $Similarity(i,j) = \frac{\lvert I_i \cap I_j \rvert}{\lvert I_i \cup I_j \rvert}$.
SCOM [20]	Ratio of the summation of the similarities between all pairs of methods to the total number of pairs of methods. The similarity between methods m_i and m_j is defined as: $Similarity(m_i, m_j) = \frac{\lvert I_i \cap I_j \rvert}{min(\lvert I_i \rvert, \lvert I_j \rvert)} . \frac{\lvert I_i \cup I_j \rvert}{l}$.
PCCC [21]	PCCC= $\begin{cases} 0 & l = 0 \text{ and } k > 1 \\ 1 & l > 0 \text{ and } k = 0 \\ \frac{NSP(G_c)}{NSP(FG_c)} & \text{otherwise} \end{cases}$ where, l=number of attributes k=number of methods NSP=number of simple paths in graph G_c FG_c= corresponding fully connected graph.
LSCC [22]	LSCC(C)= $\begin{cases} 0 & l = 0 \text{ and } k > 1 \\ 1 & l > 0 \, and \, k = 0 \\ 1 & k = 1 \\ \frac{\sum_{i=1}^{i=l} x_i(x_i-1)}{lk(k-1)} & \text{otherwise} \end{cases}$ where, l is the number of attributes, k is the number of methods, and x_i is the number of methods that reference attribute i.

Table 11. Connection magnitude-based metrics

Class Cohesion Metrics	Definition/Formula
CC [23]	Consider a set of methods $\{m_i\}$(i=1,...,m) accessing a set of attributes $\{V_j\}$(j=1,...,n). Let $\mu(V_j)$ be the number of methods that share the attribute V_j of a class. Then, $CC = \frac{\sum_1^n CV_j}{n}$, where $CV_j = \frac{\mu(V_j)}{m}$.
CO [24]	$CO = 2.\frac{\lvert E\rvert - (\lvert V\rvert - 1)}{(\lvert V\rvert - 1).(\lvert V\rvert - 2)}$ where, E and V are the edges and vertices of G from LCOM4.
CBAMU [25]	Let a be the number of attributes in the class, m be the number of methods in the class, $\mu(A_i)$ be the number of methods that access A_i and $\mu(M_j)$ be the number of methods that invoke method M_j, then, $CBAMU = \frac{1}{2}(AU(C) + MU(C))$, where, AU(C)= $\begin{cases} 0 & a = 0 \text{ or } m = 0 \\ \frac{1}{am}\sum_{i=1}^{a}\mu(A_i) & \text{otherwise} \end{cases}$ and MU(C)= $\begin{cases} 0 & m = 0 \text{ or } 1 \\ \frac{1}{m(m-1)}\sum_{j=1}^{m}\mu(M_j) & \text{otherwise} \end{cases}$
RCI [26]	Let CI(C) be the set of all data-data interactions and data-method interactions and Max(C) to be the set of all possible data-data interactions and data-method interactions. Then, $RCI(C) = \frac{\lvert CI(C)\rvert}{\lvert Max(C)\rvert}$.

Table 12. Decomposition-based metrics

Class Cohesion Metrics	Definition/Formula
CBMC [27]	Let $F_c(G) = \frac{M(G)}{N(G)}$ and $F_s(G) = \frac{1}{n}\sum_{i=1}^{n} CBMC(G^i)$ where, M(G)=the number of glue methods in graph G, N(G)=the number of non-special methods in graph G G^i = one of children of the structure tree.Then, $CBMC(G) = F_c(G) \times F_s(G)$.
ICBMC [28]	$ICBMC(G) = F_c(G) \times F_s(G)$ where, $F_c(G) = \frac{\lvert Q \rvert}{\lvert N_m(G) \rvert \times \lvert N_v(G) \rvert}$ and $F_s(G) = \frac{1}{2}\sum_{i=1}^{n} ICBMC(G^i)$ Q=number of edges in the cut set of G $N_m(G)$= number of non-special methods in graph G $N_v(G)$= number of attributes in graph G.
OL_n [29]	The average strength of the attributes, wherein the strength of an attribute is the average strength of the methods that reference that attribute. n is the number of iterations that are used to compute OL.

Now, we evaluate these metrics and compare them through three given examples. Let a_i and m_i denote attributes and methods, respectively, the first example (Fig. 2) shows four sample classes including linking methods of a class with their attributes. The cohesion of these classes increase from C1 to C4, indeed class C1 has the lowest cohesion and class C2 has the highest cohesion. Table 13 shows the cohesion value for some these metrics.

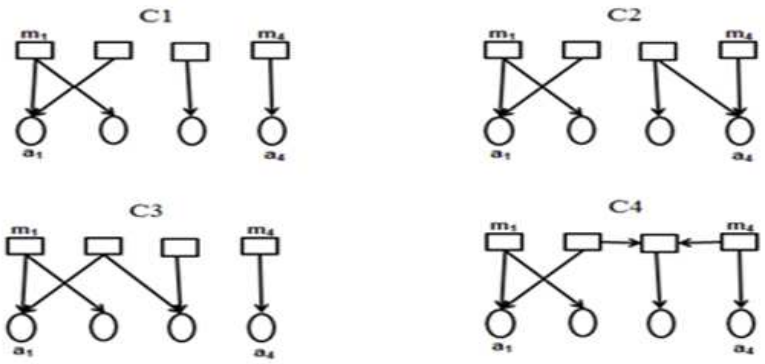

Figure 2. Example cases for capturing degree of cohesion

In Table 13, in the last column, sign (-) means that its corresponding metrics does not give the correct result about cohesion and sign (+) means that the result is acceptable. As it can be seen in Table 13, only LCOM4, TCC, DCD, DCI and CBAMU metrics can find the class cohesion correctly.

The second example (Fig. 3) shows three other classes (C1-C3) with different degrees of cohesion. In this figure, class C3 has the highest cohesion and class C1 has the lowest cohesion. In Table 14, the cohesion values for some of these metrics are presented. As it can be seen in Table 14, only LCOM1, LCOM2, LCOM5, Coh, TCC, CBMC, CCM, CC, SCOM, LSCC, CC and CBAMU metrics can find the class cohesion correctly.

Table 13. Cohesion values for Fig. 2

Metric	$C1$	$C2$	$C3$	$C4$	
$LCOM1$	5	4	4	5	-
LCOM2	4	2	2	4	-
LCOM3	3	2	2	3	-
LCOM4	3	2	2	1	+
LCOM5	0.91	0.83	0.83	0.91	-
Coh	0.31	0.37	0.37	0.31	-
TCC	0.16	0.33	0.33	0.5	+
LCC	0.16	0.33	0.5	0.5	-
DC_D	0.16	0.33	0.33	0.66	+
DC_I	0.16	0.33	0.5	0.66	+
CC	0.5	1	1	0.5	-
SCOM	0.5	1	1	0.5	-
LSCC	0.04	0.08	0.08	0.04	-
CC	0.31	0.37	0.37	0.31	-
CBAMU	0.15	0.18	0.18	0.23	+

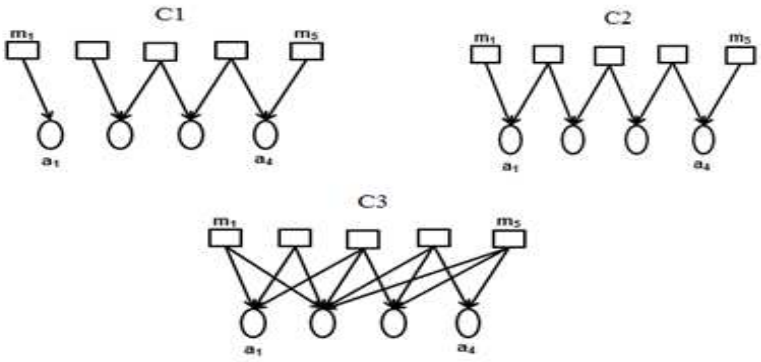

Figure 3. Example cases for capturing degree of cohesion

Table 14. Cohesion values for Fig. 3

Metric	$C1$	$C2$	$C3$	
LCOM1	7	6	0	+
LCOM2	3	2	0	+
LCOM3	2	1	1	-
LCOM4	2	1	1	-
LCOM5	0.81	0 .75	0.43	+
CCM	1	0.4	1	+
OCC	0.75	1	1	-
DC_D	0.3	0.4	0.4	-
DC_I	0.6	1	1	-
Co		0	1	-
CC	1.33	1.66	5.33	+
SCOM	1.37	1.75	6.08	+
Coh	0.35	0.4	0.65	+
LSCC	0.75	0.1	0.42	+
TCC	0.3	0.4	1	+
CC	0.35	0.4	0.65	+
LCC	0.6	1	1	-
CBAMU	0.17	0.2	0.32	+
CBMC	0	0.13	0.6	+

For the third example, consider classes in Fig. 4. In this figure, class C1 has the highest cohesion and class C4 has the lowest cohesion. The cohesion values for some of the metrics are shown in Table 15. As it can be seen in Table 15, only SCOM metrics can find the class cohesion correctly. Note that the SCOM metrics cannot find right cohesion for classes in Fig. 2.

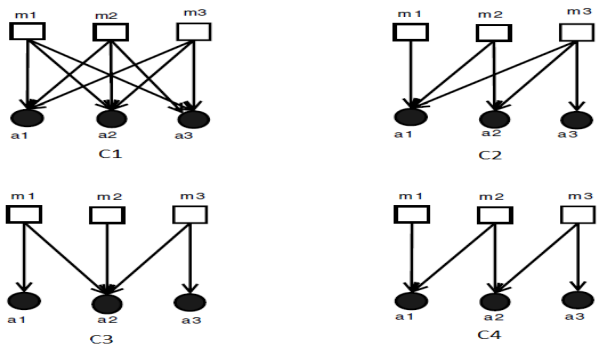

Figure 4. Examples for capturing degree of cohesion

Table 15. Cohesion values for Fig. 4

Metric	$C1$	$C2$	$C3$	$C4$	
TLCOM	0	0	0	1	-
LCOM2	0	0	0	0	-
TCC	1	1	1	0.67	-
SCOM	1	0.89	0.61	0.39	+
CBAMU	0.5	033.	0.27	0.27	-
CBMC	1	0.67	0.67	0.33	-
ICBMC	0.33	0.139	0.069	0.069	-
CCM	1	1	1	0.66	-
PCCC	1	0.2	0.11	0.11	-
RCI	1	0.5	0.41	0.41	-

From our designed different eleven classes with different cohesion, our experiment (Tables 13-15) shows that the described cohesion assessing metrics for all our designed classes in these examples do not show the right cohesion. Indeed, these metrics cannot calculate class cohesion on some classes correctly.

4 Theoretical Validations of Cohesion Metrics

For theoretical validation of the class cohesion assessing metrics a framework proposed in [17]. At this framework, four properties are presented for validation of metrics that measure cohesion values. If cohesion metrics is a true metrics, it should satisfy these properties. These four properties are:

Property 1: non-negativity and normalization: non-negative indicates that the measured class cohesion fits to a specific interval [0, Max], and using normalization the designer can compare the cohesion of different classes.

Property 2: null value and maximum value: the cohesion of a class is equal to 0 if the class has no cohesive interactions; the cohesion is equal to Max if all possible interactions within the class are present.

Property 3: monotonicity: adding cohesive interactions to the module cannot decrease its cohesion.

Property 4: cohesive modules: merging two unrelated modules into one module does not increase the individual modules cohesion. Therefore, given two classes, C1 and C2, the cohesion of the merged class C must satisfy the following condition:

$cohesion(C) \leq max\{cohesion(C1), cohesion(C2)\}$

These four properties (indicated by P1-P4) for the presented metrics (Tables 2-12) are addressed in Table 16. The sign (-) means that its corresponding metrics does not satisfy the property and sign (+) means that it satisfies the property.

Table 16. Theoretical validation results of cohesion metrics

Metric	P_1	P_2	P_3	P_4	Metric	P_1	P_2	P_3	P_4
$LCOM1$	-	+	+	+	PCC	+	+	+	+
$TLCOM$	-	+	+	+	DC_I	+	+	+	+
$LCOM2$	-	+	+	+	CC	+	+	-	+
$LCOM3$	-	-	+	+	MMAC	+	+	+	+
$LCOM4$	-	-	+	+	SCOM	+	+	-	+
CO	-	-	+	+	LSCC	+	+	+	+
$LCOM5$	-	+	+	-	CC	+	+	+	+
Coh	+	+	+	+	CAMC	-		+	+
TCC	+	+	+	+	NHD	-		-	-
LCC	+	+	+	+	CBAMU	+	+	+	+
DC_D	+	+	+	+	ICBMC	+	+	+	+
CBMC	+	+	-	+	PCCC	+	+	+	+
CCM	+	+	+	+	RCI	+	+	+	+
OCC	+	+	+	+	OL_n	+	+	+	+

5 Lack of Discrimination Anomaly

Some class cohesion metrics use normalization operation to allow for easy comparison of the cohesion of different classes, so it is possible that they get the same cohesion values for different classes that have the same number of methods and attributes but different interactions [30]. In these cases, two classes incorrectly are considered same in terms of cohesion. This leads to lack of discrimination anomaly (LDA) problem [30]. A good class cohesion metrics should avoid the LDA problem. If there is the LDA problem in a metrics, the probability that the metrics will distinguish between different classes cohesion is low; thus, the metrics is imprecise. Tables 17 and 18 show some class cohesion metrics and their LDA problems [30].

Table 17. Some class cohesion metrics and their LDA problems

Metrics	When LDA problem occur?
LCOM1	1. when the number of method pairs that share common attributes is the same in both classes, regardless of the number of shared attributes. 2. when the number of method pairs that share common attributes is the same in both classes, regardless of which attributes are shared.
LCOM2	1. same two LDA cases indicated for LCOM1 2. the two classes will have the same LCOM2 value, if $P \leq Q$. In this case, the value of LCOM2 is zero, regardless of the magnitude of the difference between P and Q in each of the two classes.
LCOM3 and LCOM4	1. same first LDA indicated for LCOM1 2. if the number of disjoint components is the same in both classes regardless of the interactions of each of the disjoint components.
LCOM5 and Coh	- two classes have the same number of attributes referenced by methods, regardless of the distribution of these references.
CAMC	- if the two classes have the same number of distinct types of parameters in their methods, regardless of the distribution of these cohesive interactions.
TCC and DC_D	- same two LDA cases indicated for LCOM1
LCC and DC_I	1. same two LDA cases indicated for LCOM1 2. the two classes have the same number of connected methods, regardless whether the methods are connected directly or transitively.

Table 18. Some class cohesion metrics and their LDA problems (continuation of Table 17)

Metrics	When LDA problem occur?
CC	- if the ratio of shared attributes between each pair of methods to the distinct number of attributes referenced by each of the corresponding pair of methods is the same, regardless of the number of shared attributes.
LSCC	1. when each attribute in Class C1 is referenced by the same number of methods that reference the corresponding attributes in Class C2, regardless of the distribution of these methods and their connectivity with other methods. 2. if the result of $\sum_{i=1}^{n} x_i(x_i - 1)$ is the same for both classes, where n is the number of attributes and x_i is the number of methods that reference attribute i, regardless of the distributions or the number of references.
NHD	1. same first LDA indicated for LSCC 2. if the result of $\sum_{i=1}^{n} x_i(n - x_i)$ is the same for both classes, where n is the number of distinct parameter types and x_i is the number of methods that reference parameter type i, regardless of the distributions or the number of references.

6 Desirable Properties of Cohesion Metrics

The addressed class cohesion metrics can be evaluated both empirically and theoretically. In the empirical validation it is determined whether the predicted value is consistent with the measured value. In [21], [26], [31]–[38] several methods are proposed to empirically validate class cohesion metrics.

In the theoretical validation it is determined whether the metrics holds the required properties. In addition to the four main properties proposed in [17] (see section 4), in literature, several properties are introduced to validate software metrics theoretically, as summarized below.

I. **Monotonicity:** adding interactions to the class must not decrease its cohesion. It means that after adding an interaction to the class, the modified class cohesion value will be the same as or higher than the cohesion value of the original class.

II. **Representation condition of measurement theory:** The metrics obtains a value that yields the same order as intuition. It means there should not be inconsistencies between empirical relations and numerical relations such that the empirical relations preserve and are preserved by the numerical relations.

III. **Lack of discrimination anomaly:** as described in section 5.

IV. **Sensitivity:** adding or removing interactions to a class should change the cohesion of the class. Sensitivity is expected to increase as:
1) the number of distinct cohesion values increases and
2) the number of classes with repeated cohesion values decreases

V. **Normalization:** allows for easy comparison of the cohesion of different classes

VI. **Prescription:** The metrics indicates how to enhance the measured element.

VII. **No equivalence of interaction:** Given three classes x, y, and z, if x and y display the same cohesion value, the classes resulting from merging z with each of x and y can show different cohesion values.

The class cohesion metrics that does not have LDA problem and satisfies the other properties is expected to be a well-defined metrics. We suggest the newly introduced metrics that, first, is evaluated from LDA problem point of view. If the metrics is found to have LDA problem, the researcher may decide to revise the metrics formula to solve the LDA problem. Software developers are advised to apply cohesion metrics that does not have LDA problem because such metrics is expected to be more reliable in indicating cohesion.

7 Conclusion and open research problem

This paper presented the state of the art in the development and evaluation of class cohesion metrics. Most existing structural class cohesion metrics is addressed and evaluated from different aspects such as on our eleven designed classes, theoretical validations, and LDA problem. Our experimental results showed that whereas important progresses have already taken place, but there is no comprehensive and reliable class cohesion metrics. Therefore, there are still many possibilities for further research that will benefit software engineers everywhere. Following cases can be considered as a research problem:
1. There are no standard benchmarks for evaluating the class cohesion metrics. Researcher needs comprehensive benchmarks for evaluating the new designed metrics and comparing it with other metrics.
2. We believe the four properties described in section 4 should be revised. For example, due to LDA problem, using the normalization and monotonicity properties, a designer necessarily cannot distinguish the cohesion level of different classes.
3. Information theory based formulas are not addressed in this area. We think that class cohesion can be assessed in terms of information loss. It seems a class with low information loss to have high cohesion. However, this is a research question.
4. The combination of structural and semantic information from a class for cohesion assessing can be considered as a research problem.

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Contrastive elements in Romanian declarative contours

Doina Jitcă

Abstract

The paper presents an intonational analysis of several Romanian declarative contours by using a functional account of the prosodic words and phrases at the communicative act level. The functional prosodic words and phrases transform the corresponding word(s) into constituents of communicative units. The intonational forms of communicative unit constituents bear functions at the information structure level. The intonational analysis has a functional description component which applies functions at information structure level to constituents of communicative units at any level of the utterance tree.

Keywords: Communicative units; information structure, topic, focus.

1 Introduction

In this paper we analyze the Romanian utterances from the communicative act perspective aiming to transform them into hierarchies of communicative units (CUs) each of them having their own Information Structure (IS). At the utterance level the intonation rebuilds (updates) the structure of the text by partially transferring the syntactic relations to the communicative level. The intonation contour of utterance transforms the word sequences into hierarchies of local and global relations between words, by using different prosodic marks for uttering them. Usually, intonational analyses are focused on identifying pitch accents, edge tones and break indices in order to characterize accentuation and phrasing elements and they are not interested in analysing hierarchies

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of these relations between words, conveyed by prosody. These relations are communicative units at the utterance level. The characterization of communicative units can be made by information structure descriptions at each level of utterance tree hierarchy. We consider that prosody supports the communicative organization of utterances. Consequently, intonational analyses have to identify the communicative units and their functional elements at the information structure level.

In our previous works [8], [7], [9] we also treated hierarchies within utterances strictly at the intonational level as prosodic unit hierarchies but we observed that each prosodic phrase is produced by a tonal contrast between two complementary functional constituents marked by certain types of melodic contours: e.g. PUSH function was related to constituent which reaches high target tone within the respective prosodic unit (on the accented syllables) and POP function was applied to the tonal contrasted constituent having lower tonal target. We considered the PUSH constituents have function in pushing forward the communicative act but other discourse function has to be investigated and related to these contrasted elements.

In [9] we based the definition of prosodic units on the existence of a tonal contrast between these two paired constituents: PUSH and POP. That is different from the perspective of AM model implemented by ToBI system which relates the existence of prosodic units to break indices in their ends. We also introduced FOCUS function for semantic focused element annotation and thus, derived PUSH+FOCUS or POP+FOCUS functions result in our functional utterance descriptions. What it was not observed in our previous approach is the existence of two types of contrast within each prosodic unit: a tonal contrast between target tones of the two paired constituents and another contrast between the slopes and shape of F0 contour variations within contrasted constituents. The second type of contrast supports the topic-focus information structure and the former one is related to a second IS level that implies other discourse functions we named in this paper PUSH and POP function, respectively.

The goal of this paper is to propose an utterance partitioning model based on communicative unit hierarchies, each of units having their

proper information structure (IS). Information Structure model has two levels related to the two types of contrasts. The Halliday's IS model is also a two level model. It defines one IS level by the contrast between a foreground component (focus) and a background component of an information unit. We consider the focus-background IS description is suitable in the particular case of a very prominent prosodic component vs. non-prominent prosodic element, but in the most general case the contrast is produced between a focus constituent and a topic constituent, both of them having moderate prominences [6].

The second IS level of the Halliday's model refers to the theme and rheme aspects related to text organization. We consider the second IS level of our model has to be related to a certain aspect of the speech mechanism for supporting a certain discourse function and not to certain aspects of text organization. The second IS level is supported at the intonational level by the contrast between target tones of paired constituents.

The paper presents an intonational analysis framework which includes an information structure view and a phonological view in utterance partitioning. The first view takes into account the melodic contour shapes of elementary units (prosodic words) with their target tone and slope and shape characterization and relates them to the functions they have within CUs. The second view takes into account the pitch events within the prosodic units in order to understand the phonological/phonetic implementation of the functional constituents. Consequently, we added a functional perspective to the AM model and to the ToBI system [12], [11], [10], [1] in order to understand and to describe the role of intonation at the communicative act level.

In the second section we present the functional categories proposed for the intonational and IS analysis. In the third section the functional analysis is exemplified by presenting several declarative contours of Romanian neutral statements from these two perspectives: i. as sequences of pitch events and ii. as paired IS functional constituents within nested communicative units.

2 Functional categories at the communicative act level

The intonational analysis presented in this paper transforms utterances into hierarchical CU structures which include local contrast at each CU level between the constituents marked with complementary functions.

The topic and focus functions are two paired functions supported by the two constituents of one CU. The topic and focus function distribution between CU components conveys the role of the related words within different semantic and IS context: e.g. old vs. new information context. The topic-focus constituents can be marked by a slow vs. sharp pitch movements during their accented syllables or the following non-accented syllables. The focus event may or may not be prosodically marked by a tonal prominence because a second IS level overlaps the topic-focus structure and focus realization is influenced by the distribution of PUSH or POP functions within the respective CU.

In this paper we do not want to propose any significance to the PUSH and POP functions of the second IS level. We keep the name used in our previous papers and relate them to the tonal contrast between the target tones of the two paired constituents of CU: PUSH function is applied to the component with the higher target tone and POP function – to the other one containing the tonal minimum.

The proposed analysis organises an utterance into a CU hierarchy which is composed of instances of a single type domain (CU domain). CU domain may be intonational phrase (IP) or intermediate phrase (ip) or Compound Prosodic Domain (CPD): compound of prosodic words, compound of ips or compound of IPs. CPD is introduced in [11] for denoting the prosodic domain of units of the same type as the prosodic domain of their constituents: e.g. two contrasted prosodic words may produce a CU which is not an ip but it can be considered a compound of prosodic words.

The description of information structure consists of a sequence of functional labels that corresponds to CU constituents. In all descriptions presented in this paper topic and focus constituents are annotated by T and F labels and the PUSH and POP function by PH and PO

labels, respectively. Each constituent has two functions at the two IS levels and is annotated by a composed label resulted by linking two simple labels by "+" and enclosing them between round parentheses. Each CU is described by a sequence of two composed labels separated by slash.

Nested IS partitions can appear in our utterance partitioning due to the corresponding nested CUs instances within the utterance hierarchy.

3 A Functional analysis of Romanian declarative contours

The sentences used for generating the declarative contours for the functional analysis were selected from [5], where they are used to exemplify the relation between focus, prosody and word order in Romanian. The statements as well as the questions which have elicited them were uttered by a female speaker.

3.1 Short broad focus statement

The intonation of a short neutral statement in Romanian has a rising-falling pattern in an asymmetrical form with a lower falling pitch movement target tone than the tone in the beginning of the rising movement [3].

Fig. 1 illustrates the F0 contours of one neutral statement with a VO syntactic structure. It is the answers of the question (1).

Ce este zgomotul acesta? (1)
[What's that noise?]
PLÂNge coPIlul.
[The baby is crying.]

The contour is an intonational phrase (IP) with two prosodic words (PWs). The rising movement during the first accented syllable *plân* in Fig. 1 aligns the peak with the post-focal syllable of the first word. The tonal jump before the post-focal syllable and the constant level during this syllable apply a focus function on the verb.

The slow fall during the $H+L^*$ pitch accent applies a topic function on the last word. Both words bear new information but the sentence-initial one bears the focus within the CU produced by the utterance. The contour in Fig. 1 is described in (2). In the indices position of each composed label we added a ToBI description of the corresponding melodic contour.

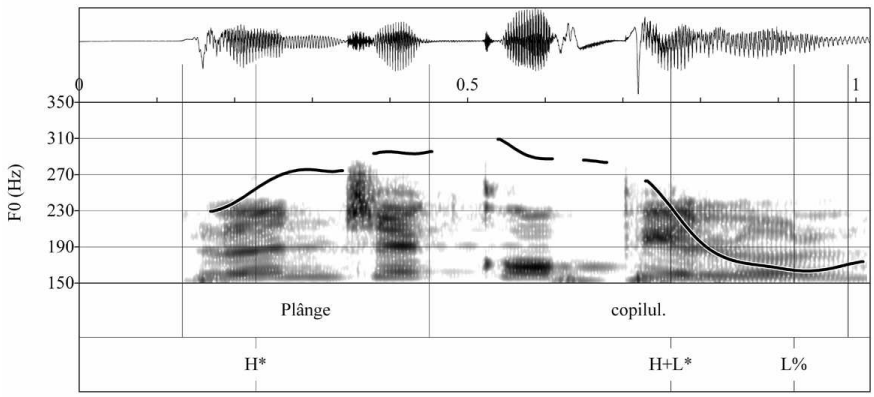

Figure 1. The F0 contour and the spectrogram of the utterance of the sentence *PLÂNge coPIlul.* 'The baby is crying.'

$$(PH + F)_{H^*}/(PO + T)_{H+L^*L\%} \tag{2}$$

In this case the contrast within CU is produced by the abrupt rising pitch movement after the first accented syllable during the H^* pitch accent and the slow falling pitch movement of the $H + L^*$ during the last prosodic word.

The $H^*/H + L^*$ pitch accent sequence describes Romanian broad focus statements [4]. This pitch accent sequence leads to a PUSH function on the first constituent and POP function on the last. The last pitch accent produces a low prominence and the last word bears the nuclear accent in Romanian broad focus statements.

3.2 Neutral statement with old-new information structure

The neutral statement (3) has a single word bearing new information (*DANEZA* 'Danish'). The F0 contour extracted from an utterance of this sentence is illustrated in Fig. 2.

Ce limbă vorbeşte bine ? (3)
[Which language does he speak well?]
VorBEŞte BIne daNEza.
[He speaks Danish well]

The F0 contour of the initial prosodic word (PW) of the verb has a high target tone followed by an abrupt fall on the last non-accented syllable down to beginning low tone level. This melodic characteristic gives it a focus function within the CU which includes the verb and the adverbial *bine* 'well'. Both verb and adverbial bear old information at the discourse level. At this local CU level, the verb is the focus element and the adverbial is the topic element. The topic function is signalled by the acoustic cue of slow rising pitch movement after the accented syllable. The end of pich movement is synchronized with the end of the last non-accented syllable.

The word *daneza* 'Danish' bears the new information at the discourse level. Its F0 pattern has an abrupt falling pitch movement (H^* pitch accent) and thus it carries the global focus at IS level. The highest level of the last pitch accent within the utterance gives it a PUSH function at the global CU level. The functional description is given in relation (4).

$$\{\{(PH+F)_{H^*}/(PO+T)_{L+<H^*}\}_{(PO+T)}/(PH+F)_{H^*L\%}\} \quad (4)$$

At the local level of the embedded CU the tonal level during the first half of the accented syllable of the adverbial is lower than that reached during the first part of the accented syllable of the verb. Consequently, a POP function is applied to the adverbial. The nuclear accent within the embedded CU is on the adverbial and it is also the global nuclear

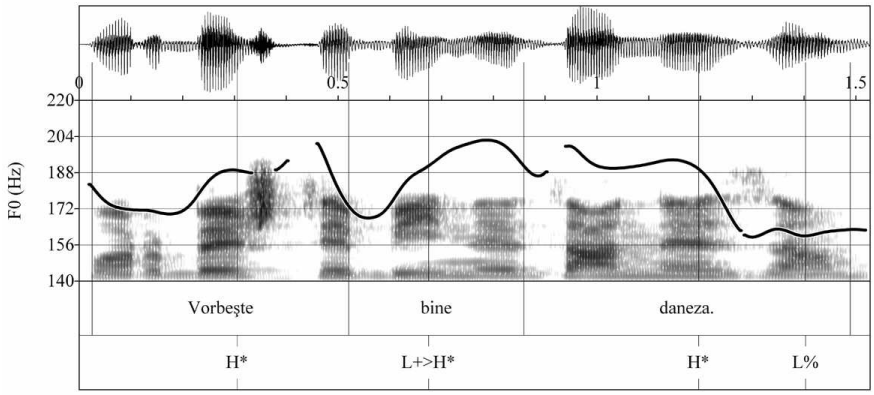

Figure 2. The F0 contour and the spectrogram of the utterance of the sentence *Vorbeste bine daneza.* 'He speaks Danish well'.

accent because it is related to the minimum tonal level of the utterance. This CU can be atomized and labelled by $(PO + T)$. It is in contrast with $(PH + F)$ constituent of the object *daneza*.

The contour presented in Fig. 2 confirms that in Romanian old-new information statements the focus corresponds to the word which bears new information [2].

3.3 Neutral all-new statement

The statement in (5) is a long all-new sentence with a SVO_1O_2 structure. The communicative structure of the utterance consists of a two level hierarchy (two nested CUs) leading to local and global topic and focus events within the utterance.

Ce se întâmplă în bucătărie? (5)
[What's going on in the kitchen?]
MaRIa face o prăjiTUră pentru GEORge. [Mary is baking a cake for George.] '

The F0 contour in Fig. 3 shows that the constituent of the subject *Maria* makes pair with a complex constituent of the verbal phrase, at

the global CU level. The topic function of the subject is encoded by the $L+H^*$ pitch accent during its accented syllable and is located in a separate phrase (an intermediate phrase). In [5] it is also claimed that topic elements may be located in a separate phrase.

In the case of the utterance in Fig. 3 the first ip isolates the topic constituent and the global focus is located in the following complex constituent. In (6) the subject is annotated by T partial label (within $PH+T$ label) and the complex constituent by F partial label (within $PO+F$ label). The global PUSH function is due to the high target tone during the first accented syllable and global POP function is produced by the lowest target tone of the utterance during the last accented syllables.

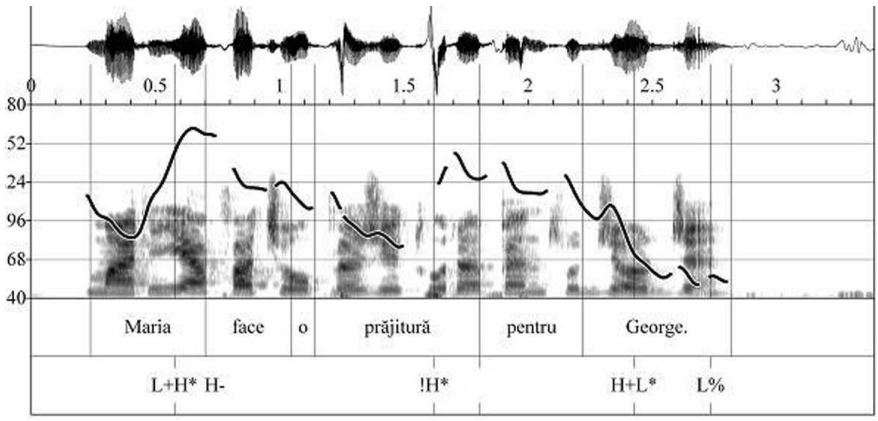

Figure 3. The F0 contour and the spectrogram of the sentence *MaRIa FAce o prăjiTUră pentru GEORge.*('Mary is baking a cake for George')

$$\{(PH+T)_{L+H^*H-}\ /\{\{(PO+F)_{!H^*}/(PH+T)_{H^*}\}_{(PH+T)}/$$
$$\{(PH+T)/(PO+F)_{H+L^*L\%}\}_{PO+F}\}_{PO+F} \quad (6)$$

The complex right constituent includes the verb, the O_1 object and O_2 object which are uttered at lower target levels than the subject.

It is compound by two complex constituents (CUs): i. the left CU of the verb and O_1 object (*face / o prăjitură*); ii. the right CU of the preposition *pentru* and the O_2 object (*George*). The left CU contains the topic at the local level of verbal phrase and the right CU contains the focus element. They are annotated by $PH+T$ and $PO+F$ labels, respectively, in (6) in the index position.

At its turn the left CU has a focus on the verb (*face*) due to the constant pitch level of its pitch movement. The noun bears a topic function marked by keeping high tone during the last syllable after the high target tone is reached. It also bears a PUSH function due to its highest target tone. The topic element of O_1 object is more prominent and it bears the nuclear accent. Consequently, the left CU is also a $PH+T$ constituent at the higher level of the verbal phrase.

The right CU (*pentru / George*) within the CU of the verbal phrase is a focus constituent and has the following two constituents: i. the preposition (*pentru*), which bears a topic function at this local level due to its slow downstepped pitch movement; O_2 object (*George*) which has a focus function marked by the abrupt falling pitch movement. The O_2 object also bears a POP function due to its minimum tonal level. The nuclear function of the last accent of O_2 object gives a POP function to the whole right part of the verbal phrase constituent. Thus, the higher level CU having the left CU and the right CU as constituents is the complementary $PO+F$ constituent of the subject within the global utterance level.

As observed in [5] in Fig. 3 the contour is declining, with compression of the pitch range. The last pitch accent is the nuclear accent of the utterance having the minimum level for its target tone.

These examples illustrate how complex utterances can be partitioned by means of a CU hierarchy. CUs are partitioned by their topic-focus and PUSH-POP information structure.

4 Conclusions

In this paper we propose an improved framework for the intonation contour analysis by adding to the phonological annotation a functional

description of local and global communicative units. We introduced the term of communicative unit in order to relate it to information structure. CUs are units produced during cognitive process while the prosodic units are produced by information packaging process during the utterance of text. There are cases when CUs overlap prosodic phrases. This view permits a Romanian utterance analysis at the communicative level in relation to the phonological events.

The goal of this paper is to present how utterances can be partitioned into a hierarchy of communicative units. At each level CUs have their own topic-focus and PUSH-POP information structure. The topic and focus functions are discussed at the pragmatic level and we illustrate their relation to the semantic significance: e.g. in the statement (3) the word *daneza* is marked as global focus at the pragmatic level as it is described in (4) but it may be also thought as one element of a set of alternatives related to the question (3).

The PUSH and POP functions are discussed in this paper only at the pragmatic level and they are related to the contrast between the target tones of CU constituents. At the semantic level the PUSH and POP functions have to convey the aboutness principle formulated for IS at the text level. A pragmatic implementation of this function has to be found in the further work.

This utterance view of nested CUs and their corresponding nested partitions of Information Structure can potentially describe any utterance partitioning more accurately, based on its intonation contour interpretation. The logic of utterance organization can be understood by identifying all paired constituents, their hierarchical organization and their local and global information structure.

The description of utterances as a hierarchical CU and IS structures can shed new light within cross-linguistic analyses of intonational contours and more particular, on the discussion concerning focus position, nuclear accent position within prosodic phrases. This model can be a basis for defining the set of the phonological categories of pitch accents by understanding their F0 patterns in respect to their IS functions. Common features and different features can be deduced for equivalent contours of different languages implied in cross-linguistic analysis.

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Fuzzy Rule-based System for Pattern Recognition and Automated Classification

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Abstract

The sorting method contains a fuzzy system that classifies waste into three classes: dangerous (toxic, flammable, harmful to the environment), recyclable and undetermined. To increase the decision weight since some waste is perilous (eg. batteries, spray bottle under pressure, oil drums, etc.), we implement a shape-signature identification algorithm.The extraction of objects contour in order to obtain the signature is done with contour tracking algorithm. The appropriate determination of contour also enables efficient calculation of shape and size features used in fuzzy system. The elaborated system aims to detect dangerous wastes that arrive among the recyclable waste because of people ignorance.

Keywords: Automated sorting, border tracing algorithm, shape-signature identification, fuzzy system.

1 Introduction

Improper waste management seriously affects the environment not only by local pollution but also contributing to global greenhouse effect gas emissions.

It is known mechanical, aeraulic sorting, infrared spectroscopy and X-ray detection for plastic (it's even possible to differentiate types PVC, PET, etc.), but the most effective sorting is currently the manual sorting. The sorting staff can separate bottles of different color, paper of different quality, dangerous waste etc. This method is costly both in cost and in time. One possibility to increase the efficiency is the

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use of semiautomatic Decision Support Systems [1], [2], [3]. There are known implementations of optical sorting through the application of algorithms for digital images recognition [4], [5], [6]. The main objective of designed SSD for waste sorting depends on the critical issues facing a certain society, country. Many researchers have applied waste management options: material recovery through recycling / reuse (paper, plastic, metals, etc.), incineration, composting and storage of perilous waste.

The automated sorting method proposed in this paper includes four basic steps. At the first step we identify if there is a danger symbol on the label using normalized correlation coefficient (CCN) and a genetic algorithm (GA). If the label contains one of the symbols of danger, the packaging is considered dangerous and is removed from the packaging tape for recycling. The second step consists in extracting the characteristics of packaging and includes a contour tracking algorithm and an algorithm for extracting the signature (the distance between the centroid and contour). At this stage we apply morphological operators: dilation and erosion to obtain a complete pattern (without holes) and we calculate the perimeter and the area of the shape. Another parameter of the fuzzy system: the shape symmetry is determined based on the signature. The 3rd step includes a fuzzy logic decision support system which, based on fuzzy input variables (shape, size and symmetry), groups the packaging into three classes: recyclable, dangerous and determined. At the fourth step based on a difference function of signatures, the decision taken at the previous step is confirmed or refuted.

2 Automated identification of objects based on NCC and GA

Dangerous symbol identification on the label is done by templates matching with the perilous symbols [7] using CCN. The genetic algorithm generates new values for rescale the image from 1/3 to 3 times and rotation from 0 to 360, so the results do not depend on the size

and image capture angle. The method is described in detail in [6].

The implementation of a GA reduces the calculation time for new values of size and rotation angle of the image, preserving only the best previous results at generation of new values. GA reduces the calculation steps thus achieving an optimal outcome time.

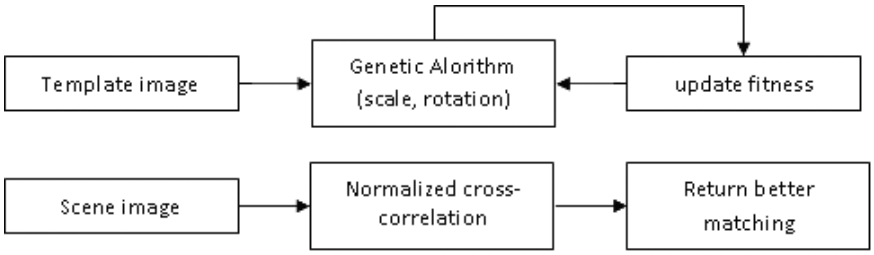


Figure 1. The block diagram of the method based on normalized cross-correlation and GA [6]

Table 1. Implementation of the method based on CCN and GA

Template and packaging picture	Proposed method implementation	Final result

The benefits of implementing AG are argued and presented in [6].

3 Features extraction

The process of the shape feature extraction comprises the following steps:

- image binarization, segmentation using a threshold histogram method;
- application of morphological operators: dilation, erosion;
- implementation of a contour tracking algorithm;
- implementation of a signature extraction algorithm;
- determination of shape, size, symmetry.

3.1 Contours determination

For image binarization it is used a threshold algorithm [8] or [9] with a single threshold. Having the binary image, we are looking for selection of the contour pixels, hence the name of the algorithm – contour tracking algorithm [10]. Contours tracking algorithm (walk of the blind) scans the image from the upper left corner until it finds a pixel (different from the background) which belongs to the object, this pixel is denoted by Ps and it is the start pixel of the contour. The current pixel is denoted by Pc. It is chosen a scroll direction of the contour. It is defined a variable dir that holds previous moving direction of the previous item along the contour to the current one. It initializes $dir = 0$ and $arm = (dir + 5)$ modulo 8.

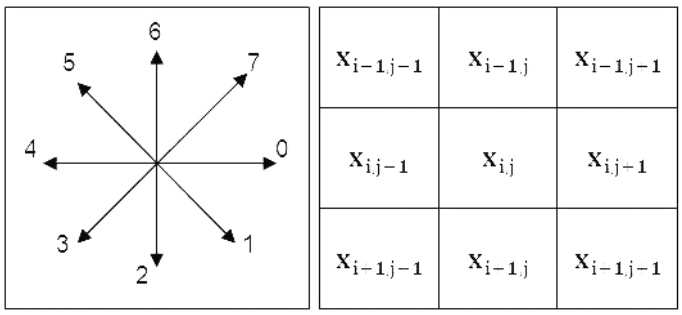
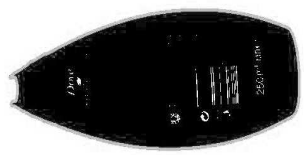
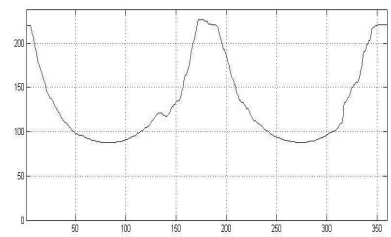
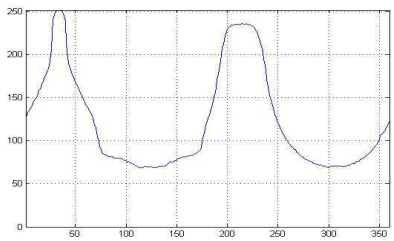


Figure 2. Representation of the direction and vicinity 3x3 of the current pixel

It scans the neighbors of current pixel in the chosen scroll direction starting from the arm position, until finding a new pixel object which is labeled and becomes current pixel Pc. Variable direction dir is updated based on the current pixel found, and the variable arm is recalculated according to dir. The previous step is repeated until the closure of contour, then until the start pixel becomes again the current pixel. Examples of contour determination are shown in Table 2, column 1.

Table 2. Edge and signatures representation

Edge determination	Shape signatures
	
	

3.2 Shape-signature extraction

A signature is a one-dimensional representation of a surface. It was chosen to define the signature as the distance from a reference point – the gravity center of shape (g_x, g_y) – to every point of the contour (Table 2, column 2) [11]. The idea is to reduce the representation of contour to a one-dimensional function, which is basically easier to describe than the original two-dimensional outline.

The shape's signature is a reversible representation – knowing the signature it is possible to reconstruct the object contour. The size of the signature vector is of 360 values, one value obtained for each angle value of half-straight line which starts from the point (g_x, g_y) and intersects the contour (the edge). If the intersection of a half-straight line with the contour which scans the shape is made at several points, then it will preserve the farthest value.

$$d(n) = \sqrt{(x(n) - g_x)^2 + (y(n) - g_y)^2}. \tag{1}$$

3.3 Shape, size, symmetry determination

The paper [12] is a review of description and representation techniques of forms in order to recognize the objects or their automatic classification. The number of distinctive features is directly proportional to the correctness of the obtained results after execution of an Pattern Recognition algorithm and indirect to computing time. For example, if the color is not a distinctive feature, it is not recommended to be included in the algorithm. For packages description there were chosen three distinctive characteristics: shape, size and symmetry. Before calculating the area of the object, the morphological operators were applied: dilatation – to fill "holes" which were obtained from segmentation due to light reflection, and then erosion – to return to the original shape (delete pixels that were added at dilatation). The area (A) is considered to be the total number of pixels that represent the object. The perimeter (P) is represented by the pixels of contour. The shape and the size are calculated according to the formulas below:

$$Shape = \frac{4\pi A}{P^2}, \tag{2}$$

$$Size = \frac{A_shape}{A_image}. \tag{3}$$

A_shape represents the area of object that we want to identify and the *A_image* – is the total number of pixels from image. The contour of shape is not a smooth line after segmentation, but contains unevenness,

therefore at the application of the formula for form calculation, we get approximate results, with a margin of error. For this reason it is done a normalization so that the values should be in the range [0, 1]. The symmetry is determined by signature. We assume that the symmetrical shapes (after horizontal axis) represent dangerous packaging – bottles of spray under pressure, batteries etc.

4 Fuzzy Rule-based System

Fuzzy logic differs from binary logic based on variables 0 or 1 ("true" or "false") by intermediate values, which is "true" or "false" with a degree of membership, 0 representing not belonging and 1 – total belonging. The fuzzy system transforms linguistic expressions "huge size", "elongated shape", "small symmetry" that include imprecision / ambiguity in numerical values. Hence, the representing degrees of membership in interval [0, 1], correspond to the partial truth between "completely true" and "completely false". There can be used modifiers / linguistic enhancers "somewhat high", "little", "pretty little", "extremely" etc. with numerical values for the system inputs. The Fuzzy logic renders values.

1) **The definition of fuzzy sets for input variables**

The fuzzification of inputs consists in determining the belonging degree of input data to fuzzy sets, by the membership function. The most usual membership functions are triangular and trapezoidal shape. We have chosen the trapezoidal membership function to define the set, because compared to the triangular membership functions, it has an interval in which the membership degree is 1, but the choice is subjective.

a) Definition of fuzzy sets for input variable *Size*

The input variable *Size* is defined by four trapezoidal fuzzy sets: Small, Medium, Large, and Extra Large with the universe of discourse [0, 0.5]. It is considered that the huge-sized objects can occupy up to 50% of the viewing area, hence the upper limit of the universe of

discourse is $0,5$. A large size object covers about 10%–15% of the viewing area. According to Figure 3, when an object occupies between 15% and 20% of the viewing area, then it can be considered as belonging also to the fuzzy set "Large" and the fuzzy set "Extra Large", but with different degrees of belonging. The 4 fuzzy sets for input variable *Size* are represented in Figure 3.

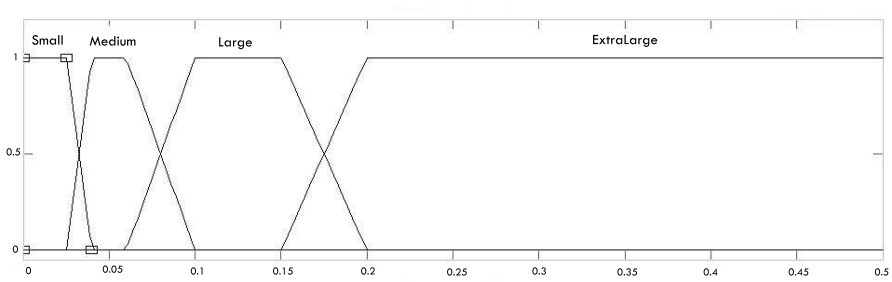


Figure 3. The fuzzy sets for input variable *Size*

b) Definition of fuzzy sets for input variable *Shape*

The input variable *Shape* is defined by four trapezoidal fuzzy sets: Elongated, Irregular, Oval and Round. According to the formula (2) the obtained values for *Shape* can be in the range [0, 1], with 0 corresponding to an elongate object similar to a line segment and 1 – to a circle. The fuzzy sets for input variable *Shape* are represented in Figure 4.

c) Definition of fuzzy sets for input variable *Symmetry*

The input variable *Symmetry* is defined by three trapezoidal fuzzy sets: Small, Medium and Big. It is expressed dimensionless based on differences between the normalized signature of the object on the intervals [0, 90], [90, 180], [180, 270], [270, 360] degrees. The definition of fuzzy sets for *Symmetry* was based on our observation: the bottles under pressure (from various sprays, lacquers), the batteries are sym-

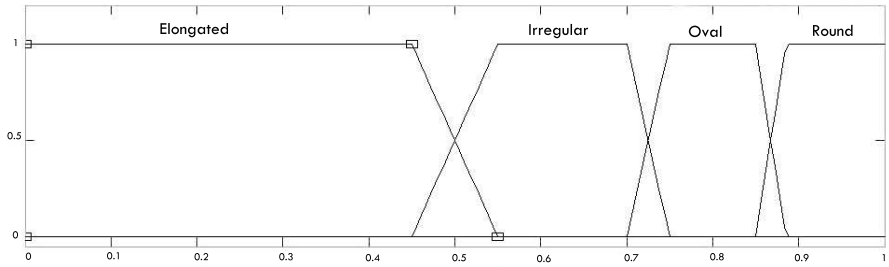


Figure 4. The fuzzy sets for input variable *Shape*

metrical towards the horizontal axis too. These packages / objects are dangerous and can't be recycled, so the membership on the Symmetry set is an important weight in decision made by the system. The fuzzy sets for input variable *Symmetry* are represented in Figure 5.

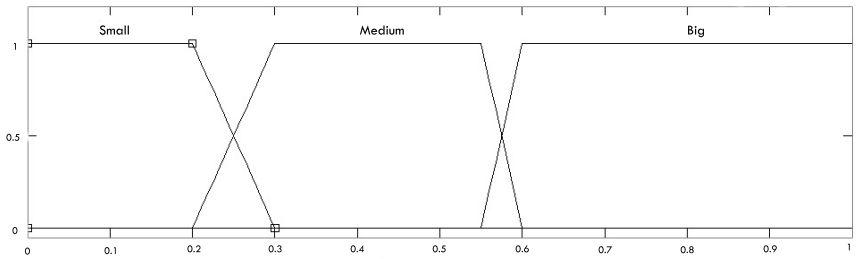


Figure 5. The fuzzy sets for input variable *Symmetry*

2) **The definition of fuzzy sets for the output variables**

Fuzzy system classifies waste into three classes: Dangerous (toxic, flammable, harmful to the environment), Undetermined and Recyclable. The output variable ObjectType is defined by fuzzy sets as in Figure 6, creating a Mamdani type system. Triangular membership functions have the small value of the triangle base (symbolical value),

therefore the system can be considered closer to a Sugeno fuzzy system. Choosing the type of system Mamdani and not Sugeno is given by the defuzzification function MoM (Moment of Maximum) type. Small base values of the 3 fuzzy sets are chosen as follows: about 0 value for Dangerous, ~ 0.5 for Undetermined and ~ 1 for Recyclable.

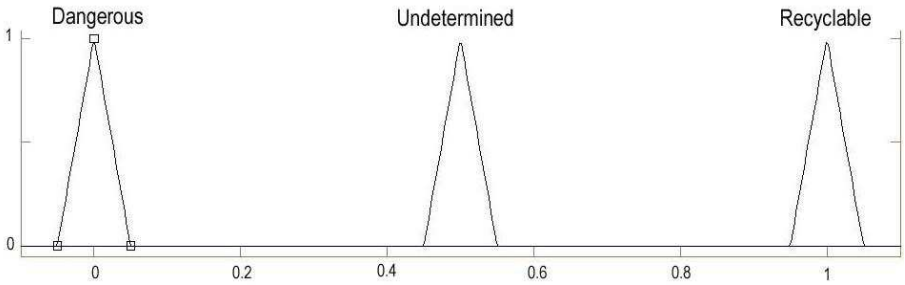


Figure 6. Fuzzy output representation

3) **Rules base of fuzzy logic system**

The rules of inference are the rules that bound fuzzy input variables of a system with fuzzy output variables of the same system. These rules are in the form as follows: IF condition / premise1 AND / OR condition / premise2 (AND / OR ...) THEN action. The entirety of rules forms the basis of rules, called Knowledge Base. The rules are qualitative statements derived from analyses made during recycling process of packaging. On this basis, an intuitive enunciation of fuzzy rules for the submitted application was defined: different weights associated to the rules can be observed. These weights are also called coefficients of certainty.

Examples of applied inference rules:

IF elongated shape AND small size AND big symmetry THEN object type is dangerous (1)

IF elongated shape AND medium size AND big symmetry THEN

object type is dangerous (1)

IF elongated shape AND small size AND medium symmetry THEN object type is dangerous (0.7)

IF elongated shape AND medium size AND small symmetry THEN object type is undetermined (1)

IF elongated shape AND small size AND small symmetry THEN object type is undetermined (1)

IF elongated shape AND extra-large size AND big symmetry THEN object type is undetermined (1) etc.

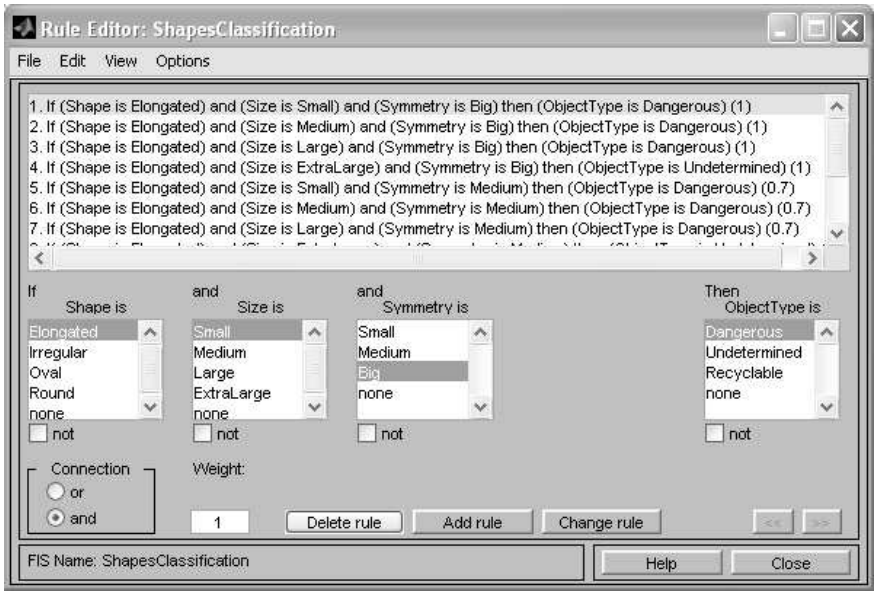


Figure 7. Inference rules applied to SF

The rules may be modified, supplemented, deleted via a graphical interface shown in Figure 7.

The intersection of two fuzzy sets is equivalent to the operation "AND logic" which is the minimum of two membership degrees. Between the premises of the rules the operator "min" is applied, the resulting value is used to truncation of output variable. When we have more active rules, a reunion of truncated fuzzy sets resulted from application of each rule is applied.

4) **Defuzzification**

In order that the developed system to classify Object Type, it is necessary that fuzzy sets of the output to be transformed into crisp data. This transformation is called defuzzification. The most common methods of defuzzification are: Centroid, COA, COG, SOM (smallest of maximum), MOM (mean of maximum), LOM (largest of maximum). For the developed system the **MoM** method was chosen. The fuzzy system for detection type of packaging applies the defuzzification operator MOM, selects from active fuzzy rules. An example of defuzzification is shown in Figure 8. Observe the active rules 7, 8 and 13. Applying MoM – the mean of maximums (of 0, 0,5 and 1), we get the result 0,5. If we have two activated rules, with the same weight, mean of 0 and 0,5 or mean of 0,5 and 1, in addition to relation MoM we apply the logical relationship AND – minimum of two values.

The achievement of this system is 95%. Some packages from the group Undetermined were identified as Dangerous. A Dangerous object has been classified as Undetermined. Most importantly, these packages are not classified as Recyclable, but to ensure an accurate automatic classification, the items from the Recyclable class are checked at the next step by identifying the signature (of the form).

5 Signatures identification

The forms signature is a functional representation. For the proposed system the signature representation by the distance from the form cen-

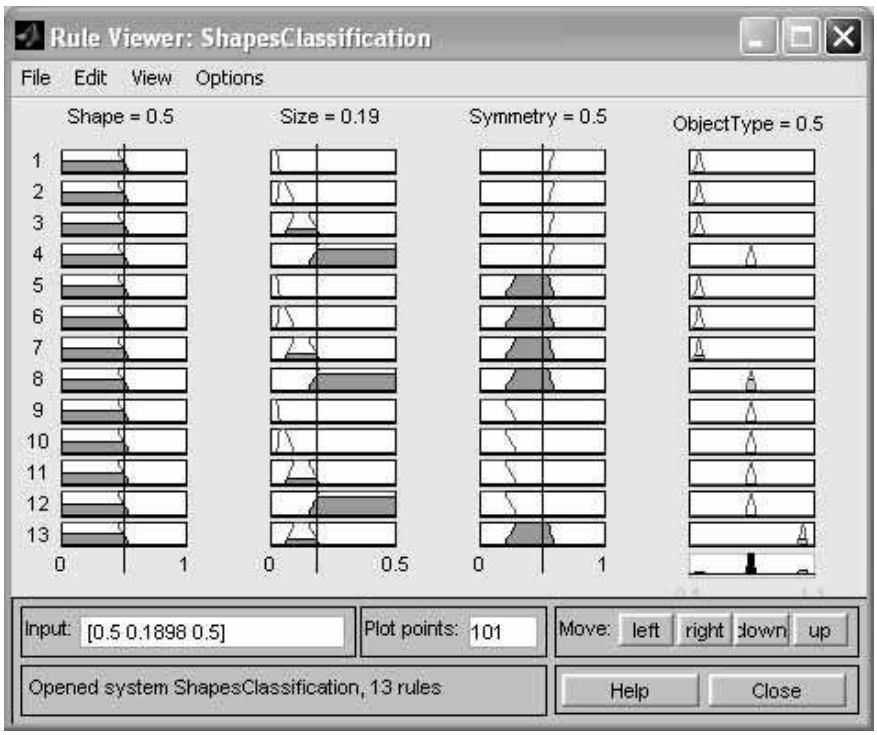
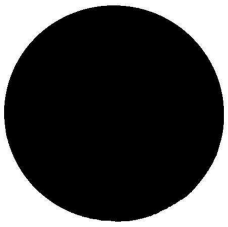
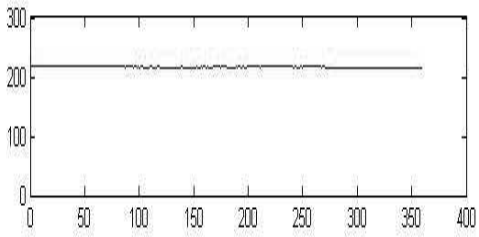
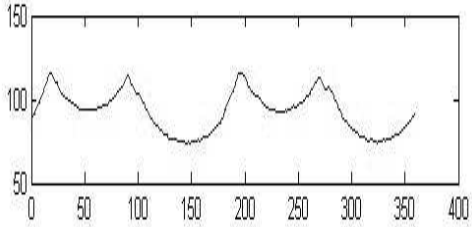


Figure 8. Example of defuzzification

ter to each point of edge was chosen. Examples of shapes and their signatures are shown in Table 3.

For objects with the same shape but different sizes, the signatures will be similar but with different amplitude (due to varying degrees of scaling). For objects with different orientation, the signatures will be similar but moved to the left / right. Applying normalization to the signature, the effect of scaling disappears. Both the correlation function and difference function ensure the invariance to rotation. Another possibility to eliminate the effect of rotation is selecting the starting point for signature generation as the farthest point from the center (if it is a single point) or the farthest point on the axis. One way to get the

Table 3. Graphic representations of forms signatures

Examples of forms	Signatures of these forms
	
	

invariance of forms size would be the normalization of obtained values: their representation between 0 and 1. This method is sensitive to noises that affect the maximum and minimum values. For the developed system the size of the packaging matters, therefore no normalization was performed. The correlation is a statistical method applied to determine a relationship between two or more variables. The correlation coefficient (r) is a quantitative value between -1 and $+1$, and describes the relationship between variables (see Table 4). Extreme values imply a correlation between variables while $r = 0$ is the total lack of linear relationship.

Examples of signatures identification by total correlation coefficient are represented in Table 5.

Defined for two variables with normal distribution (x, y), the lin-

Table 4. Guidelines to interpreting Pearson's correlation coefficient

Strength of association	r **positive**	r **negative**
Small	0.1 to 0.3	−0.1 to −0.3
Medium	0.3 to 0.5	−0.3 to −0.5
Large	0.5 to 1.0	−0.5 to −1.0

ear correlation coefficient called Pearson's coefficient (total correlation coefficient), is determined as follows [13]:

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - m_x)(y_i - m_y)}{\sqrt{(\sum_{i=1}^{n}(x_i - m_x)^2(\sum_{i=1}^{n}(y_i - m_y)^2}}, \tag{4}$$

where

m_x – the average of values $x : m_x = \sum_{i=1}^{n} \frac{x_i}{n}$,

m_y – the average of values $y : m_y = \sum_{i=1}^{n} \frac{y_i}{n}$.

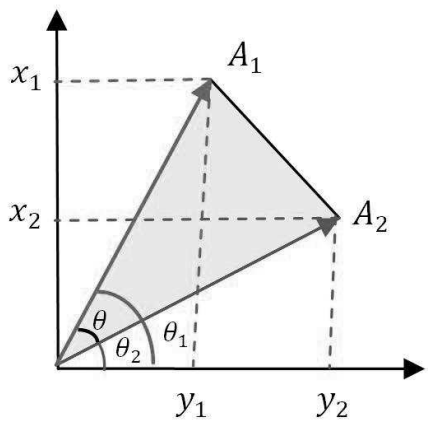
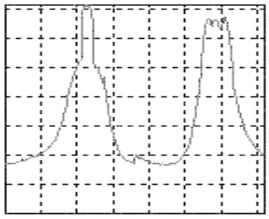
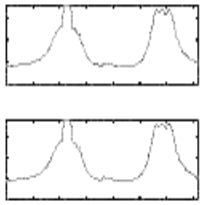
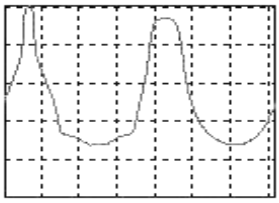
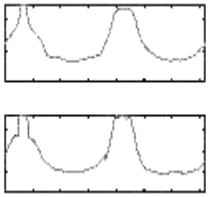
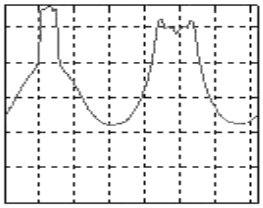
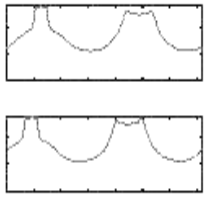


Figure 9. The cosine θ coefficient for a two-dimensional space [13]

Calculation of cosine θ coefficient (see Figure 9) in a two-dimensional space is based on trigonometric relationships of angles and is expressed

Table 5. Signatures identification by total correlation coefficient

Object shape	Shape signature	Shape signature identification
Recyclable item, included in the DB		 Recyclable (pet10.bmp)
Recyclable item, not included in the DB		 Recyclable (pet12.bmp)
Unrecycled item intentionally included		 Undetermined (ambalaj.bmp)

by the following formula:

$$cos\theta_{A_1A_2} = \cos(\theta_1 - \theta_2) = \frac{x_1y_1 + x_2y_2}{\sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)}}. \quad (5)$$

The above formula can be adapted for n independent factors:

$$cos\theta_{A_1A_2} = \frac{\sum_{i=1}^{n} x_iy_i}{\sqrt{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2}}. \quad (6)$$

The cosine θ coefficient is a measure of angular distance used for estimating similarity between the forms. This correlation coefficient indicates a complete similarity between the two forms A_1 and A_2 if $\cos\theta = 1$ and a total distinction if $\cos\theta = 0$. The both formulas were tested and we opted for the total correlation coefficient (called Pearson's coefficient). The database contains the signatures of all forms (recyclable undetermined and dangerous) to be identified as a non-recyclable object if it has been misclassified. The objects that are not included in the database were also tested and, the result is satisfactory.

6 Conclusions

The elaborated decision system aims at detection of dangerous waste that appears because of ignorance of people with respect to recyclable waste.

The Hybrid algorithm – CCN combined with GA – enables the detection of danger symbols on the label with a very high rate of success. Implementation of genetic algorithm for generating rotation and scaling values is a solution when template differs in size and rotation angle of the image scenes. GA also reduces calculation steps reaching an optimal result within a reasonable time.

Objects contour extracting in order to obtain the signature is made by the tracking contour algorithm. Correct determination of the contour also enables efficient calculation of shape and size – features used in fuzzy system.

Automatic classification of waste is carried out with a fuzzy system the inputs of which are: shape, size and symmetry. The choice of fuzzy sets is subjective and was made after analysis of packaging that should be recycled, and the dangerous ones must be collected in special containers. To increase the weight of decision that some waste is dangerous or can be recycled, an algorithm for form (packaging) signature identification is implemented.

The developed system can be adapted to sort other objects. Waste sorting was chosen to emphasize that their correct sorting would be a solution for reducing pollution, an important factor nowadays.

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Abstract of PhD Thesis

Title: Human actions recognition with modular neural networks
Author: Veaceslav Albu
Supervisor: Prof. Svetlana Cojocaru
Institute: Institute of Mathematics and Computer Science, Academy of Sciences of Moldova, Chişinău
Date of defence: November, 9, 2016

The main purpose of the dissertation is elaboration of a multimodal method for real time classification of human reactions (joining emotions and actions) into typical and non-typical in a certain environment, that ensures an effective functioning of systems destined to human actions monitoring in real time. This kind of classification task is very useful in different applications, where the number of gestures of the human is limited, such as: customers at the various types of automated machines, drivers, assembly line workers, hospital patients etc.

The proposed tool provides statistical observations and measurements of human emotional states during an interaction session with a software product (or, optionally, with a hardware plus software complex). Using computer vision and machine learning algorithms, emotions are recorded, recognized, and analyzed to give statistical feedback of the overall emotions of a number of targets within a certain time frame. Similarly, the actions of human subjects, which a user can perform during the interaction with a piece of software/hardware complex were classified. The feedback, produced by the proposed system, can provide important measures for user response to a chosen system.

The core part of the system consists of two main architectures: modular neural network architecture and deep learning architecture. An application example of this research is a infrared camera system embedded in a machine that is used frequently, such as an ATM. We use camera recordings to capture the emotional state of customers (happy, sad, neutral, etc.) and build a database of users and recorded emotions to be analyzed later. For the purposes of the study, a hardware complex was developed and tested, which was used for psychological experiments.