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# Communication Equivalence Classes in Networks

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**Abstract**

The quotient structure defined by a specific equivalence relation allows to simplify the study of complex communications between processes in computer networks. The quotient also preserves the initial algebraic structure.

This paper describes an algebraic structure modelling the communication in computer networks; it continues the study initiated in [1]. Given two communicating processes, we consider the translations between their communication ports, and introduce an equivalence relation between them in order to express that semantics of the communication between processes does not depend on the permutations of the communication ports. Considering a property $P$ preserved by this abstract semantics of the communication, we first define $P$-translations, and then provide conditions for permutation-independent communication to preserve the property $P$. The quotient structure defined by this equivalence relation allows to simplify the study of a complex communication between processes to the study of the equivalence classes representatives. The quotients preserve the initial algebraic structure.

A **communication structure** is given by the following elements:

(i) $\mathcal{P}$ – the set of processes; $p, q, s, \ldots$ range over $\mathcal{P}$;

(ii) for every process $p \in \mathcal{P}$ we associate the set of communication ports $H(p)$, and a permutation group $S^p_{H(p)}$;

(iii) for every process $p \in \mathcal{P}$, and for every subset of handles $K \subseteq H(p)$, we associate a permutation subgroup over $K$, $S^p_K = S(K) \cap S^p_{H(p)}$, where $S(K)$ is the permutation group over $K$; obviously, $S(K)$ can be viewed as a subgroup of $S(H(p))$.

The reader can find more information on the algebraic notions and results in [3] and [4]. Since $H(p)$ is the set of all communication ports of a server process $p$, the permutations of $S^p_{H(p)}$ describe the communication symmetry of $p$, which expresses the possibility to change the communication ports without affecting the communication. We can remark that:

1. If $H \subseteq K \subseteq H(p)$, then $S^p_H \leq S^p_K$.
2. If $H \subseteq K \subseteq H(p)$, $\rho \in S^p_K$ and $\rho/_{K \setminus H} = id_{K \setminus H}$, then $\rho \in S(H)$. Clearly, $\rho$ is also in $S^p_{H(p)}$; therefore $\rho \in S(H) \cap S^p_{H(p)} = S^p_H$.
3. The fact that $H \subset K$ does not imply that $S^p_H < S^p_K$; it is possible to have $S^p_H = S^p_K$.

Let $\mathcal{P}$ and $\mathcal{Q}$ be two communication structures; **a correspondence** between two processes $p \in \mathcal{P}$ and $q \in \mathcal{Q}$ is a triple $(S^p_H, \varphi, S^q_{H'})$, where $H \subseteq H(p)$, $H' \subseteq H(q)$ and $\varphi : S^p_H \to S^q_{H'}$ is a group morphism. If $\varphi$ is one-to-one, we call it a **faith** correspondence. If $\varphi$ is onto, we say that the correspondence is **full**.
We denote by $\mathcal{C}(p, q)$ the set of all correspondences between $p \in \mathcal{P}$ and $q \in \mathcal{Q}$.
$\mathcal{C}(\mathcal{P}, \mathcal{Q}) = \bigcup\{\mathcal{C}(p, q) \mid p \in \mathcal{P}, q \in \mathcal{Q}$.

Let $\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{S}$ be three communication structures. The composition of correspondences is a partial binary operation $\circ : \mathcal{C}(\mathcal{P}, \mathcal{Q}) \times \mathcal{C}(\mathcal{Q}, \mathcal{S}) \longrightarrow \mathcal{C}(\mathcal{P}, \mathcal{S})$ defined by

$$(S^p_H, \varphi, S^q_{H'}) \circ (S^q_{H'}, \varphi', S^s_{H''}) = (S^p_H, \varphi' \circ \varphi, S^s_{H''}),$$

where $p \in \mathcal{P}$, $q \in \mathcal{Q}$, $s \in \mathcal{S}$ and $H \subseteq H(p)$, $H' \subseteq H(q)$, $H'' \subseteq H(s)$. We can associate a category $Comm(\mathcal{P})$ to each communication structure $\mathcal{P}$ defined as it follows:

(i) the **objects** $pH$ are pairs $(p, H)$, where $p \in \mathcal{P}$ and $H \subseteq H(p)$;
(ii) a **morphism** $\varphi : pH \to p_1H_1$ is a correspondence $(S^p_H, \varphi, S^{p_1}_{H_1})$.

The notation $\varphi \in Comm(\mathcal{P})(pH, p_1H_1)$ means that $\varphi : pH \to p_1H_1$ is a morphism of the category. The category $Corresp(\mathcal{P}, \mathcal{Q})$ of the correspondences from the structure $\mathcal{P}$ to the structure $\mathcal{Q}$ is defined by the following elements:

(i) the **objects** are triples $(pH, \varphi, qH')$, where $pH \in |Comm(\mathcal{P})|$, $qH' \in |Comm(\mathcal{Q})|$, and $(S^p_H, \varphi, S^q_{H'})$ is a correspondence from $p$ to $q$;
(ii) a **morphism** from $(pH, \varphi, qH')$ to $(p_1H_1, \varphi_1, q_1H'_1)$ is a pair $(\alpha, \beta)$ of morphisms $\alpha : pH \to p_1H_1$, $\beta : qH' \to q_1H'_1$ such that $\beta\, \varphi = \varphi_1\, \alpha$.

Let $\mathcal{P}$ and $\mathcal{Q}$ be two communication structures, and $Comm(\mathcal{P})$ and $Comm(\mathcal{Q})$ their associated categories. We say that $\mathcal{P}$ and $\mathcal{Q}$ are isomorphic, and we denote this by $\mathcal{P} \simeq \mathcal{Q}$, if their associated categories are isomorphic, namely there is a functor $\mathcal{F} : Comm(\mathcal{P}) \to Comm(\mathcal{Q})$ that is an isomorphism of categories.

If $\mathcal{P}$ and $\mathcal{Q}$ are two communication structures, a **communication map** between $\mathcal{P}$ and $\mathcal{Q}$ is a mapping $\mathcal{F}$ from $|Comm(\mathcal{P})|$ to $|Corresp(\mathcal{P}, \mathcal{Q})|$. We say that a communication map $\mathcal{F}$ is **compatible with the correspondences** from $\mathcal{P}$ if $\mathcal{F}$ is a (covariant) functor from $Comm(\mathcal{P})$ to $Corresp(\mathcal{P}, \mathcal{Q})$. This means that

1. for each $pH$ there is a unique $\mathcal{F}(pH) = (pH, \varphi, qH') \in$ $\in |Corresp(\mathcal{P}, \mathcal{Q})|$,
2. for each morphism $\alpha : pH \to p_1H_1$ of $Comm(\mathcal{P})$ there is a unique morphism $\mathcal{F}(\alpha)$ in $Corresp(\mathcal{P}, \mathcal{Q})$, and
3. $\mathcal{F}(\alpha\, \alpha_1) = \mathcal{F}(\alpha) \circ \mathcal{F}(\alpha_1)$ for all $\alpha, \alpha_1 \in Comm(\mathcal{P})$.

Let $\mathcal{P}$ be a structure, and $Comm(\mathcal{P})$ its communication category.

(i) For two given processes $p, q \in \mathcal{P}$, a **translation** $\delta : H \to H'$ between their communication ports is a bijection from $pH$ to $qH'$, where $pH$ and $qH'$ are objects in $Comm(\mathcal{P})$. We denote such a translation by $p_H \xrightarrow{\delta} q_{H'}$ (or simply by $\delta$ whenever there is no confusion), and by $\mathcal{T}(\mathcal{P})$ the set of all translations.

(ii) two translations $p_H \xrightarrow{\delta_1} q_{H'}$ and $p_H \xrightarrow{\delta_2} q_{H'}$ are equivalent (and we denote this by $\delta_1 \sim \delta_2$) if there exist two permutations $\rho \in S_H^p$, $\rho' \in S_{H'}^q$ such that $\delta_2 \circ \rho = \rho' \circ \delta_1$.

For each communication structure $\mathcal{P}$ we can define the category $Trans(\mathcal{P})$ of its translations, with the same objects $pH$ as in $Comm(\mathcal{P})$ (namely pairs $(p, H)$ with $p \in \mathcal{P}$ and $H \subseteq H(p)$), and the translations between communication ports as morphisms between objects.

Let us consider a property $P$ of the translations, and $\Re = \{\delta \in \mathcal{T}(\mathcal{P}) \mid \delta \text{ satisfies } P\}$. We say that $\Re$ is a **$P$-translation** over the communication structure $\mathcal{P}$ if for each translation $\delta$ in $\Re$, its equivalence class $[\delta] \in \Re$, and if $p_H \xrightarrow{\delta} p_H \in \Re$, then $\delta \in S_H^p$. The following result provides a condition for $\approx_\Re$ to become an equivalence. In this way $|Comm(\mathcal{P})|/_{\approx_\Re}$ defines classes of objects having similar communication.

**Theorem 1** *If $\Re$ is a $P$-equivalence over the communication structure $\mathcal{P}$, then $\approx_\Re$ is an equivalence relation over $|Comm(\mathcal{P})|$.*

**Theorem 2** *If $pH \approx_\Re qH'$, then the groups $S_H^p$ and $S_{H'}^q$ are isomorphic.*

Let $\mathcal{P}$ be a communication structure, $\Re$ a $P$-equivalence over the communication structure $\mathcal{P}$, and $\approx_\Re$ the corresponding equivalence relation. More results of the quotient set $|Comm(\mathcal{P})|/_{\approx_\Re}$ are presented in [2]; for instance, we prove that a communication structure defined on the equivalence classes does not depend on the classes representatives. Thus any two communication structures determined by two different representative choices are isomorphic. It makes sense to talk about the quotient communication structure with respect to a $P$-equivalence. Let $\Re$ be a $P$-equivalence and $\approx_\Re$ the associated equivalence relation. The following communication structure on $|Comm(\mathcal{P})|/_{\approx_\Re}$ is well-defined:

- processes are given by the equivalence classes $\{[pH] \mid p \in \mathcal{P},\ H \subseteq H(p)\}$;
- the set of handles $H([pH])$ is $H$;
- the group of permutations $S_{H([pH])}^{[pH]}$ is $S_H^p$.

This communication structure is called the **quotient communication structure** of $\mathcal{P}$ with respect to the $P$-equivalence $\Re$, and we denote it by $\mathcal{P}/_{\Re}$.

We can affirm that such a structure reflects the complexity of the communication along the computer networks. The need of a conceptual and formal framework which make possible the study of network communication problems and properties is necessary (in our opinion). This paper is a step to such a suitable framework. It defines and studies the communication structures, formal tools which are complementary to process algebras, and can describe, analyze, and provide a semantic framework to complex distributed systems involving process communication. The algebraic structure suggests constructive solutions for communication problems by considering the group actions on the communication ports, the existence of limits in our communication categories, and so on. A wealth of research topics remains to be considered.

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# An algorithm for solving a class of knapsack problems and its generalization

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**Abstract**

A class of knapsack problems which generalizes the classical ones is studied. Algorithms based on the dynamical programming and Branch and Bound methods are proposed. The correctness and time estimation of the algorithm are given.

## 1 Problem formulation

We study a class of knapsack problems which generalizes problems from [1-2]. The main mathematical model we shall use is formulated as follows.

Let a knapsack of size $D$ and a set of items $I = \{1, 2, \ldots, n\}$ be given. For each item $j \in I$ the cost $c_j$, the size $d_j$ and the volume $d'_j$ related to partition $I$ are known. The set $I$ is divided into $p$ non-empty disjoint subsets $I_1, I_2, \ldots, I_p$,

$$I = I_1 \cup I_2 \cup \cdots \cup I_p;\ I_i \cap I_j = \emptyset,\ i \neq j.$$

For each subset $I_l$, $l = \overline{1,p}$, the size $D_l$ is known.

According to this partition, from each of the subsets $I_l$, $l = \overline{1,p}$, we include into the knapsack no more than $k_l$ items and the sum of volumes $d'_j$ of these including items from $I_l$ does not exceed $D_l$. A sequence of items $i_1, i_2, \ldots, i_q$ $(q < n)$ from $I$ we name the packing in the knapsack if the sum of items sizes $d_j$ of the sequence does not exceed $D$, each of the subsets $I_l$, $l = \overline{1,p}$, contains at most $k_l$ items of the sequence and the sum of items volumes related to the partition $d'_j$

for each $I_l$ does not exceed $D_l$, $l = \overline{1,p}$. We consider the problem of finding of the packing in the knapsack which has the maximal sum of items costs.

This problem can be represented as a boolean linear programming model, which in the case $p = n$, $D_l = D$ and $k_l = 1$, $l = \overline{1,p}$, becomes the classical knapsack problem. Therefore it is $NP$-complete. We propose an algorithm of dynamical programming method for solving the formulated problem.

In the terms of boolean programming the problem can be formulated as follows.

To maximize the object function

$$\mu(x) = \sum_{j \in I} c_j x_j$$

on the subject

$$\begin{cases} \sum\limits_{j \in I} d_j x_j \le D; \\ \sum\limits_{j \in I_l} d'_j x_j \le D_l, \ l = \overline{1,p}; \\ \sum\limits_{j \in I_l} x_j \le k_l, \ l = \overline{1,p}; \\ x_j \in \{0,1\}, \ j \in I, \end{cases}$$

where $x = (x_1, x_2, \ldots, x_n)$. Here $x_j = 1$ if the item $j$ is included into the knapsack; otherwise $x_j = 0$.

## 2 The main results and algorithms for solving the problem

To solve this problem we shall use an algorithm for solving the following auxiliary problem:

to maximize the object function

$$\mu(x) = \sum_{j \in I_l} c_j x_j$$

on the subject

$$\begin{cases} \sum\limits_{j\in I_l} d'_j x_j \le D_l; \\ \sum\limits_{j\in I_l} x_j \le k_l; \\ x_j \in \{0,1\},\ j \in I_l, \end{cases}$$

We will number the elements of $I_l$ as $1_l, 2_l, \ldots, n_l$.

**Algorithm 1**

1. Set $M_0^l = \{(\emptyset, 0)\}$.
2. For $j = \overline{1_l, n_l}$ do steps a), b) and c):
   a) Set $M_j^l = \emptyset$;
   b) Add each element $(S, c) \in M_{j-1}^l$ to $M_j^l$. Then for each $(S, c) \in M_{j-1}^l$ form the element $(S \cup \{j_l\}, c + c_{j_l})$ if $\sum\limits_{i\in S} d'_i + d'_{j_l} \le D_l$, $|I_l \cap (S \cup \{j_l\})| \le k_l$, and add $(S \cup \{j_l\}, c + c_{j_l})$ to $M_j^l$;
   c) Find in $M_j^l$ elements $(S, c)$ and $(S', c')$ with the same second components. For each pair $(S, c)$ and $(S', c)$ we delete $(S', c)$ from $M_j^l$ if $\sum\limits_{i\in S'} d'_i \ge \sum\limits_{i\in S} d'_i$ ; otherwise delete $(S, c)$.
3. Find in $M_n^l$ the element $(S, c)$ with the maximal second component. Then $S$ is a solution of the packing knapsack problem.

To solve the main problem we shall use the following algorithm.

**Algorithm 2**

1. For each $l = \overline{1, p}$ solve the corresponding auxiliary problem.

2. Denote by $M_n^l$ the sets obtained for $l = \overline{1,p}$, when the algorithm 1 is applied. Set $M_n$ will contain elements which are obtained as the possible combination of the elements from $M_n^l$: for each $(S^l, c^l) \in M_n^l$, $l = \overline{1,p}$, element $(S,c) = (\bigcup_l S^l, \sum_l c^l)$ is added to $M_n$ if $\sum_{i \in S} d_i \leq D$ .

3. Find in $M_n$ the element $(S,c)$ with the maximal second component. Then $S$ is a solution of the packing knapsack problem.

The algorithm 1 is an extension of the algorithm for a classical knapsack problem from [3]. Therefore the correctness of the algorithm can be argued in analogous way.

**Theorem 1.** *Algorithm 1 finds the optimal solution of the auxiliary problem in time $O(n_l^2 c^l)$, where $n_l$ is the number of items in the set $I_l$ and $c^l = \sum_{j \in I_l} c_j$.*

To prove this theorem we need the following lemma.

**Lemma 1.** *Let be $(S,c) \in M_j^l$ after algorithm's running. Then:*
a) $S \subseteq \{1_l, 2_l, \ldots, j_l\}$;
b) $\sum_{i \in S} c_i = c$;
c) $\sum_{i \in S} d'_i \leq D_l$;
d) $|I_l \cap S| \leq k_l$;
e) *if* $(S',c) \in M_j^l$ *then* $S' = S$;
f) *if* $S' \subseteq \{1_l, 2_l, \ldots, j_l\}$ *and* $\sum_{i \in S'} c_i = c$, $|I_l \cap S'| \leq k_l$ ,
*then* $\sum_{i \in S} d'_i \leq \sum_{i \in S'} d'_i$;
g) *moreover, if* $S \subseteq \{1_l, 2_l, \ldots, j_l\}$ *and* $\sum_{i \in S} d'_i \leq D_l$, $\sum_{i \in S} c_i = c$, $|I_l \cap S| \leq k_l$ , *then there exists* $(S',c) \in M_j^l$;

**Proof**. We use the induction principle on the iteration number $j$. The statement for $j = 0$ is evident. Now we consider that $j > 0$ and $(S, c) \in M_j^l$. Here two cases may be:

*Case 1.* $j \notin S$. Then the element $(S, c)$ has been transferred from $M_{j-1}^l$ in $M_j^l$ at step 2(b). Therefore the properties a), b), c), d), and e) follow from the induction principle.

*Case 2.* $j \in S$. Then $(S\backslash\{j\}, c - c_j) \in M_{j-1}^l$ and the properties a), b), c), d), and e) hold too.

Now let us prove *f)*. Suppose that $S \neq S'$ and analyze the following three cases:

*Case 1.* $j \notin S, S'$. Then $(S, c)$ and $(S', c) \in M_{j-1}^l$. So, according to the induction principle $S = S'$.

*Case 2.* $j \in S, S'$. Then $(S\backslash\{j\}, c - c_j)$, $(S'\backslash\{j\}, c - c_j) \in M_{j-1}^l$. Therefore $S = S'$.

*Case 3.* $j \in S, j \notin S'$ or $j \notin S, j \in S'$. Then $(S', c)$ was eliminated at the step 2(c). Therefore f) holds.

Now let us prove the property *g)*. We shall use the induction principle on the number $k = maxS$, were $maxS$ is the maximum number of the elements of the subsets $S$. This property holds for $S = \oslash$. Let us consider $k = maxS > 0$. Then according to the induction principle in $M_{k-1}^l$ there exists element $(S\backslash\{k\}, c - c_k) = (S', c - c_k)$. Therefore the element $(S' \cup \{k\}, c)$ has been added to $M_k^l$ at the step 2(b). Then either $(S' \cup \{k\}, c) \in M_j^l$. So, the property *f)* holds $\square$.

**Proof of Theorem 1**. The correctness of the algorithm 1 follows from Lemma 1.

Since at the step 3 we find the element $(S, c)$ with the maximal second component and take the first component $S$ as the solution of the packing knapsack problem, the set $S$ is an admissible solution (conform properties c) and d) ), its cost is equal to $c$ (conform b)) and a better admissible solution does not exist (conform g)).

Let us find the estimation time of the algorithm 1. Observe, that the power of each set $M_{j-1}^l$ does not exceed $c$, because two elements with the same second component don't exist in $M_{j-1}^l$ (conform e)). Each modifier operations at the step 2(b) can be implemented in the

time $O(n^l)$, and must be repeated for all $O(c^l)$ elements from $M_{j-1}^l$. The step 2(c) needs $O(n^l c^l)$ time too, because it may be implemented simultaneously with the step 2(b): for that all elements of $M_j^l$ are memorized in a table with the length equal to $c^l$, are numbered by second component, and when an other element is tried to be put at a place, the sums of the volumes $d_j'$ are calculated and compared, condition $|I_l \bigcap S| \leq k_l$ is verified and the element, that does not implement this condition or, otherwise, element with the greater volumes sum, is eliminated. Therefore the algorithm finds the optimal solution in time $O(n_l^2 c^l)$ $\square$.

**Theorem 2.** *Algorithm 2 finds the optimal solution of the packing knapsack problem in time* $O(pn^2c + c^p)$, *where* $c = \sum_{j \in I} c_j$.

**Proof** . The correctness of the algorithm 2 can be argued in following way.

We obtain the sets $M_n^l, l = \overline{1,p}$, when the algorithm 1 is applied to solve $p$ auxiliary problems. The first components of elements of the sets $M_n^l$ form all admissible solutions for the corresponding auxiliary problem. At the step 2 all possible combination of elements from different sets $M_n^l, l = \overline{1,p}$ are formed if the sum of items volumes from $\bigcup_{l=1}^p S^l$ does not exceed $D$. Therefore, on the grounds of the theorem 1 and of the condition from step 2, sets $M_n$ contain all admissible solutions of the main problem. Thus, the correctness of the algorithm is argued.

Since at the step 3 we find the element $(S, c)$ with the maximal second component and take the first component $S$ as the solution of the packing knapsack problem, the set $S$ is an admissible solution, its cost is equal to $c$ and a better admissible solution does not exist.

Let us find the estimation time of the algorithm 2. At the step 1 we solve p auxiliary problems, which have the estimation time $O(n^2c)$ each of them. Therefore the step 1 needs time $O(pn^2c)$.

At the step 2 we form all possible combinations of $p$ elements from $c$ elements, therefore we need time $O(c^p)$.

Then, the estimation time of algorithm 2 is $O(pn^2c + c^p)$ $\square$.

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# On stability of an optimal situation in a finite cooperative game with a parametric concept of equilibrium (from lexicographic optimality to Nash equilibrium)*

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**Abstract**

A parametric concept of equilibrium (principle of optimality) in a finite cooperative game in normal form of several players is introduced. This concept is defined by means of the partition of players into coalitions. Lexicographically optimal situation and Nash equilibrium situation correspond to two special cases of this partition. The quantitative analysis of stability of an optimal situation for the independent perturbations of players' payoff functions is performed. The maximum level of such perturbations which save the optimality of a situation is found.

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**Key words and phrases:** finite cooperative game, concept of equilibrium, lexicographically optimal situation, Nash equilibrium situation, stability radius.

## 1 Introduction

The aim of a game-theoretical model is to find classes of the solutions which are rational (coordinated) for the players (participants) and organizations (coalitions) included into this model. There are cooperative and non-cooperative principles of optimality in a normal form game.



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Usually these principles lead to different optimal situations. There is no unified approach to determining such concepts in the theory of non-antagonistic games. The most famous one is Nash equilibrium concept and its different generalizations. In this work a parametric concept of equilibrium in a normal form game is considered. Such concept leads to generally-optimal situations and its parameter is the way of the partition of players into coalitions. Lexicographically optimal situation and Nash equilibrium situation correspond to two special cases of such a partition (where there are one coalition of all players and the set of one-element coalitions). The analysis of stability of the situation optimal on a given partition for independent perturbations of players' payoff functions is performed. A formula of the stability radius is derived. Thus the maximum level of perturbations which save the optimality of a situation is found.

Observe that a formula of the stability radius of a generally-efficient situation in a normal form game was earlier derived in [1] for another parametric concept of equilibrium. In two special cases that concept lead to Pareto optimal situation and Nash equilibrium situation. Besides that, the questions of stability of vector discrete problems with parametric principles of optimality are discussed in [2–5].

## 2 Basic definitions, denotations and properties

Let's consider the main subject of investigations in game theory – a finite non-cooperative game in normal form of several players [6,7], where each player $i \in N_n = \{1, 2, \ldots, n\},\ n \geq 2$, has the finite set of available strategies $X_i \subset \mathbf{R}, 2 \leq |X_i| < \infty$. Game realization and its outcome (situation) is unambiguously determined by the strategy choice of each player. Let on the game situation set which is the Cartesian product of the sets $X_j,\ j \in N_n$,

$$X = \prod_{j=1}^{n} X_j$$

the linear payoff functions of players

$$f_i(x) = C_i x, \quad i \in N_n,$$

are determined, where $C_i$ is the $i$-th row of the matrix $C = [c_{ij}] \in \mathbf{R}^{n\times n}$, $x = (x_1, x_2, \ldots, x_n)^T$, $x_j \in X_j$, $j \in N_n$. In the result of the game, which further will be called the game with a matrix $C$, each player $i$ receives the profit $f_i(x)$, which it tries to maximize.

Now we introduce a concept of a generally-optimal situation.

Any nonempty subset $J \subseteq N_n$ is called a coalition of players. For any coalition $J$ and any situation $x^0 = (x_1^0, x_2^0, \ldots, x_n^0) \in X$ we determine the set

$$W(x^0, J) = \prod_{j=1}^{n} W_j(x^0, J),$$

where

$$W_j(x^0, J) = \begin{cases} X_j & \text{if } j \in J, \\ \{x_j^0\} & \text{if } j \in N_n \backslash J. \end{cases}$$

Thus $W(x^0, J)$ is the set of the situations reachable from a situation $x^0$ by the coalition $J$ .

We define the binary relation of the lexicographic order " $\prec$ " in the space $\mathbf{R}^d$ of any dimension $d \in \mathbf{N}$, assuming that for any two vectors $y = (y_1, y_2, \ldots, y_d)$ and $y' = (y'_1, y'_2, \ldots, y'_d)$

$$y \prec y' \Leftrightarrow y_k < y'_k,$$

where $k = \min\{i \in N_d \ : \ y_i \neq y'_i\}$.

Then the following property is obvious.

**Property 1** *Let $y, y' \in \mathbf{R}^d,\ d \in \mathbf{N}$. If $y_1 < y'_1$, then $y \prec y'$.*

Let $s \in N_n,\ N_n = \bigcup\limits_{r\in N_s} J_r$ is a partition of set $N_n$ into $s$ coalitions, i. e. $J_r \neq \emptyset,\ r \in N_s;\ p \neq q \Rightarrow J_p \cap J_q = \emptyset$. For this partition we define the set $G^n(C, J_1, J_2, \ldots, J_s)$ of generally-optimal, or $(J_1, J_2, \ldots, J_s)$-optimal, situations in the game with a matrix $C$ as follows

$$G^n(C, J_1, J_2, \ldots, J_s) = \{x \in X \ : \ \forall\, r \in N_s \ (\zeta(x, C, J_r) = \emptyset)\},$$

where

$$\zeta(x, C, J_r) = \{x' \in W(x, J_r) \ : \ C_{J_r} x \prec C_{J_r} x'\},$$

$C_{J_r}$ is the submatrix of the matrix $C$ consisting of the rows numbered by $J_r$.

It is evident that any $N_n$-optimal situation $x \in G^n(C, N_n)$ (all the players are in one coalition) is lexicographically optimal. Thus all the players are ordered by importance in such way, that any previous player is more important then the next players. This corresponds to the general statement of the optimization problem with sequentially applied criterion [8–10].

Another special case is an $(\{1\}, \{2\}, \ldots, \{n\})$-optimal situation $x \in G^n(C, \{1\}, \{2\}, \ldots, \{n\})$ (individually-optimal), which is called Nash equilibrium situation [11] (see also [6, 7]). In this case the game is non-cooperative and the rationality of an equilibrium situation lies in the fact that any deviations of one player from this situation (while all the others keep to it) gives him no profit.

Thereby the introduction of such coalition characteristics of a situation which allows to generalize such classical concepts as the lexicographic optimality and Nash equilibrium herein is considered as the parametrization of the principle of optimality.

Without loss of generality we suppose that the partition $N_n = \bigcup\limits_{r \in N_s} J_r$ has the following form

$$J_1 = \{1, 2, \ldots, t_1\},\ J_2 = \{t_1 + 1, t_1 + 2, \ldots, t_2\}, \ldots,$$

$$J_s = \{t_{s-1} + 1, t_{s-1} + 2, \ldots, n\}.$$

Then taking into account the separation property of the linear payoff functions $C_i x,\ i \in N_n$, the following property can be directly obtained from the definition of $(J_1, J_2, \ldots, J_s)$-optimal situations set.

**Property 2** *For any partition* $(J_1, J_2, \ldots, J_s)$

$$G^n(C, J_1, J_2, \ldots, J_s) = \prod_{r=1}^{s} L^{|J_r|}(C^r, X_{J_r})$$

*holds, where each factor* $L^{|J_r|}(C^r, X_{J_r})$ *is the set of the lexicographically optimal solutions of the* $|J_r|$*-criterion vector problem*

$$C^r x_{J_r} \to \operatorname{lex} \max_{x_{J_r} \in X_{J_r}},$$

*i. e.*

$$L^{|J_r|}(C^r, X_{J_r}) = \{x_{J_r} \in X_{J_r} \ : \ \lambda(x_{J_r}, C^r) = \emptyset\},$$

*where*

$$\lambda(x_{J_r}, C^r) = \{x'_{J_r} \in X_{J_r} \ : \ C^r x_{J_r} \prec C^r x'_{J_r} \ \},$$

*$C^r$ is the square matrix of the size $|J_r| \times |J_r|$ formed by the intersection of the rows and columns of a matrix $C$ numbered by $J_r$; $X_{J_r}$ is the projection of the set $X$ onto $J_r$, i. e.*

$$X_{J_r} = \prod_{j \in J_r} X_j \subset \mathbf{R}^{|J_r|}.$$

It is easy to see that $x_{J_r}$ is the projection of a vector $x$ onto the coordinate axis of the space $\mathbf{R}^n$ numbered by $J_r$.

It is a well known fact [8–10] that the set $L^{|J_r|}(C^r, X_{J_r})$ is a result of the solution of the sequence of scalar problems

$$L_i = \text{Arg max}\{C_i^r x_{J_r} \ : \ x_{J_r} \in L_{i-1}\}, \quad i \in N_{|J_r|}, \tag{1}$$

where $L_0 = X_{J_r}$; $C_i^r$ is the $i$-th row of the matrix $C^r$. Therefore $L^{|J_r|}(C^r, X_{J_r}) = L_{|J_r|} \neq \emptyset$ for any index $r \in N_s$. Then from property 2 it follows that

**Property 3** *For any matrix $C \in \mathbf{R}^{n \times n}$ and any partition of the player set into $s$ coalitions $J_1, J_2, \ldots, J_s$ the set of $(J_1, J_2, \ldots, J_s)$-optimal situations $G^n(C, J_1, J_2, \ldots, J_s)$ is non empty.*

Moreover, one can state obvious

**Property 4** *A situation $x \notin G^n(C, J_1, J_2, \ldots, J_s)$ if and only if there exists such an index $r \in N_s$, that $x_{J_r} \notin L^{|J_r|}(C^r, X_{J_r})$.*

We define the norm $l_\infty$ for any natural number $q$ in the $q$-dimensional space $\mathbf{R}^q$ as follows

$$||z||_\infty = \max\{|z_i| \ : \ i \in N_q\}, \ z = (z_1, z_2, \ldots, z_q) \in \mathbf{R}^q.$$

Under the norm $||C||_\infty$ of a matrix $C = [c_{ij}] \in \mathbf{R}^{n\times n}$ we understand the norm of the vector $(c_{11}, c_{12}, \ldots, c_{n\,n-1}, c_{nn}) \in \mathbf{R}^{nn}$. We also will use the norm $l_1$ :

$$||z||_1 = \sum_{i\in N_q} |z_i|.$$

For arbitrary number $\varepsilon > 0$ we define the set of perturbation matrices

$$\Omega(\varepsilon) = \{B \in \mathbf{R}^{n\times n} \ : \ ||B||_\infty < \varepsilon\}.$$

## 3 The formula of stability radius

By analogy with [1, 2, 5, 12] under the stability radius of a $(J_1, J_2, \ldots, J_s)$-optimal situation $x \in G^n(C, J_1, J_2, \ldots, J_s)$ we will understand the number

$$\rho^n(x, C, J_1, J_2, \ldots, J_s) = \begin{cases} \sup\ \Xi & \text{if } \Xi \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\Xi = \{\varepsilon > 0 \ : \ \forall\, B \in \Omega(\varepsilon) \quad (x \in G^n(C + B, J_1, J_2, \ldots, J_s))\}.$$

Such a situation always exists by virtue of property 3.

**Theorem 1** *Let $C \in \mathbf{R}^{n\times n}$, $s \in N_n$, $n \geq 2$. For the stability radius $\rho^n(x, C, J_1, J_2, \ldots, J_s)$ of a generally-optimal situation $x \in G^n(C, J_1, J_2, \ldots, J_s)$ in the game with a matrix $C$ the following formula holds*

$$\rho^n(x, C, J_1, J_2, \ldots, J_s) = \min_{r\in N_s} \min_{x'_{J_r}\in X_{J_r}\setminus\{x_{J_r}\}} \frac{C^r_1(x_{J_r} - x'_{J_r})}{||x_{J_r} - x'_{J_r}||_1}. \qquad (2)$$

**Proof.** Let us denote the right part of formula (2) as $\varphi$ and also let $\rho := \rho^n(x, C, J_1, J_2, \ldots, J_s)$ for shortness of the subsequent text.

First we prove the inequality $\rho \geq \varphi$. If $\varphi = 0$, then this inequality is evident.

Let $\varphi > 0,\ B \in \Omega(\varphi)$. Directly from the definition of the number $\varphi$ it follows that for any index $r \in N_s$ and any vector $x'_{J_r} \in X_{J_r} \setminus \{x_{J_r}\}$ the inequalities

$$||B^r||_\infty \ \le\ ||B||_\infty \ <\ \varphi \ \le \frac{C_1^r(x_{J_r} - x'_{J_r})}{||x_{J_r} - x'_{J_r}||_1}$$

take place. Then we derive

$$(C_1^r + B_1^r)(x_{J_r} - x'_{J_r}) = C_1^r(x_{J_r} - x'_{J_r}) + B_1^r(x_{J_r} - x'_{J_r}) \ge$$

$$\ge C_1^r(x_{J_r} - x'_{J_r}) - ||B_1^r||_\infty ||x_{J_r} - x'_{J_r}||_1 > 0.$$

Therefore, according to property 1, we get $(C^r+B^r)x'_{J_r} \prec (C^r+B^r)x_{J_r}$, i. e. $\lambda(x_{J_r}, C^r + B^r) = \emptyset$. Thus

$$\forall r \in N_s \quad \left(x_{J_r} \in L^{|J_r|}(C^r + B^r, X_{J_r})\right).$$

Taking into account property 4 we conclude

$$\forall B \in \Omega(\varphi) \quad (x \in G(C + B, J_1, J_2, \ldots, J_s)),$$

what proves the inequality $\rho \ge \varphi$.

Turn to the proof of the inequality $\rho \le \varphi$. According to the definition of the number $\varphi$ there exists such an index $k \in N_s$ and such a vector $x^0_{J_k} \in X_{J_k} \setminus \{x_{J_k}\}$, that

$$C_1^k(x_{J_k} - x^0_{J_k}) = \varphi ||x_{J_k} - x^0_{J_k}||_1. \tag{3}$$

For any number $\varepsilon > \varphi$ construct the perturbation matrix $\widehat{B} = \left[\widehat{b}_{ij}\right] \in \Omega(\varepsilon)$ with the elements

$$\widehat{b}_{ij} = \begin{cases} \alpha & \text{if } (i,j) \in \{t_{k-1}+1\} \times J_k,\ x_j \ge x^0_j, \\ -\alpha & \text{if } (i,j) \in \{t_{k-1}+1\} \times J_k,\ x_j < x^0_j, \\ 0 & \text{if } (i,j) \in N_n \times N_n \setminus \{t_{k-1}+1\} \times J_k, \end{cases}$$

where $\varepsilon > \alpha > \varphi$. Then in view of (3) we get

$$(C_1^k + \widehat{B}_1^k)(x_{J_k} - x^0_{J_k}) = C_1^k(x_{J_k} - x^0_{J_k}) + \widehat{B}_1^k(x_{J_k} - x^0_{J_k}) =$$

$= C_1^k(x_{J_k} - x^0_{J_k}) - \alpha||x_{J_k} - x^0_{J_k}||_1 = \varphi||x_{J_k} - x^0_{J_k}||_1 - \alpha||x_{J_k} - x^0_{J_k}||_1 < 0.$

From the proved inequality by virtue of property 1 we derive

$$(C_1^k + \widehat{B}_1^k)x_{J_k} \prec (C_1^k + \widehat{B}_1^k)x^0_{J_k},$$

i. e.

$$x_{J_k} \notin L^{|J_k|}(C^k + \widehat{B}^k, X_{J_k}).$$

In view of property 4 it follows that the formula

$$\forall \varepsilon > \varphi \quad \exists \widehat{B} \in \Omega(\varepsilon) \quad (x \notin G(C + \widehat{B}, J_1, J_2, \ldots, J_s))$$

is valid and, hence, the inequality $\rho \leq \varphi$ holds.

Theorem is proved.

## 4 Corollaries

Let's introduce the set of strict situations for a given partition $(J_1, J_2, \ldots, J_s)$ :

$$S^n(C, J_1, J_2, \ldots, J_s) = \big\{x \in X \ : \ \forall\, r \in N_s \quad \forall\, x'_{J_r} \in X_{J_r} \setminus \{x_{J_r}\}$$

$$\big(C_1^r x_{J_r} > C_1^r x'_{J_r}\big)\big\}.$$

It is easy to see that only two cases are possible

$$S^n(C, J_1, J_2, \ldots, J_s) = \emptyset \quad \text{or}$$

$$S^n(C, J_1, J_2, \ldots, J_s) = G^n(C, J_1, J_2, \ldots, J_s).$$

A situation $x \in G^n(C, J_1, J_2, \ldots, J_s)$ is called stable if $\rho^n(C, J_1, J_2, \ldots, J_s) > 0$.

**Corollary 1** *A $(J_1, J_2, \ldots, J_s)$-optimal situation is stable if and only if it is strict.*

Clearly, $G^n(C, J_1, J_2, \ldots, J_s) = \{x^0\}$ if $x^0 \in S^n(C, J_1, J_2, \ldots, J_s)$. Therefore the following corollary takes place.

**Corollary 2** *If a $(J_1, J_2, \ldots, J_s)$-optimal situation $x^0$ is stable, then*

$$G^n(C, J_1, J_2, \ldots, J_s) = \{x^0\}.$$

Taking into account formulas (1) and (2) it is easy to see that the reverse statement is not valid in the general case, i. e. if a $(J_1, J_2, \ldots, J_s)$-optimal situation $x^0$ is a unique, then it can be either stable or unstable.

Corollary 2 implies

**Corollary 3** *If $|G^n(C, J_1, J_2, \ldots, J_s)| > 1$, then no one situation $x \in G^n(C, J_1, J_2, \ldots, J_s)$ is stable.*

**Corollary 4** *For the stability radius of a lexicographically optimal situation $x \in G^n(C, N_n)$ of the game with a matrix $C$ the formula*

$$\rho^n(x, C, N_n) = \min_{x' \in X \setminus \{x\}} \frac{C_1(x - x')}{||x - x'||_1}$$

*holds.*

**Corollary 5** *For the stability radius of a Nash equilibrium situation $x \in G^n(C, \{1\}, \{2\}, \ldots, \{n\})$ of the game with a matrix $C$ the formula*

$$\rho^n(x, C, \{1\}, \{2\}, \ldots, \{n\}) = \min\{|c_{ii}| \ : \ i \in N_n\}$$

*holds.*

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# On stability in game problems of finding Nash set

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**Abstract**

A finite game of several players in the case of linear payoff functions is considered. Stability of the problem of finding the set of Nash equilibrium situations is investigated. The formula of the stability radius of this problem is derived.

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**Key words:** non-cooperative finite game, Nash equilibrium situation, stability radius.

Consider the main subject of investigations in game theory – a finite non-cooperative game of some players in normal form [1], [2], when each player $i \in N_n = \{1, 2, \ldots, n\}$, $n \geq 2$, has finite number of available strategies $X_i \subseteq \mathbf{R}$, $2 \leq |X_i| < \infty$. Game realization and its outcome (situation) is unambiguously determined by the strategy choice of each player. They make this choice independently from each other.

Let on the set of all situations $X = \prod_{i \in N_n} X_i$ the linear payoff functions of players

$$f_i(x) = C_i x, \qquad i \in N_n,$$

is defined. Here $C_i$ is the $i$-th row of matrix $C = (c_{ij})_{n \times n} \in \mathbf{R}^{nn}$, $x = (x_1, x_2, \ldots, x_n)^T$, $x_i \in X_i$, $i \in N_n$. In the result of the game which is called the game with matrix $C$, each player $i$ gets the profit $f_i(x)$.

Consider the problem $Z^n(C)$ of finding the set of Nash equilibrium situations $NE^n(C)$ in the game with matrix $C$ [3]:

$$NE^n(C) = \{x \in X : \ \forall\, i \in N_n \ \ (\pi(x, C_i) = \emptyset)\},$$

where

$$\pi(x, C_i) = \{x' \in W(x, i) : \ C_i x' > C_i x\},$$

$$W(x, i) = \prod_{j \in N_n} W_j(x, i),$$

$$W_j(x, i) = \begin{cases} X_j & \text{if } j = i, \\ \{x_j\} & \text{if } j \neq i. \end{cases}$$

Thus $W(x, i)$ is the set of situations reachable by player $i$ from situation $x$.

As it is known, rationality of equilibrium situation lies in the fact, that any deviations of one player from this situation give him no profit.

It is easy to show (see, for example, [4]), that for any matrix $C \in \mathbf{R}^{nn}$ the set of Nash equilibrium situations $NE^n(C)$ is not empty.

By analogy with [5] — [8], under the stability radius of the problem $Z^n(C)$ we mean the number

$$\rho^n(C) = \begin{cases} \sup \Xi & \text{if } \Xi \neq \emptyset, \\ 0 & \text{if } \Xi = \emptyset, \end{cases}$$

where

$$\Xi = \{\varepsilon > 0 : \ \forall \ B \in \mathcal{B}(\varepsilon) \ \ (NE^n(C + B) \subseteq NE^n(C))\},$$

$$\mathcal{B}(\varepsilon) = \{B \in \mathbf{R}^{nn} : \ ||B||_\infty < \varepsilon\},$$

$$||B||_\infty = \max\{|b_{ij}| : \ (i, j) \in N_n \times N_n\}.$$

In other words, the stability radius is the maximum level of perturbations of elements of matrix $C$, which does not lead to appearance of new equilibrium situations. It is reasonable to define the stability radius to be equal to infinity, if $NE^n(C+B) \subseteq NE^n(C)$ for any matrix $B \in \mathbf{R}^{nn}$. Therefore we have

**Property 1.** *If* $\rho^n(C) = \infty$, *then* $\overline{NE^n}(C) := X \setminus NE^n(C) = \emptyset$.

Denote

$$I(x, C) = \{i \in N_n : \ \pi(x, C_i) \neq \emptyset\},$$

$$K(C) = \{i \in N_n : \ c_{ii} \neq 0\}.$$

**Property 2.** *For any index $k \in N_n$, such that $c_{kk} \neq 0$, there exists such situation $x^0 \in \overline{NE^n}(C)$, that $I(x^0, C) = \{k\}$.*

Since by virtue of the inequality $c_{kk} \neq 0$ for any situation $x \in NE^n(C)$ there exists situation $x^0 \in W(x, k)$, that $C_k(x^0 - x) < 0$, i. e. $\pi(x^0, C_k) \neq \emptyset$, then for any index $i \in N_n \setminus \{k\}$ and any situation $x' \in W(x^0, i)$ one easily derives

$$C_i(x' - x^0) = c_{ii}(x'_i - x^0_i) = c_{ii}(x'_i - x_i) = C_i(x' - x) \leq 0.$$

It follows that $\pi(x^0, C_i) = \emptyset$, $i \in N_n \setminus \{k\}$. Summing up, one obtains property 2.

**Property 3.** *Set $K(C)$ is empty if and only if set $\overline{NE^n}(C)$ is empty.*

Necessity of this statement follows directly from the definitions of sets $K(C)$ and $\pi(x, C_i)$, $i \in N_n$, and sufficiency can be easily shown, if we assume contrary and apply property 2.

**Theorem.** *For stability radius $\rho^n(C)$ of the problem $Z^n(C)$, $n \geq 2$, the following formula is valid*

$$\rho^n(C) = \begin{cases} \min\{|c_{ii}| : \ i \in K(C)\} & \text{if } \ K(C) \neq \emptyset, \\ \infty & \text{if } \ K(C) = \emptyset. \end{cases}$$

*Proof.* If $K(C) = \emptyset$, then by virtue of property 3 the equality $\overline{NE^n}(C) = \emptyset$ holds and, hence, according to property 1 we have $\rho^n(C) = \infty$.

Let further $K(C) \neq \emptyset$. Then

$$\psi := \min\{|c_{ii}| : \ i \in K(C)\} > 0$$

and by virtue of property 3 we obtain $\overline{NE^n}(C) \neq \emptyset$.

At first, let us prove inequality $\rho^n(C) \geq \psi$. For any situation $x \in \overline{NE^n}(C)$ there exist index $l \in N_n$ and situation $x' \in W(x, l)$ such that inequality

$$C_l(x' - x) > 0$$

holds. From this for any matrix $B \in \mathcal{B}(\psi)$ we easily derive

$$(C_l+B_l)(x'-x) = |C_l(x'-x)|+B_l(x'-x) = |c_{ll}||x'_l-x_l|+b_{ll}(x'_l-x_l) \geq$$

$$\geq |c_{ll}||x'_l - x_l| + |b_{ll}||x'_l - x_l| \geq (|c_{ll}| - ||B||_\infty)|x'_l - x_l| > 0,$$

i. e. $x' \in \pi(x, C_l + B_l)$. Therefore $x \in \overline{NE^n}(C + B)$. Thus, we obtain

$$\forall\ B \in \mathcal{B}(\psi) \qquad (NE^n(C + B) \subseteq NE^n(C)).$$

Hence, estimation $\rho^n(C) \geq \psi$ is valid.

Let us now prove inequality $\rho^n(C) \leq \psi$. We choose arbitrary number $\varepsilon > \psi$ and index $k \in N_n$, for which equality $c_{kk} = \psi$ holds. We define matrix $B^* = (b^*_{ij})_{n\times n} \in \mathcal{B}(\varepsilon)$ with elements

$$b^*_{ij} = \begin{cases} -c_{kk} & \text{if } i = j = k, \\ 0 & \text{otherwise.} \end{cases}$$

Property 2 ensures (by virtue of $c_{kk} \neq 0$) that there exists such situation $x^0 \in \overline{NE^n}(C)$, for which $I(x^0, C) = \{k\}$, i. e. for any index $i \in N_n \backslash \{k\}$ set $\pi(x^0, C_i)$ is empty. Therefore, taking into account the structure of matrix $B^*$, we have

$$\pi(x^0, C_i + B^*_i) = \pi(x^0, C_i) = \emptyset, \qquad i \in N_n \setminus \{k\},$$

$$\pi(x^0, C_k + B^*_k) = \pi(x^0, (c_{k1}, c_{k2}, \ldots, c_{kk-1}, 0, c_{kk+1}, \ldots, c_{kn})) = \emptyset.$$

It follows that $x^0 \in NE^n(C + B^*)$, i. e.

$$NE^n(C + B^*) \not\subseteq NE^n(C).$$

Further, since $B^* \in \mathcal{B}(\varepsilon)$ the inequality $\rho^n(C) < \varepsilon$ becomes obvious for any number $\varepsilon > \psi$. Hence, $\rho^n(C) \leq \psi$.

The theorem is proved.

The problem $Z^n(C)$ is called stable if $\rho^n(C) > 0$.

**Corollary 1.** *The problem $Z^n(C)$, $n \geq 2$, is stable for any matrix $C \in \mathbf{R}^{nn}$.*

Following [5], by rigidity radius of the problem $Z^n(C)$ we understand the number

$$\widetilde{\rho^n}(C) = \begin{cases} \sup \Omega & \text{if } \Omega \neq \emptyset, \\ 0 & \text{if } \Omega = \emptyset, \end{cases}$$

where

$$\Omega = \{\varepsilon > 0 : \ \forall\ B \in \mathcal{B}(\varepsilon) \ \ (NE^n(C+B) = NE^n(C))\}.$$

Taking into account the result of work [4], where the formula of stability radius of Nash equlibrium situation in the game with matrix $C$ is obtained, we easily state

**Corollary 2.** *For rigidity raduis $\widetilde{\rho^n}(C)$ of the problem $Z^n(C)$, $n \geq 2$, the followng formula is valid*

$$\widetilde{\rho^n}(C) = \min\{|c_{ii}| : \ i \in N_n\}.$$

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# The maximum flow in dynamic networks

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**Abstract**

The dynamic maximum flow problem that generalizes the static maximum flow problem is formulated and studied. We consider the problem on a network with capacities depending on time, fixed transit times on the arcs, and a given time horizon. The corresponding algorithm to solve this problem is proposed and some details concerning its complexity are discussed.

**Mathematics Subject Classification 2000:** 90B10, 90C35, 90C27.

**Keywords and phrases:** dynamic networks, network flow, dynamic flows, flows over time, maximum flows.

## 1 Introduction

In this paper we study the dynamic version of the maximum flow problem on networks that generalizes the well-known static one [1, 2]. This basic combinatorial optimization problem has a large implementation for many practical problems. Its solution is needed in order to solve other more complex problems, like the minimum cost circulation problem and the parametric maximum flow problem. Our dynamic model is based on the classical maximum flow problem on static networks and some generalization from [3, 4, 5, 6, 7, 8].

Despite being closer to reality, dynamic flow models have not been investigated in such a detailed form as classical flow models because of the complexity of the dynamic network flow models in comparison with the static ones. Dynamic flows are widely used to model network-structured, decision-making problems over time: problems in electronic communication, production and distribution, economic planning, cash

flow, job scheduling, and transportation (see, for example, [9]). In the considered dynamic models the flow passes an arc with time and it can be delayed at nodes. The flow values on arcs and the network parameters in this problem can change with time. While very efficient solution methods exist for static flow problems, dynamic flow problems have proved to be more difficult to solve.

The object of the maximum flow problem is to send a maximum amount of flow within a given time from supply nodes to demand nodes in such a way that link capacities are not exceeded. This problem has been studied extensively in the context of static networks. In this paper, we study the maximum flow problem in dynamic networks. We assume that capacities of edges depend on time. We propose an algorithm for finding the maximum dynamic flow, which is based on reducing the dynamic problem to the classical maximum flow problem on a time-expanded network. The complexity of this algorithm depends on the complexity of the algorithm used for the maximum static flow problem.

## 2 Problem formulation

We consider a directed network $N = (V, E, u, \tau, V^-, V^+)$ with set of vertices $V = V^- \cup V^+ \cup V^0$, where $V^-, V^+$ and $V^0$ are sources, sinks and intermediate nodes, respectively, and set of edges $E$. Without loosing generality, we assume that no edges enter sources or exit sinks. Our aim is to find a maximum flow over time in the network $N$ within makespan $\mathbb{T} = \{0, 1, 2, \ldots, T\}$ while respecting the following restrictions. Each edge $e \in E$ has a nonnegative time-varying capacity $u_e$ which bounds the amount of flow allowed on each arc in every moment of time. Moreover, edge $e$ has an associated positive transit time $\tau_e$ which determines the amount of time it takes for flow to travel from the tail to the head of that edge. The objective is to find a dynamic flow that sends in time $T$ as much flow as possible.

We start with the definition of static flows. A static flow $x$ on the static network $N = (V, E, u, V^-, V^+)$ assigns to every arc $e$ a non-negative flow value $x_e$ such that the following flow conservation

constraints are obeyed:

$$\sum_{e\in E^+(v)} x_e - \sum_{e\in E^-(v)} x_e = \begin{cases} -y_v,\ v \in V^-, \\ 0,\ v \in V^0, \\ y_v,\ v \in V^+, \end{cases}$$

$$y_v \geq 0,\ \forall\, v \in V,$$

where $E^+(v) = \{(u,v) \,|\, (u,v) \in E\}$, $E^-(v) = \{(v,u) \,|\, (v,u) \in E\}$.

A static flow $x$ is called feasible if it obeys the capacity constraints

$$0 \leq x_e \leq u_e,\ \forall\, e \in E.$$

A feasible dynamic flow on $N = (V, E, u, \tau, V^-, V^+)$ is a function $x$: $E \times \mathbb{T} \to R_+$ that satisfies the following conditions:

$$\sum_{\substack{e\in E^+(v) \\ t-\tau_e\geq 0}} x_e(t-\tau_e) - \sum_{e\in E^-(v)} x_e(t) = \begin{cases} -y_v(t),\ v \in V^-, \\ 0,\ v \in V^0, \\ y_v(t),\ v \in V^+, \end{cases} \quad \forall\, t \in \mathbb{T},\ \forall\, v \in V; \tag{1}$$

$$y_v(t) \geq 0,\ \forall\, v \in V,\ t \in \mathbb{T};$$

$$0 \leq x_e(t) \leq u_e(t), \quad \forall\, t \in \mathbb{T},\ \forall\, e \in E; \tag{2}$$

$$x_e(t) = 0,\ \forall\, e \in E,\ t = \overline{T - \tau_e + 1, T}. \tag{3}$$

Here the function $x$ defines the value $x_e(t)$ of flow entering edge $e$ at time $t$. It is easy to observe that the flow does not enter edge $e$ at time $t$ if it will have to leave the edge after time $T$; this is ensured by condition (3). Capacity constraints (2) mean that in a feasible dynamic flow, at most $u_e$ units of flow can enter the arc $e$ within each integral time step. Conditions (1) represent flow conservation constraints.

We consider the discrete time model, in which all times are integral and bounded by horizon $T$. Time is measured in discrete steps, so that if one unit of flow leaves node $u$ at time $t$ on arc $e = (u, v)$, one unit of flow arrives at node $v$ at time $t + \tau_e$, where $\tau_e$ is the transit time of arc $e$. The time horizon (finite or infinite) is the time until which the flow can travel in the network and defines the makespan $\mathbb{T} = \{0, 1, \ldots, T\}$ of time moments we consider.

The value of the flow $x$ is defined as follows

$$|x| = \sum_{t=0}^{T} \sum_{v \in V^+} y_v(t). \tag{4}$$

Our aim is to find a feasible flow that maximizes the objective function (4).

In the case of many sources and sinks the maximum flow problem can be reduced to the standard one by introducing one additional artificial source and one additional artificial sink and edges leading from the new source to true sources and from true sinks to the new sink. The transit times of these new edges are zero and the capacities of edges connecting the artificial source with all other sources are bounded by the capacities of these sources; the capacities of edges connecting all other sinks with the artificial sink are bounded by the capacities of these sinks.

It is easy to observe that if $\tau_e = 0, \forall\, e \in E$ and $T = 0$ then the formulated problem becomes the classical maximum flow problem on a static network.

## 3 Main results

In this paper we propose a new approach for solving the formulated problem, which is based on its reduction to a static maximum flow problem which is a well studied problem in operation research and other fields. We show that our problem on network $N = (V, E, u, \tau)$ can be reduced to a static problem on auxiliary static network $N^T = (V^T, E^T, u^T)$; we name it the time-expanded network. The advantage of this approach is that it turns the problem of determining an optimal flow over time into a classical static network flow problem on the time-expanded network.

The maximum dynamic flow problem can be solved by maximum flow computation on the corresponding time-expanded network, which is a static representation of the dynamic network. Such a time-expanded network contains a copy of the node set of the underlying

network for each discrete interval of time, building a time layer. Moreover, for each arc with transit time $\tau_e$ in the given network, there is a copy between each pair of time layers of distance. We define this network as follows:

1. $V^T$: $= \{v(t) \,|\, v \in V,\ t \in \mathbb{T}\}$;
2. $E^T$: $= \{(v(t), w(t+\tau_e)) \,|\, e = (v,w) \in E,\ 0 \le t \le T - \tau_e\}$;
3. $u^T_{e(t)}$: $= u_e(t)$ for $e(t) \in E^T$;

The essence of the time-expanded network is that it contains a copy of the vertices of the dynamic network for each time $t \in \mathbb{T}$, and the transit times and flows are implicit in the edges linking those copies.

Let $e(t) = (v(t), w(t+\tau_e)) \in E^T$ and let $x_e(t)$ be a flow on the dynamic network $N$. The corresponding function $x^T_{e(t)}$ on the time-expanded network $N^T$ is defined as follows:

$$x^T_{e(t)} = x^T_{(v(t),w(t+\tau_e))} = x_e(t),\ \forall\, e(t) \in E^T. \tag{5}$$

We show in the following lemma that dynamic flows $x_e(t)$ with time horizon $T$ are equivalent to static flows $x^T_{e(t)}$ in the time-expanded network.

**Lemma 1.** *The correspondence (5) is a bijection from the set of feasible flows on the dynamic network $N$ onto the set of feasible flows on the time-expanded network $N^T$.*

**Proof.** It is obvious that the correspondence above is a bijection from the set of $T$-horizon functions on the dynamic network $N$ onto the set of functions on the time-expanded network $N^T$. It is also easy to observe that a feasible flow on the dynamic network $N$ is a feasible flow on the time-expanded network $N^T$ and vice-versa. Indeed,

$$0 \le x^T_{e(t)} = x_e(t) \le u_e(t) = u^T_{e(t)},\ \forall\, e \in E,\ \ 0 \le t < T.$$

Therefore it is enough to show that each dynamic flow on the dynamic network $N$ is put into the correspondence with a static flow on the time-expanded network $N^T$ and vice-versa.

Henceforward we define $d_v(t) = \begin{cases} -y_v(t),\ v \in V^-, \\ 0,\ v \in V^0, \\ y_v(t),\ v \in V^+, \end{cases} \quad \forall v \in V.$

Let $x_e(t)$ be a dynamic flow on $N$ and let $x^T_{e(t)}$ be a corresponding function on $N^T$. Let's prove that $x^T_{e(t)}$ satisfies the conservation constraints on its static network. Let $v \in V$ be an arbitrary node in $N$ and $t$: $0 \le t < T$ an arbitrary moment of time:

$$d_v(t) \overset{(i)}{=} \sum_{\substack{e \in E^+(v) \\ t-\tau_e \ge 0}} x_e(t-\tau_e) - \sum_{e \in E^-(v)} x_e(t) =$$

$$= \sum_{e(t-\tau_e) \in E^+(v(t))} x^T_{e(t-\tau(e))} - \sum_{e(t) \in E^-(v(t))} x^T_{e(t)} \overset{(ii)}{=} d^T_{v(t)}. \qquad (6)$$

Note that according to the definition of the time-expanded network the set of edges $\{e(t-\tau_e) | e(t-\tau_e) \in E^+(v(t))\}$ consists of all edges that enter $v(t)$, while the set of edges $\{e(t) | e(t) \in E^-(v(t))\}$ consists of all edges that originate from $v(t)$. Therefore, all necessary conditions are satisfied for each node $v(t) \in V^T$. Hence, $x^T_{e(t)}$ is a flow on the time-expanded network $N^T$.

Let $x^T_{e(t)}$ be a static flow on the time-expanded network $N^T$ and let $x_e(t)$ be a corresponding function on the dynamic network $N$. Let $v(t) \in V^T$ be an arbitrary node in $N^T$. The conservation constraints for this node in the static network are expressed by equality (ii) from (6), which holds for all $v(t) \in V^T$ at all times $t$: $0 \le t < T$. Therefore, equality (i) holds for all $v \in V$ at all times $t$: $0 \le t < T$ and $x_e(t)$ is a flow on the dynamic network $N$. □

**Corollary 1.** *The following condition is true:*

$$\sum_{t \in \mathbb{T}} \sum_{v \in V^-} y_v(t) = \sum_{t \in \mathbb{T}} \sum_{v \in V^+} y_v(t).$$

**Proof.** The proof of this corollary can be obtained by examining three sets of vertices $V^-, V^+$ and $V^0$ and summing equations (1) in all

moments of time. In such a way, we obtain the considered condition. □

If we define a flow correspondence to be $x^T_{e(t)} := x_e(t)$, the maximum flow problem on dynamic networks can be solved by applying network flow optimization techniques for static flows directly to the expanded network. Thus to obtain the solution, we construct the time-expanded network, after that we solve the classical maximum flow problem on the static network and then we reconstruct the solution of the static problem to the dynamic problem. Therefore, we can solve the dynamic maximum flow problem by reducing it to the maximum flow problem on static networks.

# 4 Algorithm

Let the dynamic network $N$ be given. Our object is to solve the maximum flow problem on $N$. Proceedings are following:

1. Building the time-expanded network $N^T$ for the given dynamic network $N$.

2. Solving the classical maximum flow problem on the static network $N^T$, using one of the known algorithms (see, for example, [1, 10, 11, 12]).

3. Reconstructing the solution of the static problem on $N^T$ to the dynamic problem on $N$. □

Now let us examine the complexity of this algorithm including the time necessary to solve the resulting problem on the static time-expanded network. The process of building the time-expanded network and of reconstruction the solution of the static maximum flow problem to the dynamic one has the complexity $O(nT + mT)$, where $n$ is the number of nodes in the dynamic network and $m$ is the number of edges in this network. The complexity of step (2) depends on the complexity of the algorithm used for maximum flow problem in static networks.

If such an algorithm has complexity $O(f(n', m'))$, where $n'$ and $m'$ are number of nodes and edges in the network, then the complexity of solving the maximum flow problem on the time-expanded network employing the same algorithm is $O(f(nT, mT))$.

## 5 The dynamic problem on the given time interval

In the above sections we have discussed the problem of maximum dynamic flow from the zero time moment to the fixed time horizon $T$. In such problems we find the maximum amount of flow until the time $T$. In many practical cases it is necessary to know the maximum flow in the time period from $t_1$ to $t_2$, where $t_1 < t_2$. The solution of this problem is based on the Ford-Fulkerson theorem about the maximum flow and minimal cut [1]. We have to construct a time-expanded network, the discrete time moments of which form the following makespan $\mathbb{T} = \{t_1, t_1 + 1, \ldots, t_2 - 1, t_2\}$. In that way, constructing such a time-expanding network and finding the maximum flow in this network we can obtain the maximum flow in the dynamic network for the time period from $t_1$ to $t_2$.

## 6 Generalization

We would like to note that the same argumentation can be held to solve the maximum flow network problem on the dynamic networks in the case when, instead of the condition (2) in the definition of the feasible dynamic flow, the following condition takes place:

$$r_e^1(t) \leq x_e(t) \leq r_e^2(t), \quad \forall\, t \in \mathbb{T},\ \forall\, e \in E,$$

where $r_e^1(t)$ and $r_e^2(t)$ are lower and upper boundaries of the capacity of the edge $e$ correspondingly. In this case we introduce one additional artificial source $b_1$ and one additional artificial sink $b_2$. For every arc $e = (u, v)$, where $r_e^1 \neq 0$ we introduce arcs $(b_1, v)$ and $(u, b_2)$ with $r^1$ and 0 as the upper and lower boundaries of the capacity of the edges.

We reduce $r^2$ to $r^2 - r^1$, but $r^1$ to 0. We also introduce the arc $(b_2, b_1)$ with $r^2_{(b_2,b_1)} = \infty$ and $r^1_{(b_2,b_1)} = 0$. The transit times of all introduced arcs are zero. In such a mode we obtain a new network, on which we can solve the standard maximum flow problem.

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# The multiobjective transportation fractional programming model

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**Abstract**

In this paper the multiobjective transportation problem of linear-fractional type with one nonlinear time constraint criterion is investigated. Particularly, the case of the identical denominators is studied. The concrete procedure to find the set of all basic efficient solutions for this model is proposed. This algorithm is tested on an annexed example.

**Keywords and phrases:** multiobjective problem, optimization, linear-fractional criterion, basic efficient solution, bottleneck restriction

The transportation problem dealing with the total cost minimizing criterion, considered as a classical one, is well-known and sufficiently analyzed in the respective sources.

The transportation model of a "bottleneck" type is a specific problem within the transportation classical issue, the objective function of which is a non-linear one. Special cases of these types of problems are investigated in many paper-works like [1], [2], [5], [6], where concrete algorithms in order to solve them are carried out. The transportation model of the "bottleneck" type with 2 criteria, where the first one is providing the total transportation cost minimization and the second one, that is non-linear, is strangling in time, is studied in article [7], where the authors propose the concrete algorithm to solve it. The special algorithm for solving transportation model of the "bottleneck" type with 3 criteria is presented in paper [8], where it is tested on a concrete example.

One should mention that in our daily life the multiobjective fractional programming models are of great interest. We are often concerned about the optimization of the ratios like the summary cost of the total transportation expenditures to the maximal necessary time to satisfy the demands, the total benefits or production values into time unit, the total depreciation into time unit and many other important similar criteria, which may appear in order to evaluate the economical activities and make the correct managerial decisions. These problems led to the "bottleneck" transportation model with multiple fractional criteria, where the "bottleneck" criteria appear as a "minmax" time constraining. The common characteristic of these objective ratios is the identical denominators. Concrete algorithms for solving of special models of transportation type with one criterion, where the objective function is a fractional one, are proposed in papers [3],[4].

The multicriterial transportation model of "bottleneck" type with two fractional criteria is defined as follows:

$$\min z_1 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}}{\max\limits_{i,j}\{t_{ij}|x_{ij}>0\}} \tag{1}$$

$$\min z_2 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij}}{\max\limits_{i,j}\{t_{ij}|x_{ij}>0\}} \tag{2}$$

$$\min z_3 = \max_{i,j}\{t_{ij}|x_{ij}>0\} \tag{3}$$

in conditions (4)-(7)

$$\sum_{j=1}^{n} x_{ij} = a_i, \forall i = \overline{1,m} \tag{4}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \forall j = \overline{1,n} \tag{5}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{6}$$

$$x_{ij} \geq 0, i = \overline{1,m}, j = \overline{1,n} \tag{7}$$

where $c_{ij}$ - cost of transporting a unit from source $i$ to destination $j$, $d_{ij}$ - deterioration of a unit while transporting from source $i$ to destination $j$, $a_i$ - availability at source $i$, $b_j$ - requirement at destination $j$, $x_{ij}$ - amount transported from source $i$ to destination $j$, $t_{ij}$ - time of transporting a unit from source $i$ to destination $j$.

A non traditional algorithm of building numerous efficient solutions of the models is carried out here. There is no sense to look for an optimal solution to settle the multicriterial mathematical models. As it often occurs, there are no solutions at all.

That is why, one should better determine the multitude of non-dominant solutions, which are known as efficient solutions or optimal in the terms of Pareto.

In order to solve the multi criteria model the notion of an efficient solution has been introduced.

**DEF:** The feasible solution for the multiple criteria model is considered to be efficient if and only if another feasible solution, for which we obtain a better value at least for one criterion while the values of the rest criteria remain unmodified, doesn't exist.

In order to solve the problem (1)- (7) by finding the set of the efficient basic solutions, we reduce it to the following model:

$$\min z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{8}$$

$$\min z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij} \tag{9}$$

$$\min z_3 = \max_{i,j} \{t_{ij} | x_{ij} > 0\} \tag{10}$$

in conditions (4)-(7)

The authors Y.P. Aneja and K.P.K. Nair in their paper: "Bicriteria transportation problem" [1] propose an algorithm to solve the model (8)-(10) in conditions (4)-(7). The algorithm determines the multitude of extreme non-dominant solutions within the admissible space of

solutions. The algorithm is theoretically and scientifically tested and proved in a concrete case.

The algorithm of solving the model (1)-(7) develops a procedure of building a multitude of all efficient, basic solutions. This set coincides with the set of the efficient basic solutions for the model (8)-(10) in conditions (4)-(7). That is why we reduce the multicriterial fractional transportation model of "bottleneck" type (1)-(7) to the problem (8)-(10) in the restrictions (4)-(7) in order to find the set of its basic efficient solutions.

**Theorem 1.** *The set of the efficient basic solutions of the model (1)-(7) and the model (8)-(10) coincide.*

Proof. Let $X^1$ be an efficient basic solution for the model (1)-(7), and $T^1 = \max\limits_{i,j}\{t_{ij}/x^1_{ij} > 0\}$. We state that for each available solution $X^2$ of this model and corresponding $T^2$, where $T^2 = \max\limits_{i,j}\{t_{ij}/x^2_{ij} > 0\}$, based on the definition of the efficient solution, the following inequalities are true:

$$\begin{array}{c} Z_1(X^1) < Z_1(X^2) \text{ and } Z_2(X^1) \le Z_2(X^2) \\ \text{or} \\ Z_1(X^1) \le Z_1(X^2) \text{ and } Z_2(X^1) < Z_2(X^2) \end{array} \tag{11}$$

where $T^2 \le T^1$, $T^1 \ge 0$, $T^2 \ge 0$.

We suppose that the solution $X^1$ is not efficient for the model (8)-(10) in the conditions (4)-(7). Analogously to the antecedent reasoning, using the definition of the efficient solution, it follows that there exists the available solution $X^2$ of this model and corresponding $T^2$, for which the following inequalities are true:

$$\begin{array}{c} \frac{Z_1(X^2)}{T^2} < \frac{Z_1(X^1)}{T^1} \text{ and } \frac{Z_2(X^2)}{T^2} \le \frac{Z_2(X^1)}{T^1} \\ \text{or} \\ \frac{Z_1(X^2)}{T^2} \le \frac{Z_1(X^1)}{T^1} \text{ and } \frac{Z_2(X^2)}{T^2} < \frac{Z_2(X^1)}{T^1} \end{array} \tag{12}$$

where $T^2 \le T^1$, $T^1 \ge 0$, $T^2 \ge 0$.

Multiplying inequalities (12) by $T^1$ and supposing $k = \frac{T^1}{T^2}$, we obtain that the following inequalities are true:

$$\begin{array}{c} kZ_1(X^2) < Z_1(X^1) \text{ and } kZ_2(X^2) \leq Z_2(X^1) \\ \text{or} \\ kZ_1(X^2) \leq Z_1(X^1) \text{ and } kZ_2(X^2) < Z_2(X^1) \end{array} \tag{13}$$

where $T^2 \leq T^1$, $T^1 \geq 0$, $T^2 \geq 0$.

As it is obvious that $k \geq 1$, from (13) we conclude that for the solution $X^2$ the following inequalities are true:

$$\begin{array}{c} Z_1(X^2) < Z_1(X^1) \text{ and } Z_2(X^2) \leq Z_2(X^1) \\ \text{or} \\ Z_1(X^2) \leq Z_1(X^1) \text{ and } Z_2(X^2) < Z_2(X^1) \end{array} \tag{14}$$

where $T^2 \leq T^1$, $T^1 \geq 0$, $T^2 \geq 0$,
that contradicts (11).

It can be proved analogously that each efficient solution of the model (8)-(10) is also an efficient solution for the model (1)-(7).

The theorem is proved.

Generalizing this idea for the model with multiple number of fractional criteria with the "bottleneck" constraining criterion, we conclude that it may be reduced to the model (15) in order to find the set of its efficient basic solutions, that is defined as follows:

$$\begin{array}{c} \min z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^1 x_{ij} \\ \min z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^2 x_{ij} \\ ..... \\ \min z_r = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^r x_{ij} \\ \min z_{r+1} = \max_{i,j}\{t_{ij} | x_{ij} > 0\} \\ \text{in conditions (4)-(7)} \end{array} \tag{15}$$

Values $C_{ij}^k$, $k = 1, ..., r$, $i = 1, ..., m$, $j = 1, ..., n$ correspond to the concrete interpretation of the respective criteria.

If there are some criteria of "max" type among the set of criteria from the model (15), it is not difficult to reduce this case to the initial one. It is obvious that the model (8)-(10) is a particular case of the model (15), and so the algorithm to solve the model (15) in conditions (4)-(7) can be used to solve the model (8)-(10).

The truthfulness of the above theorem for the model (15) is proved similarly.

The algorithm of finding the set of the efficient basic solutions for the model (15) is an interactive one. Initially we consider at least $(m+n-1)$ cells from the tables $C^k$, $k = 1, 2, \ldots, r$, in order to find the first efficient basic solution of model (15). The indexes' order is maintained the same as in the table $T$, where the cells are numbered according to the respective time values well arranged in the increasing order. Each iteration supposes a deep levels' exploration and a completion of the multitude of efficient basic solutions for a new unblocked stochastic time-variable.

The exploration procedure of each time instant chain is finite in depth and ends on every branch, in the case when the same solutions have been found at upper level of other branch or when all possibilities of improvement have been spent at this level.

At the time when the solution of a certain configuration detains the form recorded in another link, which has been investigated earlier, its depth exploration has no justification, that is why it is eventually stopped.

We propose the logic scheme to construct the algorithm for solving the multiple criteria transportation models of "bottleneck" type with a finite number of criteria, which is presented at Fig.1, where $\Delta_{ij} = (u_i + v_j) - c_{ij}$, $n_i < p$ ($p$ is defined by the dimension of the problem, $n_i$ is an index of ordering the cells by data from the table $T$).

The logical blocks are to be verified before every logical ramification according to Fig.1.

## ALGORITHM

1. Table $T$ with the increasing order of time values which uses the $k$ index is being well arranged. The index order is maintained for the

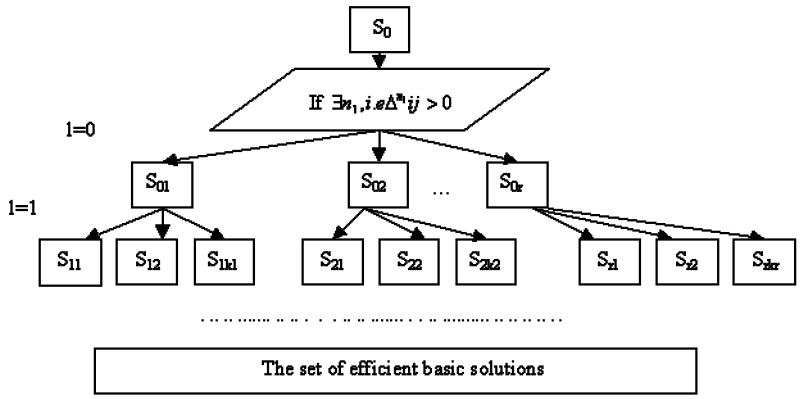


Figure 1.

respective cells from the tables $C^k$, $k = 1, ..., r$.

2. The adoption of an initial, basic solution in the first $p = (m+n-1)$ cells from the table $C$ is performed. The other cells are considered to be blocked.

3. All configurations of basic solutions can be recorded at the level $l = 0$, using only the non-blocked cells and providing the dozing in all those cells with $(i, j|x_{ij} > 0)$, for which the relation $\Delta_{ij} \geq 0$ to be true at least for one criterion.

Each configuration of the solution is iteratively investigated, obtaining in such a way the following records at the next level: $l = l + 1$.

If a certain level of a basic solution, which was previously obtained, is found, the latter won't be further studied. Since the problem covers a finite dimension, the multitude, consequently, of all basic solutions for the unblocked cells will be obtained by exploring a finite number of levels in depth.

4. If $p < m*n$, the following $p = p+1$ cell is unblocked, and for this purpose the exploration of the basic solutions is revived, then we start with the level 0. The 4th step will be repeated until we get $p = m * n$.

The basic efficient solutions set is selected out of the multitude of the basic solutions.

**Theorem 2.** *The set of all efficient basic solutions for the multiple criteria transportation problem of "bottleneck" type is found by applying the above algorithm.*

Proof. Let $L$ be a list of efficient basic solutions of model (15) being found by applying the above algorithm. We suppose, that the efficient basic solution $S_1$, that was not found using the above algorithm exists and $S_1 \notin L$. Let $S_1$ corresponds to $T_1$. We will fix it on the branch that corresponds to the $T_1$ beginning with the level 0, when corresponding cells from table $T$ are cleared. Wide exploration of the fixed branch leads to the registration of all basic solutions of branch $T_1$. So, all the basic solutions that correspond to time $T_1$ are contained in this set. We will separate from the set $L_{T_1}$ the efficient basic solutions corresponding to time $T_1$. It is obvious that $L_{T_1} \subset L$. As a result, if $S_1 \in L_{T_1}$, then $S_1$ is a basic efficient solution found by applying the above algorithm or if $S_1 \notin L_{T_1}$, then $S_1$ is not a basic solution and moreover it is not a basic efficient solution. So, iether $S_1$ is not a basic solution or it is contained in list $L$. The theorem is proved.

**Example**

Consider the following 3-criteria problem.

Time, Supply, Demand=

| | | | | |
|---|---|---|---|---|
| 10 | 95 | 73 | 52 | 8 |
| 68 | 66 | 30 | 21 | 19 |
| 37 | 63 | 19 | 23 | 17 |
| 11 | 3 | 14 | 16 | $b_j \vert a_i$ |

Cost 1,2=

| | | | |
|---|---|---|---|
| 1 4 | 2 4 | 7 3 | 7 4 |
| 1 5 | 9 8 | 3 9 | 4 10 |
| 8 6 | 9 2 | 4 5 | 6 1 |

Using the above proposed algorithm we have found the following 11 efficient basic solutions:

S1=(176,207,68); S2=(164,276,68); S3=(178,203,68);
S4=(172,213,68); S5=(158,283,68); S6=(208,167,73);
S7=(202,173,73); S8=(156,200,95); S9=(176,175,95);
S10=(143,265,95); S11=(186,171,95).

The authors of the article [1], using their own algorithm for this example, have obtained 9 efficient extreme solutions.

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# A New Approach in Agent Path-Finding using State Mark Gradients

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**Abstract**

Since searching is one of the most important problem-solving methods, especially in Artificial Intelligence where it is often difficult to devise straightforward solutions, it has been given continuous attention by researchers. In this paper a new algorithm for agent path-finding is presented. Our approach is based on environment marking during exploration. We tested the performances of Q-learning and Learning Real-Time A* algorithm for three proposed mazes and then a comparison was made between our algorithm, two variants of Q-learning and LRTA* algorithm.

**Keywords:** artificial intelligence, path-finding, maze, reinforcement learning, Q-learning, LRTA*, agents

## 1 Introduction

A very important problem of distributed computing is to increase the asynchronicity of communication in order to decrease communication costs. Following that direction of thinking, in the last decade the agent paradigm was developed, although the asynchronous approach increased the rate of use for any single or multiple known resource. The basic idea of multiagent systems was to achieve better performance of data search in distributed databases due to its inherent asynchronous work model. At this hour those systems have many different applications e.g. data mining and complex simulations. However, agent-based systems have the same problem inherited from their distributed systems support. This is the problem of finding the path in the network. We propose a new approach in this direction beginning with the idea

that we can make the agent act somehow like simple biological beings that can mark the discovered (followed) path in order to later remind it.

A path-finding problem has two main components [8]:

- a set of nodes $N$, where each node $n \in N$ represents a state;
- a set of directed links $L$, where each link $l \in L$ is an operator available to the problem solver.

In general, if the problem solver is an agent in a multiagent environment, an operator is an action that can be performed by the agent in the current state. Usually, there is a unique start node $s \in N$, representing the initial state of the agent, and a set of goal nodes $G \subset N$, representing the desired states, i.e. the solutions of the path-finding problem.

Each link has a corresponding weight $w_l$, which represents the cost of applying the operator (or performing the action). Sometimes, the link weight is called the distance between the two nodes. The nodes that have directed links from a node are its neighbors.

The maze is an example of a path-finding problem in a grid state space. The states can be either free states (where the agent can move) or obstacle states (inaccessible to the agent). The allowed operators are moves along $x$ and $y$ axes (north, east, south, west); diagonal moves are forbidden. Classically, the start node is the upper-left corner and the goal node is the bottom-right corner of the maze. In this paper, we used a more general approach, allowing the goal node to be placed anywhere in the maze.

## 2 Path-Finding Using Q-Learning

Reinforcement learning [6, 3] is a learning technique based on the maximization of a (usually numerical) reward signal, given as a consequence for taking a certain action in a certain state. The learning agent is not told directly what to do, but it must discover the actions that will give it the highest reward. In some cases, actions may affect not only the

immediate reward, but also the next situation and, through that, all subsequent rewards. These two characteristics, trial and error search and delayed reward, are the two most important distinguishing features of reinforcement learning.

The Q-learning algorithm [7] is a popular reinforcement technique, often used in multiagent learning systems. The main idea of the algorithm is to learn an action-value function, $Q : S \times A \rightarrow R$, that estimates the long-term discounted rewards for each pair state-action. If an action $a \in A$ in state $s \in S$ produces a reward $r \in R$ and a transition to state $s' \in S$, the corresponding $Q$ value is modified as follows:

$$Q(s,a) = Q(s,a) + \alpha \cdot (r + \gamma \cdot \max_{a' \in A} Q(s',a') - Q(s,a)), \tag{1}$$

where $\alpha$ is the learning rate and $\gamma$ the discount factor. If $\gamma = 0$, the agent is interested only by maximizing its immediate reward. Such a strategy ignores future rewards, so that the total gain (value) may be smaller. The closer $\gamma$ is to 1, the more important future rewards are for the learning agent.

The optimal policy the agent must use is:

$$\pi^* = \arg\max_a Q(s,a). \tag{2}$$

In our maze problem, the set of actions is:

$$A = \{north, east, south, west\}, \tag{3}$$

and the set of states:

$$S = \{s_{ij}, 1 \leq i \leq mazeheight, 1 \leq j \leq mazewidth\}, \tag{4}$$

$$s_{ij} \in \{free, obstacle, start, stop\}. \tag{5}$$

The rewards we used were:

- 100 for reaching the stop state;

- -100 for trying to move to an obstacle state;
- 0 for moving to another free state.

We first did not place any constraints on the agent movement. Thus, the agent could move to an obstacle state, it would be considered a terminal state and the algorithm would be restarted in a new learning episode. However, in order to speed up the learning, we decided that if the agent chose to move to an obstacle state, it would be given a -100 penalty but then it would keep its former position. This approach is more realistic considering a real situation with a robot moving through a physical maze. When it touches a wall, it doesn't necessarily break, so that the learning should proceed again from the start position.

One of the challenges that arise in reinforcement learning is the tradeoff between exploration and exploitation [6]. To obtain a bigger reward, a reinforcement learning agent must prefer actions that it has tried in the past and found to be effective in producing reward. But to discover such actions it has to try actions that it has not selected before. The agent has to exploit what it already knows in order to obtain reward, but it also has to explore in order to make better action selections in the future.

The initial exploration rate must be high, and then it is gradually decreased each time the agent finds the solution. These values may differ according to the problem [1]. We chose an initial exploration rate $\epsilon_0 \in \{0.8, 0.9, 1\}$ and performed several performance tests. When the agent finds a solution, this rate is decreased by 0.05. However, it cannot become less than 0.1. At the beginning, the agent doesn't know its path (the $Q$ values are initialized at small random values in order not to bias any direction) and it must rely on exploration. As it finds solutions, the exploration rate is decreased down to 0.1. If, after several episodes, the $Q$ function has not converged, the agent must still have a chance to find a solution through exploration.

Two cases were considered: in the first one, the agent simply applies Q-learning, in the second, it marks the states that it has visited and receives a -10 penalty if it goes back again to an already visited state. This is done to discourage the agent to enter any loops.

The performance of the agent is given by the number of learning episodes in which it can learn the path. It must be mentioned that learning the path is not the same thing as finding the path. The agent will find the path many times, but it is important to adjust its Q-values in order to be able to eventually find the solution in one single try, relying only on these Q-values.

We tested the algorithm on three mazes: a small one (11x11) with only one possible solution, a bigger one (25x25) also with only one possible solution, and another big one (19x30), which allowed multiple solutions (see figures 7, 8, and 9). We measured the number of learning episodes and the total steps needed for the agent to reach the solution. A statistical processing of the data was then made, and mean value and standard deviation were computed:

$$m_X = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \tag{6}$$

$$\sigma_X = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - m_X)^2}. \tag{7}$$

Table 1 presents these results for different values of the initial exploration rate $\epsilon_0$, obtained after 20 learning trials:

It must be noted that in both cases, the initial exploration rate has a great influence on the number of learning episodes. When the agent doesn't remember the visited states, learning is the fastest when $\epsilon_0$ is 1 (figure 1).

However, the number of steps in every learning episode is bigger when $\epsilon_0$ is bigger. If the behavior of the agent is driven only by exploration, it performs in a random manner, and so the number of states it goes through is bigger. When $\epsilon_0$ is smaller, we have a greater number of learning episodes, but in each episode the agent finds the solution faster, because it relies more on "real" knowledge (figure 2).

After every successful learning episode, the exploration rate is decreased and the agent begins to use its previously acquired information, therefore the number of steps to reach the solution in each learning

Table 1. The performance of Q-learning with different initial exploration rates in the presence and absence of state marks.

| **Maze type** | **State marks** | $\epsilon_0$ | $m_{episodes}$ | $\sigma_{episodes}$ | $m_{totalsteps}$ | $\sigma_{totalsteps}$ |
|---|---|---|---|---|---|---|
| small (11x11) single solution | No | 1 | 9.1 | 3.375 | 19,727 | 5,880 |
| | | 0.9 | 13.15 | 4.028 | 13,516 | 3,067 |
| | | 0.8 | 17.75 | 7.569 | 14,814 | 6,134 |
| | Yes | 1 | 8.9 | 4.625 | 21,671 | 9,627 |
| | | 0.9 | 8.25 | 2.385 | 8,830 | 2,203 |
| | | 0.8 | 9.85 | 2.475 | 5,907 | 1,573 |
| big (25x25) single solution | No | 1 | 18.55 | 4.717 | 1,368,121 | 285,868 |
| | | 0.9 | 191.8 | 26.658 | 749,443 | 60,625 |
| | | 0.8 | 295.65 | 37.936 | 779,560 | 137,781 |
| | Yes | 1 | >1000 | | ~73,900,000 | |
| | | 0.9 | 12.6 | 4.363 | 106,253 | 30,783 |
| | | 0.8 | 13.45 | 3.263 | 57,640 | 14,753 |
| big (19x30) multiple solutions | No | 1 | 43.7 | 10.194 | 138,489 | 26,510 |
| | | 0.9 | 99.55 | 22.675 | 106,341 | 16,196 |
| | | 0.8 | 151.3 | 34.134 | 98,464 | 15,771 |
| | Yes | 1 | 101.3 | 51.424 | 329,483 | 169,084 |
| | | 0.9 | 68.8 | 23.134 | 74,729 | 19,438 |
| | | 0.8 | 59 | 18.501 | 41,541 | 8,003 |

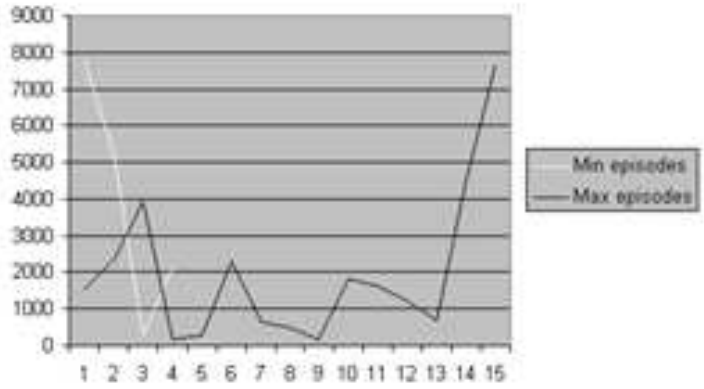


Figure 1. The number of steps per learning episode for a simple single solution maze without state marks and initial exploration rate 1.

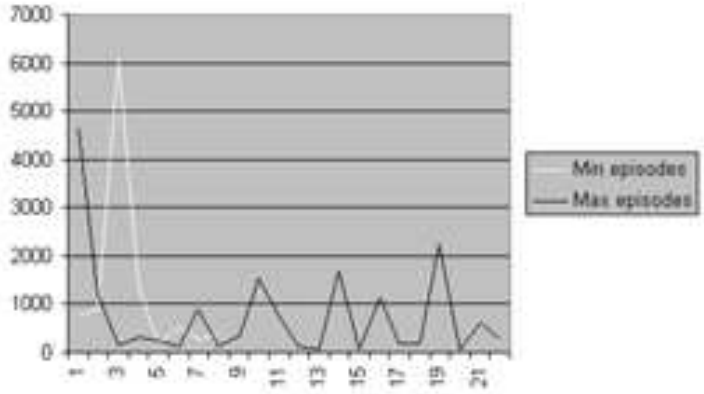


Figure 2. The number of steps per learning episode for a simple single solution maze without state marks and initial exploration rate 0.9.

episode gradually diminishes (figure 3). There are still peaks, because the exploration rate is never null, and a random decision in a key state (e.g. a bifurcation where one way leads to the solution and the other to a dead end) can greatly influence the outcome of the learning process.

A different phenomenon appears when the agent marks the states and it receives a penalty for coming back to an already visited state. In such a case, the best performance is obtained with $\epsilon_0 = 0.9$, rather than $\epsilon_0 = 1$ (figure 4). The randomization of the learning process impedes over the performance of the agent, because the chances are greater to return to a marked state. The agent modifies its Q-values and tries to avoid that state, although it may be a useful one. When the initial exploration rate is less than 1, the algorithm has better performance than in the simple case. Also, when $\epsilon_0 = 0.9$, the number of steps per episode is usually smaller than in the no-marks case.

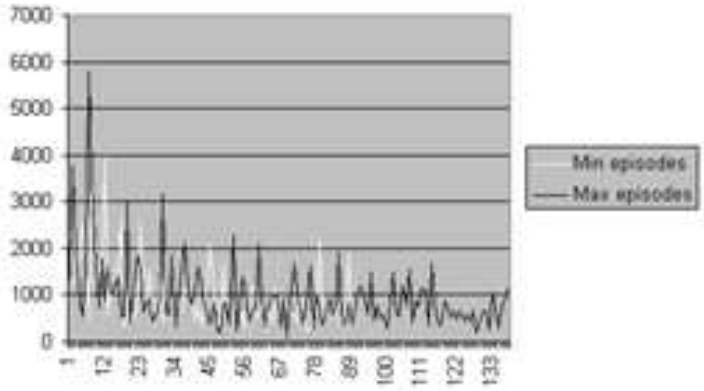


Figure 3. The number of steps per learning episode for a complex single solution maze without state marks and initial exploration rate 0.9.

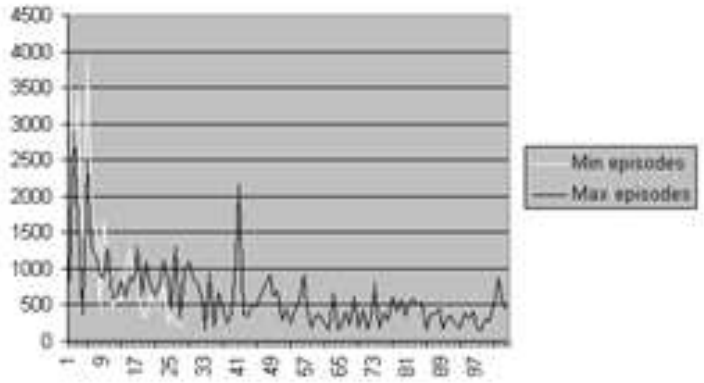


Figure 4. The number of steps per learning episode for a big multiple solution maze with state marks and initial exploration rate 0.8.

## 3 The Influence of Learning Rate and Discount Factor

After demonstrating that the agent learns quicker if it uses state marks with an initial exploration rate of 0.9, we tried to analyze the influence of learning rate $\alpha$ and discount factor $\gamma$ (see equation 1) on the path-finding efficiency. Table 2 presents the results for different values of $\alpha$ and $\gamma$, obtained after 20 learning trials.

Although for simple problems a medium learning rate can yield good results, in general a lower learning rate produces better outcomes. As equation (1) shows, if $\alpha$ is big, the current value of $Q(s, a)$ becomes less important, and its next value is mainly determined by the reward and the next state: $r + \gamma \cdot \max_{a' \in A} Q(s', a')$. When the maze is simple, a big discount factor produces better results, but in complex mazes, for small learning rates, a medium value is preferred.

Figure 5 shows the performance change for a small (11x11) single solution maze, when the learning rate is constant (0.1) and the discount factor varies. One can notice that the number of learning episodes is smaller when the discount factor is small, i.e. the agent concentrates on immediate reward. However, the number of total steps to find the solution has an opposite behavior, as shown in paragraph 2.

Figure 6 shows the performance change for the same maze, this time keeping the discount factor constant (0.5) and varying the learning rate. In this case, both the number of episodes and the number of total steps to the solution are minimum when the learning rate is medium.

## 4 Path-Finding Using Learning Real-Time A* (LRTA*)

A classical approach for determining the minimal cost path is A* [3], an off-line search algorithm which computes an entire solution path before executing the first step in the path. However, in a multiagent system, it is not often possible to perform local computations for all nodes, due to time limitations. The LRTA* algorithm [5] addresses this problem

Table 2. The performance of Q-learning with state marks for different learning rates and discount factors.

| **Maze type** | $\alpha$ | $\gamma$ | $m_{episodes}$ | $\sigma_{episodes}$ | $m_{totalsteps}$ | $\sigma_{totalsteps}$ |
|---|---|---|---|---|---|---|
| small (11x11) single solution | 0.1 | 0.1 | 7.6 | 2.458 | 14,780 | 6,048 |
| | | 0.5 | 8.15 | 2.574 | 8,374 | 2,797 |
| | | 0.9 | 12.8 | 4.082 | 9,368 | 2,284 |
| | 0.5 | 0.1 | 5.5 | 2.674 | 6,272 | 2,734 |
| | | 0.5 | 5.7 | 2.61 | 4,124 | 1,230 |
| | | 0.9 | 6.95 | 1.658 | 4,292 | 724 |
| | 0.9 | 0.1 | 13.55 | 13.204 | 11,861 | 12,152 |
| | | 0.5 | 21.8 | 17.882 | 15,513 | 12,753 |
| | | 0.9 | 6.05 | 1.687 | 4,042 | 1,139 |
| big (25x25) single solution | 0.1 | 0.1 | 20.95 | 7.074 | 369,139 | 147,502 |
| | | 0.5 | 14.95 | 4.189 | 131,756 | 50,757 |
| | | 0.9 | 47.15 | 16.135 | 183,783 | 32,608 |
| | 0.5 | 0.1 | >1000 | | ~5,500,000 | |
| | | 0.5 | >1000 | | ~8,300,000 | |
| | | 0.9 | >1000 | | ~4,500,000 | |
| | 0.9 | 0.1 | >1000 | | ~18,300,000 | |
| | | 0.5 | >1000 | | ~12,800,000 | |
| | | 0.9 | >1000 | | ~8,100,000 | |
| big (19x30) multiple solutions | 0.1 | 0.1 | 71.7 | 19.662 | 78,764 | 19,135 |
| | | 0.5 | 70.4 | 32.469 | 73,511 | 26,015 |
| | | 0.9 | 78.15 | 19.764 | 78,814 | 13,126 |
| | 0.5 | 0.1 | >1000 | | ~1,400,000 | |
| | | 0.5 | >1000 | | ~1,200,000 | |
| | | 0.9 | 46.8 | 9.026 | 46,736 | 6,517 |
| | 0.9 | 0.1 | >1000 | | ~1,600,000 | |
| | | 0.5 | >1000 | | ~1,400,000 | |
| | | 0.9 | 391.7 | 385.956 | 319,561 | 306,239 |

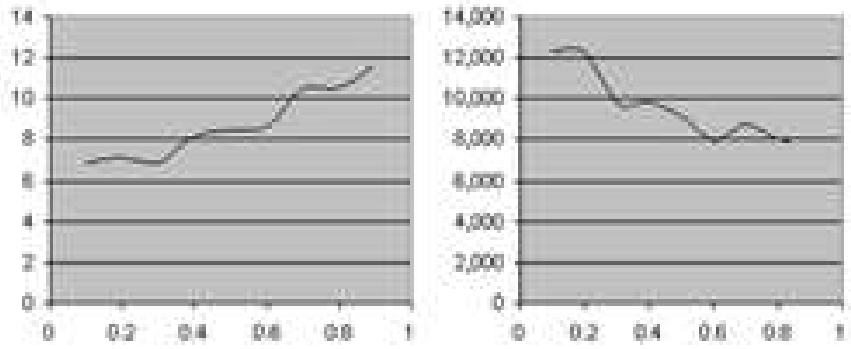


Figure 5. a) The number of episodes when $\alpha = 0.1$ and $\gamma$ varies; b) The number of total steps when $\alpha = 0.1$ and $\gamma$ varies.

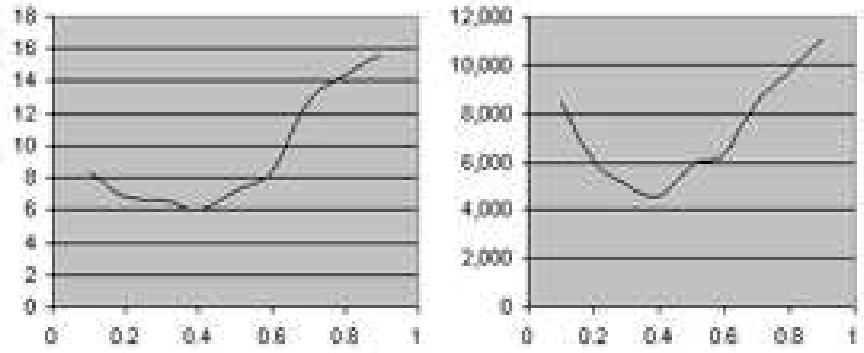


Figure 6. a) The number of episodes when $\gamma = 0.5$ and $\alpha$ varies; b) The number of total steps when $\gamma = 0.5$ and $\alpha$ varies.

by imposing that an agent should perform computations only for its neighboring nodes. Using multiple iterations, it has been proven that the solution eventually converges to the optimal one. The agent must estimate the distance to the goal $h(i)$ for each node $i$ it visits. The LRTA* algorithm is presented bellow [8]:

`Look-ahead: calculate` $f(j) = k(i,j) + h(j)$ `for each neighbor` $j$ `of the current node` $i$`, where` $h(j)$ `is the current estimate of the shortest distance from` $j$ `to the goal node, and` $k(i,j)$ `is the link cost from` $i$ `to` $j$`;`
`Update the estimate of node` $i$`:` $h(i) = \min_j f(j)$`;`
`Action selection: move to the neighbor` $j$ `that has the minimum` $f(j)$ `value. Ties are broken randomly.`

Because the agent determines the next action in a constant time, LRTA* is considered a real-time, on-line algorithm. It updates the estimations with the best so far for not overestimating the actual cost. Therefore, the initial value of $h(i)$ must be optimistic i.e. never overestimate the true value. It was demonstrated [2] that real-time search methods are powerful sub-optimal search methods that can outperform off-line search methods in terms of total running time.

We used LRTA* to find the solutions for our proposed mazes. We measured the number of episodes needed to reach the goal node, the number of learning steps per episode and the number of total steps in which an agent can find the solution. The results are displayed in table 3.

It is evident that LRTA* has better performance than Q-learning for these path-finding problems. LRTA* is an algorithm specially designed for this type of problems, whereas reinforcement learning methods are general learning solutions.

## 5 Learning Using State Mark Gradients

If the agent marks the states it has visited, by analogy with the real world, the marks should depreciate and this could be used by the agent

Table 3. The performance of LRTA* for the three proposed mazes.

| **Maze type** | $m_{episodes}$ | $\sigma_{episodes}$ | $m_{steps/ep}$ | $\sigma_{steps/ep}$ | $m_{totalsteps}$ | $\sigma_{totalsteps}$ |
|---|---|---|---|---|---|---|
| small (11x11) single solution | 15.3 | 0.461 | 47.111 | 23.428 | 720.8 | 7.369 |
| big (25x25) single solution | 111.2 | 0.402 | 213.439 | 87.273 | 23734.4 | 45.026 |
| big (19x30) multiple solutions | 96.6 | 23.02 | 77.172 | 81.232 | 7454.8 | 951.227 |

in finding its path to the solution. The agent still goes through an exploration and an exploitation phase. The main idea is that in the first phase, it constantly tries to go to a minimum mark state, which correspond to an unvisited state. In the latter, it tries to follow the ascending state marks gradient, which leads it from the initial state to the goal state.

We propose the following algorithm:

$episode = 1$;
initialize state marks with 0;
initialize state $s$ with the initial state;

while ($s$ is not terminal)
  // choose action
  get state marks for the neighbors: $north, east, south, west$, ignore the walls;
  $wantedDirection = \min(north, east, south, west)$;
  choose the action corresponding to the $wantedDirection$;
  if (more actions possible)
    favor the previous direction with a probability given by $consistencyRate$;

```
      other ties are broken randomly;
   end if

//perform action
   get next state s';

// update state marks
   for all states t
      marks(t) = marks(t)/decayRate;
   end for

marks(s) = 1;
   s = s';
   if (s = initial state and for all neighbors
   sn, marks(sn) > 0)
      episode = episode + 1;
      restart
   end if
end while
```

As stated before, the agent must choose an action that will place it in an unvisited state, or at least a state visited long before. After finding the solution, the agent has the trace of its path, and therefore it must only follow it from the initial state, using the gradient ascent of the state marks. Due to the depreciation, newly visited states have greater marks. The last visited state is the solution.

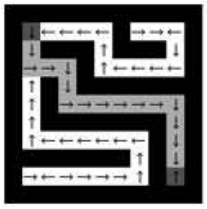

Figure 7. The map of the solved simple single solution maze.

The last test of the algorithm prevents the cases in which the agent

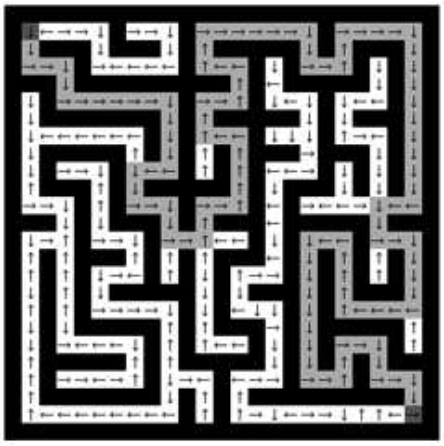

Figure 8. The map of the solved complex single solution maze.

falls into a loop. This situation is not possible for single solution mazes (figures 7 and 8), but it is inevitable for multiple solution mazes (figure 9). If, after a loop, the agent comes back to the initial state, the state marks are reset and a new learning episode begins.

The exploitation phase is done using the same algorithm, but in this case the action at a certain moment is chosen by considering the *max* function instead of *min*:

$$wantedDirection = \max(north, east, south, west);$$

Two constants are used: the decay rate and the consistency rate. The decay rate shows how quickly state marks degrade. The value we used is 1.01. Its purpose become clearer if the agent ignores the marks states below a certain limit. The second constant represents the probability that the agent will keep its current direction, if it is possible. Table 4 shows the results of learning with different consistency rates, after 100 learning trials.

It is obvious that in the case of a single solution maze, where the path is very strict, the consistency rate is not important. The agent will be constrained by the maze design and the only decisions it can make

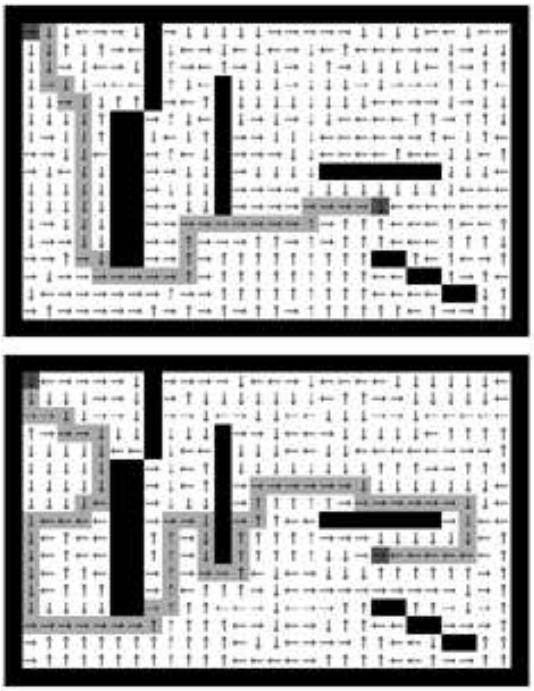

Figure 9. The map of the solved big multiple solution maze: two possible solutions.

Table 4. The performance of state mark gradients algorithm with degrading state marks for different consistency rates.

| **Maze type** | **Consistency rate** | $m_{episodes}$ | $\sigma_{episodes}$ | $m_{totalsteps}$ | $\sigma_{totalsteps}$ |
|---|---|---|---|---|---|
| small (11x11) single solution | 0.9 | 1 | 0 | 68.8 | 13.717 |
| | 0.8 | 1 | 0 | 58.8 | 22.454 |
| | 0.5 | 1 | 0 | 50.6 | 21.514 |
| | 0.1 | 1 | 0 | 50.32 | 23.981 |
| big (25x25) single solution | 0.9 | 1 | 0 | 260.4 | 80.039 |
| | 0.8 | 1 | 0 | 261.6 | 87.447 |
| | 0.5 | 1 | 0 | 237.2 | 76.578 |
| | 0.1 | 1 | 0 | 242.48 | 86.238 |
| big (19x30) multiple solutions | 0.9 | 2.9 | 2.385 | 614.56 | 505.517 |
| | 0.8 | 3.25 | 2.875 | 612.76 | 462.663 |
| | 0.5 | 4.25 | 3.708 | 637.92 | 450.715 |
| | 0.1 | 6.36 | 5.572 | 811.28 | 544.777 |

refer to cross points. For big multiple solution mazes, if the consistency rate is high, the number of total steps to reach the solution is lower, because the agent will travel less in a random manner, thus increasing its chances to find its goal more quickly.

Our algorithm proves to be more efficient than Q-learning and LRTA*, as it discovers the solution in much fewer learning episodes (only one episode if there are no loops possible in the maze). Also, the number of steps in which the agent reaches the goal node is up to ten times smaller than the number of steps necessary for the LRTA* algorithm.

## 6 Conclusions

As shown, the proposed method has better performance than the other presented algorithms. This is validated both by the theoretical approach as the practical results that are depicted in tables 1 to 3. The main advantage of this method is that in a multiagent system the other agents can use the results about discovered paths by simply inspecting the current location marking. The method can be easily modified in

order to be used over generalized graphs.

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# A Case-Based Reasoning for Regulation of an Urban Transportation Network

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**Abstract**

This paper presents a classification-based approach to case-based reasoning. This approach has been implemented in a decision-making system for regulating an urban transportation network. Planning relies on two classification processes: strong classification to retrieve a similar planning perturbation and smooth classification when the former fails. Smooth classification is an original mechanism that can become of general use in case-based reasoning. We discuss in this paper the two processes from general and applicative point of view.

**Key words:** Case-Based Reasoning, regulation, transportation network, strong classification, smooth classification.

## 1 Introduction

The principle of Case-Based Reasoning (CBR) is based on an analogical reasoning [3][5], where previous experiences are used to define a solution to the present problem. This artificial reasoning proposes solutions to a problem which belongs to a class of problems frequently, encountered by the decision-maker. It rests on a case bases where are recorded the situations already met and on a measure of similarity between the situations which allow, for a given situation, to find the nearest cases in the case bases. The present case is composed of a problem (disturbance in our case), in need of a solution, it is taken by the Automatic Vehicle Monitoring (AVM) system or telephone (figure 1). This case is compared to former ones in the cases-base. A disturbance and its solution compose each one of them.

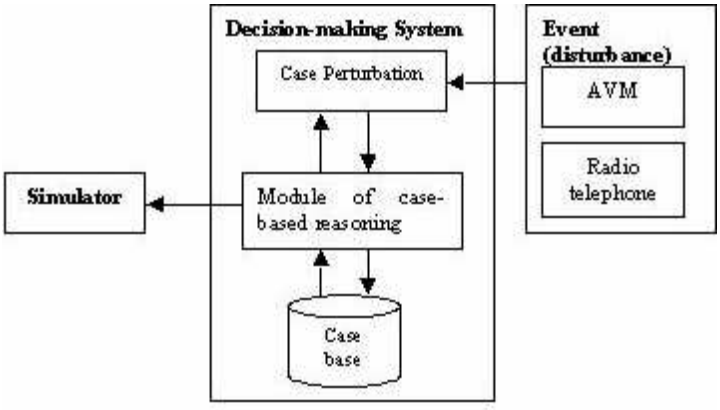


Figure 1. Global Architecture

The execution of the cycle allows to create a knowledge base for the resolution of real problems and reuse this knowledge to solve future problems. This reasoning is used by man in various environments and plays an important role in the expertise.

## 2 Presentation of the CBR

In our Decision-making system for regulation, this module contains a case-base reasoning organized in the form of databases. The disturbances which occur are compared thanks to a partial order $<$ such as if db1<db2. We can say that the disturbance db1 is more specific than db2. With each recorded disturbance noted source an index idx(source) is associated such as idx(source)>source : an index is a generalization of a disturbance source. The index is organized in a hierarchy Hidx for the order $<$. To remind a scenario (recorded disturbance) by the means of Hidx. We use two processes of classification. Strong classification seeks the index of the sources problems, which are more general than the target problem. More precisely, given that a target problem "noted target", our CBR seeks a disturbance source such as idx(source)>target.

The source<idx(source) is a strong classification characterized by the equation:

$$\text{Source<idx(source)>target} \quad (1)$$

In the event of failure of strong classification, when there is not treated disturbance source such as idx(source)>target, smooth classification is activated. The latter is based on an "approximate" pairing. In other words, the CBR module seeks functions of modification $\varphi$ and $\psi$ such as:

$$\text{Source<idx(source)} \cong \varphi(\text{idx(source)}) > \text{ target} \quad (2)$$

Such that $\cong$ is read "similar to" and means intuitively "equal to near modification". The function of modification $\varphi$ checks the following property: if a treatment treatment(idx(source) is known, then we can build a treatment($\varphi$(idx(source))). Unlike the function of modification, $\psi$ is such, that if a treatment treatment($\psi$(target)) is known, then a treatment treatment(target) can be built. Reminder in the CBR model seeks a similar treatment to the target disturbance and establishes how they are similar. Source is similar to target if one of the sequences of relations (1) and (2) between source and target is checked. These sequences of relation (1) and (2) are called "ways of similarity". The similarity in our case is based on six data:

- name of the nearest stop where the dysfunction is located,
- distance to the next stop
- distance to the next terminus
- Source<idx(source)>target
- Source<idx(source)>target
- Mode of transportation (bus or tramway)

Given a target target disturbance, the case source which is the nearest to target is calculated with a distance. This distance is based on a cost function which, with a way of similarity Sim(source, target), associates a numerical value cost(Sim(source, target)). Given two disturbances, source and target, the distance from source to target is defined as:

$$\text{d(source, target)} = \min\{\text{cost(Sim(source, target))}\} \quad (3)$$

The minimum being taken on the whole of the ways of similarity Sim(source, target) from source to target. This distance is similar to an edition distance [3]. In the resolution of our disturbance, the cost is evaluated by using three criteria: regularity, punctuality and transfer

## 3 Planning starting from case

There are two principal stages in the process of planning in our decision-making system for regulation. Knowing that the regulator builds the solution of regulation to leave basic actions such as (injects a vehicle, delay a vehicle or advances a vehicle): the remind processes lean on the strong classification, followed if necessary by a smooth classification. Initially, the target disturbance is classified in the database of the disturbances with the identified parameters of the urban transportation network (strong classification). Two cases are possible

(1) there is an analogy of the target disturbance with which a source plan Plan is associated (in this case, Plan can be adapted in a plan for the target disturbance);

(2) it does not have an obvious analogy (in this case, the system must modify the target disturbance p and a quite selected disturbance P from database in order to find a relation of compatibility between P and p) (smooth classification).

The result of the remind is a couple (Plan(pk), Sim(pk, p)), where Plan (pk) is a source plan associated with P and where Sim(pk, p) is a way of similarity between pk and p, that is to say a way of the form pk<P>p for strong classification and pk<P $\cong$ $\varphi$(P)> $\psi$ (p) $\cong$ p for smooth classification. The fact that a couple (Plan(pk), Sim(pk, p)) goes back by the reminding process assures that the system will be able to adapting a plan Plan(pk) in a plan Plan(p) for the disturbance p.

## 4 Discussion

The principal objective of our system of regulation which is based on a CBR approach is to propose to the regulator in a tiny time a solution nearest possible to the disturbance in progress. This is so that it could

be discharged from this task given that it passes more than 50% of time with the radiotelephone. This approach is applicable insofar as the consultation of the incident sheets carried out by the services of the maintenance and regulation raises thousands of incidents per year, whose majority is similar, as much on the context that on the strategies applied to solve them.

Many works in the context of regulation were done. Besma [2] uses a multi-agent approach integrating a genetic algorithm. The genetic algorithm makes it possible to give an optimal solution. But it is not the purpose of the regulator, which needs instantaneous answers taking into account the workload of regulation of a transportation system. The latter on average includes more than one hundred of vehicles. Optimising a solution of regulation is interesting during the off-peak hours. At this period, the regulator can be allowed to await the response of an advanced algorithm beyond a response time necessary for the correct operation of the network. Laichour [5] uses a multi-agent approach, but he is interested only in the regulation of connections. On the other hand, Chihaib [4] uses techniques of the artificial intelligence and in particular the propagation of constraints and fuzzy logic. All this works are available but not complete.

## 5 Conclusion

In this article, we presented a Case-based approach starting from a case which is based on classification. This approach was established in a Decision-making system for regulation of an urban transportation network and shows the following characteristics: (1) the cases are indexed by structures of graphs [9], and are organized in a hierarchy according to an analogy relation; (2) the remind and adaptation are interdependent processes: if the remind succeeds, the adaptability is assured; (3) the objective of the system is in record time to propose plans of solution for the occurring disturbances. Work on the regulation was undertaken and is always in hand with reference to the proposal for a computerized decision-making system for regulation. The approach that we dealt with is a step to increase the number of choices concerning approaches

used in such systems.

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# Experience of Macroeconomic Models Realization in Transition Economy

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**Abstract**

This article represents an issue about macroeconomic model realization under transition economy of Moldova Republic. Different macroeconomic model beginning with simple monetary model and growth model, and more complicated model as financial programming models, general equilibrium model, describing complex functioning of national economy and forecasting effect of various economic policies on economy development were considered. Some comparative analysis and conclusions were presented.

## 1 Introduction

Macroeconomic models, dealing with aggregated indicators, describe working of the economy as a whole. Such models are accessible to the deep mathematical analysis and give an opportunity of economic system research at small volume of the initial data. Macroeconomic models are effective tools for theoretical researches of economic processes and for working out the direction of economic development and forecasting a lot of national economy indicators. Regular use of macroeconomic models in the countries with the settled market relations began in the first half of the 20-th century. At present time many macroeconomic models having both theoretical and applied value are developed. In the countries which are in the process of transition to market relations the wide use of macroeconomic models concerns to the second half of the 20-th century. In these countries macroeconomic modelling is used for creation of some analytical framework with the purpose

of co-ordinating the set of macroeconomic policies and the structural transformations necessary for maintenance the transition of the country to market economy. Wide use of macroeconomic models is explained by their simplicity from the mathematical point of view, being simultaneously useful and important tool displaying real economy. Macroeconomic models are used for support and for analysis of macroeconomic policies, and for improvement of a sequence of performances of these initiatives.

The formalised macroeconomic models represent the simple and clear analytical framework consisting of a range of economic identities and well-specified assumptions, determining interrelation between economic variables (the so-called, behavioural equations). At the same time the process of model creation and calibration for concrete economy is rather complex. It requires special economic knowledge and a certain experience: in definition of necessary changes of the models structure reflecting specific features of concrete economy, in a choice of the important economic variables in quality of exogenous. The purpose of present article is to give the summary of the macroeconomic models realised in Institute of Mathematics and Computer Science during last decade, which can be used for the co-ordination of economic policies and structural reforms at Republic Moldova transition to market relations.

## 2 Simple Monetary and Growth Models

For construction of the financial programming model or the financial program it is necessary to define accounting macroeconomic framework, which would cover the basic sectors of economy: private, government, monetary and foreign. Economic identities make a basis of accounting framework. The behavioural equations determining interrelation between economic variables, are specified so that to add accounting identities up to the closed system of equations. The variables identified in this framework are subdivided into exogenous, endogenous and policy. The combination of variables, economic relations and identities forms economic model, which is called to prove policy decisions. For

the realisation of financial programming model (development of the financial program) it is necessary to execute the forecast of exogenous variables, to define precisely values of target variables and to solve the model for policy variables which will provide desirable values for the target variables. In the simple monetary model of open economy [1-2] it is usually supposed that prices and output are set exogenous, and the inverse of the income velocity of money is constant. The balance of payments in such model can be represented as a difference between the flow of demand for money of the private sector and the flow of internal credits. The flow of internal credits is a policy variable. In this model the gain of the money credits offered by banking system, in a limit will be compensated due to decrease in the international credits calculated on the same base. For the formulation of the financial program the required value of the balance of payments is set, and the model is solved in scope of finding necessary increase (reduction) of internal credits. Such type of models provides motivation of substantial use of marginal credits as key policy tools. Monitoring of internal credits expansion allows to determine, whether the financial program promotes the achievement of required output at the given change in international reserves.

One of the lacks of presented model is that the nominal income is set as exogenous. In practice, certainly, financial programs treat a level of the internal prices, as endogenous one. Endogenous price level, and, hence, the nominal income demands small updating in a simple monetary framework, as now in model there are two target variables, namely the balance of payments and inflation.

One of ways of the problem solution consists in the assumption, that both prices and international reserves are crucial for violation of the money market balance. However the purposes of the balance of payments and inflation can not be achieved independently as there is only one policy variable (internal credits).

Use of the exchange rate as tool policy provides a way for achievement of independence of the balance of payments values and inflation. Change of the exchange rate has two interconnected effects: first, it improves the international competitiveness of economy; second, it in-

creases the initiative of production of the goods for export. The exchange rate policy, hence, is intended for change of production structure and expenditures in economy, decrease of import and increase of export, improve of trade balance. Having available two tools, namely, rate of inflation and rate of internal credit expansion, authorities are able to achieve the balance of payments purpose, having received expected rate of inflation.

Exogenous nominal income (in simple monetary model) is the potential lack of monetary model when the construction of the program has the important participation in economic growth. So, for example, it is necessary to consider models of growth in addition to the monetary analysis. Many from available, standard models of growth accent a role of savings, investments and offers of external financing. Despite of the significant success achieved in this area, this particular type of growth model is still widely used, especially for countries where the information base is insufficient. Though models which correlate economic growth extremely to increasing of physical capital stocks and therefore with availability of internal and external financing, having advantage of simplicity, are not capable to receive positive effect of growth from increasing efficiency of policies. Accordingly, these models can not provide analysis of the total growth in the countries with transitive economy.

In simple model of growth [1] the change in output is related with investments by means of the ratio of capital gain to output (ICOR). For settled value of ICOR, the increase in the level of investments will follow from the appropriate increase in growth of the real gross domestic product. As alternative, it is possible to use this ratio for reception of a "necessary" level of investments, which are consistent with desirable rate of growth. The model includes two behavioural equations. One is that import is the steady function from output. The second reflects the fact, that private savings are positively dependent from the level of available income. The model becomes closed with identity asserting that the total accumulation (equal to investments) is made of private and foreign savings.

Working of this open economy model is rather simple. We shall

assume, for example, that authorities increase a level of investments, increasing the state accumulation through reduction of the state consumption. As the change of private and state investments is assumed to be constant, the growth of state accumulation will increase the total accumulation and, hence, investments. At given ICOR, growth of investments further will be transformed into higher rate of growth of real GDP. Provided that authorities carry out some control above private inflow of capital or if the offer of foreign capital is completely elastic, the purpose of output can be co-ordinated with the purpose of the balance of payments. The practical advantage of this simple model of growth, obviously, is caused by realism of these various assumptions, as some of these assumptions are rather rigid.

Now we shall consider, how the model will be modified, if the level of net foreign inflow was fixed. In practice, certainly, acceptance of internal policies, which result in financial stability, and which raise perspective of growth, would assume production of additional external financing for the country, as well as more attractive terms of this financing. Really, even in a case when foreign creditors are not ready to expand borrowing, absence of the capital can be reduced or even inverted, if the country has provided following suitable adjustable strategy. In this sense, the assumption of a constancy of net capital inflow is used here more for an illustration of a role of other policies in liquidation of a so-called gap of foreign currency receipts, than for the description of a reality.

In condition when foreign financing is given, any additional accumulation is insufficient to achieve simultaneously both the purpose of growth, and the purpose of balance of payments, private accumulation will be adjusted to equate the total accumulation and investments. But this "necessary" level of private accumulation can be improbable in sense of concordance with a level, which follows from behavioural ratio between private accumulation and the available income. In such chance, the iterative sequence of adjustments can be accomplished either above parameters or above the growth goal until the required level of private accumulation will come in conformity with a historical behavioural ratio between private accumulation and the available income.

The alternative way of the problem solution consists in inclusion of the exchange rate change effect in trading balance. Influence of the exchange rate on export and import allows to co-ordinate the purpose of growth with an accessible level of foreign inflow actions.

The approach of financial programming described above can be generalised by introducing into it the simple growth analysis. The resulting merged approach implicitly means three fundamental purposes of financial programs: the balance of payments, inflation and rate growth of real gross domestic product inside the concordance framework. Merge of monetary and growth model [1] is rather simple. The most suitable growth model for the countries with adjustable programs is that with limited foreign financing. Suitable monetary model could be that in which both balance of payments and prices are determined inside the model. Gross domestic product can be determined from ICOR ratio. The prices are defined from monetary model. So nominal gross domestic product can be determined directly. The monetary model also determines demand for money (through a ratio income - income velocity of money), and supply for money from the balance sheet of the banking system. Definition of the balance of payments is common for both models: growth model and monetary model. Inflow of foreign capital is supposed to be exogenous, import is the subject to be influenced by internal output and real exchange rate, but export is the subject to be influenced by the real exchange rate. Meaning, that prices are exogenous in simple growth model while gross domestic product is exogenous in the simple monetary model, the merged model can be solved by it representation as two ratios between change in gross domestic product and change in prices.

In the growth model this ratio is positive. The gain of the internal level prices increases nominal gross domestic product, and through ratio accumulation - increases the income. There will be also changes in nominal accumulation. The private sector can use these savings for accumulation of the physical capital or money. While all marginal additional nominal savings are not designated for accumulation, some gain of the physical capital will take place. At positive investments, in concordance with ICOR, gross domestic product will increase also.

From the monetary model, on the other hand, the ratio between change in prices and change in gross domestic product is negative. Increasing in gross domestic product increases demand for money, thus, creating discrepancy of the monetary market. For the given rate of internal credit expansion, this discrepancy will create inflow of foreign reserves and will cause falling in internal prices to clear monetary market.

In summary, the basic idea consists in the simultaneous solution of these two ratios so that gross domestic product and internal prices could be expressed as function of exogenous and political variables. The structure of the merged model can be represented graphically.

Realisation of simple monetary model and incorporated model [3-4] was executed in Microsoft Office Excel application on given by Republic of Moldova data. As base year 2000 was chosen. Models are supplied with the simple and convenient interface, which help the user carry out simulations and iterative calculations, changing target, policy, exogenous and endogenous variables. Calculations were carried out also on model of World Bank RMSM-X [5], which is a version of Harrod-Domar growth model with two financial gaps.

In the approach described above, in merged monetary and growth model the simplified form of the behavioural equations is used with the purpose to receive a simple method for solution. However, model, which formed a basis for economic program creation, should be exhaustive to reflect adequate economy structure, but such a model can be received only endowing analytical solution. In that case to obtain model solution the numerical methods are used.

## 3 More Complicated Financial Programming Models

For operative work, including long-term planning, it would be desirable, if not necessary, to have a small calculating model. In such a model certain behavioural relation between key macroeconomic variables and control parameters are precisely determined by means of the consistent

framework. At the absence of precisely determined model it's very difficult to develop enough desagregated middle term economic program which will be co-ordinated to behavioural relationships. Moreover, a key element of the economic program designing is the ability to change an assumption, interrelations and policies to provide a number of alternatives even if the purpose of these alternatives is granting wide basis for discussion of the offered policies. Precise, completely integrated model is necessary for such type of exercises.

The represented in [6] model, though it was applied to Turkey, is sufficiently general that could be applied to a lot of developing economy. It is intended for creation of a framework for middle terms analysis with certain connections between tax, monetary, the exchange rate policies and key macroeconomic variables. The model has some considerably differing features. It unites production determined by demand, with supply that is reflected in policies, which influence investments. It allows determining relative shares of internal and external factors of production by means of their relative prices. Adjustment and growth interact with fiscal, monetary and external policies. It gives a flexible policy of the external debt.

The national economy yields unique goods, domestic goods, which either are exported, or consumed at home, or invested. It is supposed, that all import goods are intermediate, i.e. on average there is some gain of the added cost on the imported goods before their final sale. Output is carried out by means of internal factors of production and intermediate import goods. The added cost or gross domestic product minus intermediate import goods is equal to volume of output. Relative values of each used factor, factorial shares, are functions from their relative prices. The level of output is determined by a final demand for it. If output is less than capacity, so that demand does not prevail over prices, the domestic goods prices are determined by cost of two factors of production: industrial taxes and import duties. If output is more than capacities, prices are higher than cost and reflect pressure of abundant cumulative demand. The model thus is suitable for determination of the inflation caused by both demand, and cost.

The final demand consists of two parts: exogenous part, which con-

tains government consumption, government investments, and change in stocks and export. Real private consumption is the function of real available income. Real private investments are the function of real cost of internal credit flow directed to private sector. Export of goods is the function of internal key variables: real gross domestic product and relative prices. The real added cost is made due to increase in total output (endogenously determined, sensitive to the relative price), received due to internal factors of production. By means of the added cost price index, set exogenous, used basically for all calculations of wages, payments for the ground, the capital etc., the real added cost will be transformed to the nominal added cost. From total value of the nominal added cost the certain part goes directly to the private sector as the income, the rest is collected at the state. The added cost collecting in a private sector, with taking into account profit from export and factors income, transfers from abroad and state transfers, determines total private income.

After subtraction of direct taxes the available private income is determined. Then real private consumption as function from the real available income pays off. Budgetary deficit pays off as a difference between state expenditures and state revenue. The state expenditure consists of exogenously determined consumption and changes in stocks, interest payments on unpaid external public debt, transfers to private sector and endogenously determined interest payments for credit which should be received after basic year. The public revenue consists of endogenously determined direct and indirect tax incomes, transfers from abroad and parts of all added cost which concerns to the state. Subtracting from budgetary deficit the exogenously determined part of internal non banking financing, the flow of internal credit given to the state by banking system, and known external financing (disbursement minus amortisation payments), we will obtain the unpaid external financial debts. The external debt service module includes the detailed description of loan categories, grace and maturity period of loans, interests rate change, calculates foreign financing necessary to serve this deficit and government expenditure on interest payments for again acquired debt.

The current account balance can be represented as resources balance (i.e. a difference between export and import of goods and non-factor services plus net factor income and transfers to the government and private sectors from abroad and minus interest payments for the external debt service).

The capital account of balance payments consists of a set of exogenous entrance indicators for the state and private sectors, of endogenously defined sum of the state deficit external financing and the private capital flows caused by destabilisation of the money market. Change of foreign reserves is simply the sum of the current and capital accounts. Change of reserves will reflect change in current account, in expansion of external financing of state deficit, increasing in internal monetary market discrepancy.

The real supply of money is determined by a function of real gross domestic product and inflation. The capital is in permanent movement. If, for example, the supply of money exceeds demand for money, there will be an outflow of the private capital, as local residents will try to reduce available deficit of monetary balances to balance their portfolios.

Given model was adapted to economy realities of Republic Moldova [7], and realised in system of economic modelling Javelin-Plus, using an iterative method for solving of the system of non-linear equations and econometric estimation of six behavioural equations being a component part of the model. The realised model is endowed with a set of multilevel user menus, which provides simple and effective interface for the user. The program interface enables the user to change values for exogenous, policy and target variables; to change validity of model; to modify behavioural equations, historical and base year data. Thus, the user can carry out multiple simulating calculations and to carry out on their basis the multilateral economic analysis.

Also in Javelin-Plus there was realised the W.Bier's idea [9] about creation of a set of macroeconomic models, using various updating of behavioural functions.The program and the appropriate interface was created, with their help the control calculations were carried out, according to economic development of Moldova Republic.

Further the financial programming model developed by U.S.A.D.

for Romania in 1993 [10] was considered. The model contains 82 equations, the 16 equations from which are the behavioural ones. As the purpose of model is granting the detailed and concrete data concerning intersectoral interrelations, the high degree of simultaneity was entered at the solution of the model. The model is solved taking into account the target variable gross domestic product, which is determined in nominal and in real values. Also as a target variable any other endogenous variable can be chosen. Thus, the set of decisions received with the help of the model depends on the purposes of the application.

The main indicator – gross domestic product – is calculated from the production and use side of the basic sectors and from the expenditure side on the other hand. The model gives the solution for two main blocks: the balance of payments block and the block concerning national accounts.

The balance of payments block delivers the data on the basic components and determines balance of goods and non-factor services, which then are used in the block concerning national accounts. To catch feedback effects in economy, the block of national accounts contains an essential set of relations between endogenous variables of system.

The balance of payments block. Volumes of export depend on external economic activity, and export prices are connected to world trade. Besides this, relative prices of export are important at definition of export volumes, reflecting competitiveness of national economy, thus, influencing the amount of local goods required by foreign markets, according to competitiveness of local and foreign supplies in these markets.

The imported goods are subdivided into the power goods and other goods. Import of fuel appears basically in trading exchanges. Price indexes on export were developed on the aggregated total exports, and price indexes on import were constructed for combustible and other import.

As soon as export and import of goods and non-factor services are received, the model calculates balances for goods, non-factor services, balance of material services and current account balance. Change in foreign inflow is almost always set exogenously. To the moment as

foreign reserves are established it is necessary to enter policy into the model, used for maintenance of this offered level. Model calculates total need for import, volume of external debt and capital account deficit.

The model can give information concerning necessary external financing; model solution gives total loans necessary for financing of current account deficit. It is easy to enter estimations into the model concerning need for loans, except for existing commitments. For calculation of additional loans, from the total necessary loans, calculated in the model, disbursement and amortisation payments will be taken away.

**Investments.** Total investments are broken into gross investments and change in stocks both of them are represented by behavioural equations. Total investments are determined by gross surpluses of exploitation, by general level of economic activity and by interest rate. General level of economic activity also determines changes in stocks. As total gross surpluses of exploitation represent difference between gross domestic product and net profit of domestic economy to which taxes from income are added, investments are closely interconnected to economic activity of the state and private sectors.

Private consumption depends on net profit of household, relative prices and uncertainty in prices. Net profit of households consists of salary and other incomes. Wage rate depends on the added cost which, in turn, depends on the general level of economic activity.

**The government consumption and transfers.** The public sector consists of state and local budgets and social insurance fund. As the model assumes the representation of an opportunity of the analysis of tax policies at a national level, the basic desagregation takes place at the state level. Concerning to incomes, direct taxes are subdivided into taxes from associations and taxes from households. Modes of the taxation for these two sectors are policy variables, and they can be used for the analysis of tax initiatives on a national economy. Indirect taxes are subdivided into the tax from the added cost, excises and taxes to import. Tax tariffs also are a policy variable within the model framework. The State expenditure consists of consumption and sub-

sidy. Subsidies are subdivided into what are given to business sector and subsidy, given to household. The historical data on long prospect are available only for the state expenditures, rather than for the state consumption, and as consequence, behavioural function for the general expenditures was appreciated. And, thus, the state consumption is a result of subtraction of the subsidies, transfers and interest payments on an external and internal debt from the general expenditure.

**The added cost.** The system of equations simultaneously gives solution concerning total production and general expenditure in national economy. The secondary sector, which accumulates almost half of total added cost of the country, is endogenous; its level depends on investment activity and consumption. Though the tertiary sector is contained basically in the models, the initial sector is endogenous, as its level is defined by a global level of expenditure and economic activity of the remaining two sectors.

The considered model of Romanian economy was adapted for economy of Republic Moldova [11]. In Javelin-Plus system the modelling computer program having multilevel user interfaces was realised. Enclosed user menus are represented. With its help it is possible: to modify initial historical and base year data; to modify and estimate behavioural functions; to set values for exogenous variables for all period of model action; to establish values for policy variables also for all period of model action; to receive results of calculations as diagrams and tables. There is an opportunity to analyse intermediate results. The base year co-ordination matrix, for all years previous to base year is realised. Tables of results on all considered in model sectors are stipulated. There is an opportunity of viewing of intermediate results and fast diagrams. At the desire, it is possible to print out all necessary graphic and digital information. The model is realised using consecutive iteration method. Therefore in an interactive mode it is possible to change amount of iterations and accuracy of calculations. The opportunity of change of validity of model is stipulated. Calculations for the five years' period and longer forecast periods were carried out.

## 4 A General Equilibrium Model

In continuation, we shall state simple general equilibrium computing model [12] for small open two-sector economy. Changes in an external environment and economic policies become tool in definition of progress in economic development for less developed countries. These models give good mechanism with the help of which the external shocks and economic policies pass through all economy. Shocks include the external sector trading shocks: repeated increase of the mineral oil prices or falling of the prices for the intermediate goods in the inflow of external capitals. The reciprocal policies most frequently offered also are aimed at external sector: real exchange evaluation for the adaptation to adverse trading shocks or to reduction of foreign borrowing; reduction of the deformed taxes for increase of economic efficiency and to make economy competitive in the world markets.

Hence the model which covers the marked shocks and the appropriate policies, attaches special meaning to external sector of economy. Moreover, the decision of many problems is connected to relations between foreign sector and other economy. Thus, the model will cover, at least, two industrial sectors: one, making goods on export, and other making goods for a home market. The goods participating in circulation are useful to subdivide into imported and exported goods. Such division enables to look at trading shocks, and also on influence of tool policies, such as tariffs for import and grants for export.

The minimal model, which includes these features though is small, covers a large area of results. It is possible to investigate influence of increase in prices on mineral oil (or other prices for import and/or export). Besides these model enables to look at use of trading and financial policies: grants for export, tariffs for import and internal indirect taxes. Increase or decrease in foreign capital inflow also can be investigated within the framework of this model.

The biggest advantage of small models consists in their simplicity. They represent transparent mechanism thanks to which change in international shocks or policies influence economy.

Base model concerns one country with two industrial sectors and

three goods. There are two products made in the country: export of goods that are sold to foreigners and have not been demanded inside the country, and internal goods sold only inside the country. In the model there is one consumer which receives the income. The country, being small, in the world market, faces with the fixed world prices for export and import.

In the model three economic agents operate: producer, household and rest of the world. Marginal opportunities of local production are determined by the greatest possible combination of commodities between export and demand for the internal goods, which economy can offer. Function is supposed to be concave and will be specified as function with constant elasticity of transformation (CET). The volume of the aggregated production is fixed. As in the model the intermediate commodities are not examined, the volume of production corresponds to gross domestic product. The assumption of the fixed level of production volume is equivalent to full employment of all initial factors of production. Export-demand for internal goods ratio is function of the relative prices. The price of the composite good is determined as dual estimation for the equation determining export. The price of aggregated supply corresponds to gross domestic product deflator.

The composite goods consist of the local goods and the import goods. It is supposed, that import and the local goods are imperfect substitutes (are not interchangeable). Following this treatment, it is supposed, that the composite goods are defined as aggregated function with constant elasticity of substitution (CES) dependent on volume of import and volume of internal goods. Consumers maximise composite goods utilisation. Volume of import is defined from relationship between import and internal supply, which is function of the relative prices. Price of the composite goods is defined as the dual evaluation appropriate to equation for import. Price of composite goods corresponds to aggregated consumer prices index.

Separate equation determines income of domestic economy and demand of domestic economy for composite good. We shall note that income is spent for the unique composite good.

Price equations define interrelation between seven prices. In the

model world prices for export are fixed, local prices for export and import are determined; price of internal supply is calculated; price of aggregated output and price of composite good pays off. As only relative prices are considered, it is necessary to define scaling price – the exchange rate in this case.

There are determined three equations specifying markets clearing conditions. Internal demand is equal to the internal supply, composite demand is equal to composite supply and trading balance restrictions should be satisfied. The full model contains fourteen equations and thirteen endogenous variables. However three balance conditions are not independent. Any of them can be omitted and the resulting model becomes completely determined. To prove, that three conditions of equilibrium are not independent, it is enough to show that the model satisfies the Walraas law. Such model "is closed" in the sense that there is no outflow of funds inside and outside the economy. It is easy to prove it using three identities: for aggregated supply, for aggregated composite supply and aggregate profit in nominal prices.

The considered model, known as (1-2-3) model, differs from standard neo-classic models in which all commodities are sold and all sold goods are interchangeable. The commodities in it have no property of perfect substitution and perfect transformation. All commodities made inside the country which are not exported, are really treated as commodities not subject to sale and purchase. The share of the goods not subject to sale and purchase in gross domestic product is equal to difference between unit, (as all parameters are considered in shares from gross domestic product) and share of export, and all sectors are treated symmetrically.

The expanded version of model includes public revenues and expenditures, and also accumulation and investments. In new statement four tax tools are included: the tariff for import, grants for export, indirect taxes sales, direct taxes. Besides that, accumulation and investments are included. The unique representative of domestic economy saves up fixed share of income. Public savings (budgetary deficit or proficit) are balances between income from tax both foreign grants and government expenditure (all exogenous) such as the government consumption and

transfers to domestic economy. Current account balance, entered for foreign accumulation presentation, is a difference between export and import in world prices, adjusted to grants and transfers from abroad. As it was noticed earlier, output is fixed, so the model is controlled with the help of accumulation: aggregated accumulation is adjusted under the aggregated investments. In total, there are 20 equations and 19 endogenous variables. However, by means of law Walraas one of the equations – the identity of the accumulation-investment – follows from others and can be omitted.

Computing framework construction needs to carry out at modelling usual steps: announcement of parameters and variables, to find out input data; assignment of initial values to parameters and variables; equation specification. And in addition, model should be precisely determined as a set of equations; in some cases the necessity of scope function optimisation can appear. In summary computing simulation calculations will be carried out with using the built-in procedure. Other convenient feature of (1-2-3) model is the small volume of necessary data. The data on national income, financial and balance of payments accounts, which are usually enabled for publication by national governments, are sufficient.

At calibrating all the data are scaled and indexed according to output, which is accepted equal to 1 in base year. Calibrating of parameters and variables will be carried out; it is interconnected with base year data so, that calibrating will be carried out automatically when elasticity values or base year are changed. Exponent parameters of CET and CES functions are calculated under the formulas containing appropriate elasticity. Having pre-set values for export, import, supply and demand at the base year, the share parameters of internal commodities are determined. Formulas for their calculations turn out from expressions for demand for export and import functions accordingly. Other pair of share parameters is determined directly with the help of functions CET and CES. Alternative procedure for calibrating consists in establishing base year data and solving system of equations for definition the values of parameters satisfying base year equilibrium condition. In this case there is no necessity to receive explicit expres-

sions for parameter that is very useful property when more complex functional forms are considered.

The considered model was adapted to economic conditions of Moldova Republic [12]. With its help simulation calculations were spent, using as policy variables tariffs for import, direct and indirect tax rates. On the basis of these calculations the quantitative analysis was carried out.

The calculating general equilibrium model was realised in Excel application. It is endowed with multilevel user menus and user rulers that does work with model simply and conveniently even for the persons, not having special teaching. There is stipulated built-in Help for users, it contains brief description of the model and user's guide.

## 5 Behavioural Functions

For financial programming model (example of Turkey) the following behavioural equations were used.

Real private consumption is defined as a function of a trend component of real disposable income and a transitory or deviation from trend component of real disposable income.

$C_t = \overline{C} + k_1 * YDRP_t + k_2 * YDRT_t.$

Real private investments (in log terms) depend on the logarithm of one-two period lags of real private investments and the logarithm of real flows of credit to the private sector from the banking system $\Delta DCR_t/P_t$,

$Ln(I_t) = \overline{I} + k_3 * Ln(I_{t-2} + k_4 * Ln(I_{t-1} + k_5 * Ln(DeltaDCR_t/P_t).$

Export volume (in log terms) is determined as a function of the logarithm of real gross domestic product and the logarithm of price of exports in domestic currency terms, determined as the product of the exchange rate, and the foreign price of exports $e_t^{*E} P_t$, relative to the price of the home good $P_t$

$Ln(E_t^v = \overline{E} + k_6 * Ln(GDP_t/PGDP_t) + k_7 * Ln(e_t P_t^{*E}).$

Import volume $M_t^v$ is defined by multiplying the contribution of imports to a unit of output $m_t$, by the level of output. The contribution of the logarithm of imports to output depends on the logarithm of price of imports in domestic currency terms $m_t X_t$, which is determined as the product of the

exchange rate, the foreign currency price of import$P_t^{*M}$ and one plus the import duty rate $t^M$, relative to the price of the home good:

$$Ln(m_t) = \overline{M} + k_8 * Ln(e_t P_t^{*M}(1 + t^M)/P_t).$$

Capacity output $\bar{X}_t$, (in log terms) is a function of a time trend and the logarithm of real investments :

$$Ln(P_t^{*M}) = \overline{X} + k_{10}t + k_{11} * Ln(I_t + G_t^I).$$

The logarithm of the real demand for real money $M_t^D/P_t$ is specified to be a function of the logarithm of real GDP and the rate of inflation:

$$Ln(M_t^D/P_t) = \overline{M^D} + k_{12} * Ln(GDP_t/PGDP_t) + k_{13} * ((P_t - P_{t-1})/P_{t-1}).$$

Behavioural Equations of the Romanian Model.

All dependencies, except those for prices, are expressed in real terms.

Real private consumption depends on the size of net profit of households and of the relative prices.

$$Ln(NPCNR_t) = \overline{CN} + g_1 * Ln(NYHSR_t) + g_2 * Ln(NPCNP_t/GDPLP_t) + \\ + g_3 * NICNP_t.$$

$$Ln(NICNP_t) = g_4 + g_5 * TREND + \nu_t.$$

Real investments consist of the total accumulation of a fixed capital and the change in stocks. Total accumulation of a fixed capital is determined as a function of the GDP, gross surplus of exploitation, and interest rate.

$$Ln(NGFIVR_t) = \overline{GFIVR} + g_6 * LN(NGDPR_t) + g_7 * Ln(NPRFTR_{t-1}) + \\ + g_8 * Ln(NINTER_{t-1}).$$

Real change in stocks is determined as a function of real gross domestic product. There was introduced one dummy variable in scope to consider some production problems in year 1989

$$Ln(NDINVR_t) = \overline{DIN} + g_9 * Ln(NGDPR_t) + g_{10} * dummy1_{98}.$$

The state operational expenditure includes consumption, percentage payments for an external and internal duty, transfers and subsidies.

$$Ln(NGCER_t) = \overline{GCE} + g_{11} * Ln(NGDPR_t) + g_{12} * dummy2_{91}.$$

Real government expenditure includes consumption, interest payments for external and internal debt, transfers and subsidies. Government expen-

diture, naturally, are defined by a level of real gross domestic product. The binary variable was put into consideration to take into account an unnatural diminishes in real government expenditures in year 1991.

Real total value added in secondary sector is determined as a function of total real consumption and of the gross fixed capital.

$$Ln(VLAD2_t) = \overline{VLAD} + g_{13} * Ln(NCONR_t) + g_{14} * Ln(NGFIVR_t).$$

Real rate of wage is determined as a function of the real value added in all sectors of economy and of one lug rate of wage.

$$Ln(NWAGER_t) = \overline{WAGER} + g_{15} * Ln(VLAD2_t) + g_{16} * Ln(NGFIVR_t).$$

Deflators of investments, private consumption and the government expenditures - everyone are considered functions of gross domestic product deflator.

Investment deflator

$$Ln(NINVLP_t) = \overline{INVLP} + g_{17} * Ln(GPLP_t.$$

Private consumption deflator

$$Ln(NPCNP_t) = \overline{PCNPP} + g_{18} * Ln(GPLP_t).$$

Government expenditures deflator

$$Ln(NGCNP_t) = \overline{GCNP} + g_{19} * Ln(GPLP_t).$$

Total export volume of goods is determined as a function of foreign markets economic activity, and simultaneously as a function of the time variable

$$Ln(XMRCH_t) = \overline{XMRCHR} + g_{20} * Ln(FGDPR_t) + g_{21} * TREND.$$

Price for the export is determined as a function of the international prices, and of the time variable

$$Ln(XMRCH\$P_t) = \overline{XMRCH\$P} + g_{22} * Ln(TMUVP\$_t) +$$
$$+g_{23} * TREND.$$

Total import is subdivided into two categories: petroleum and other import. These two kinds of import depend on real gross domestic product and on the relative price for import. The relative prices include the exchange rate, and also can include import tariffs. As consequence, it is possible the consideration of the effects of exchange rate policy on economic activity of particular sectors and economy as a whole.

Import of petroleum is determined as

$$Ln(MOILR_t) = g_{24} * Ln(NGDPR_t) +$$

$+g_{25} * Ln(MOIL\$P_t/(GDPLP_t/REXCHI_t))$.

Other import is determined as

$Ln(MOTHR_t) = g_{26} * Ln(NGDPR_t)+$
$+g_{27} * Ln(MOTH\$P_t/(GDPLP_t/REXCHI_t))$.

Import price depends of the international markets price.

Petroleum price is determined by the following equation:

$Ln(MOIL\$P_t) = \overline{MOIL\$P} + g_{29} * Ln(TMUVPS\$_t)$.

Other import price equation is determined by the following:

$Ln(MOTH\$P_t) = \overline{MOTH\$P} + g_{30} * Ln(TMUVPS\$_t)$.

# 6 Conclusions

The basic problem appearing at the realisation of enumerated model is the problem of information guarantee. We shall start with financial programming and growth models. As the basis of these models is made of calculating identities, and parameters of these models, as a rule, are estimated by calculation of simple relations between researched indicators, at the nearest moment of time, very seldom these parameters are estimated by econometric means. Therefore the basic problem at realisation of these indicators is connected to definition of exogenous variables, and, first of all, definition of growth for the short-term period on the basis of expert estimations of possible development. Comparative advantages of these models consist in their simplicity and ability to create the co-ordinated framework for estimation and monitoring of the short-term economic development allowing repeatedly and timely to overestimate both policy variables and offers which include forecasting stages of economic growth.

In economic models for Turkey and Romania the complication of the models goes on a way to completing a set of computing identities not only by the simple ratios determined by means of behavioural parameters, but also by the more complex econometric dependencies. With its help there are determined:

- prices;
- basic components of the aggregated demand, monetary and private sectors and balance of payments.

With introduction of the econometric dependencies in macroeconomic models, there are very serious problems in estimation of the coefficients of econometric equations, and especially for economies in transition to market relations. And

it is clear that at attempt to carry out econometric estimations we face with lack of the data as quantitative (a sufficient number of observation is lacking), and also qualitative (statistics on the main macroeconomic indicators are weak). The result is that a consistent and reliable time series is frequently lacking. To such indicators it is possible to attribute deflators of: gross domestic product, investments, consumption, government expenditures, added value etc. And other problem - transition economy is subject to structural reforms, which represent a particular problem.

Taking it into account, the statistical interrelations at reception of the effective econometric estimations of the model's parameters or at forecasting the future change in evolution of considered macroeconomic variables are necessary to be considered cautiously. Calculating general equilibrium models less are subject to data restrictions in comparison with other considered models. They use, basically, national accounts base year data and are constantly specified from year to year. Being based on the data of base year, on knowledge of elasticity of substitution and elasticity of transformation, factors of the behavioural functions used in model pay off. Hence, in this case it is necessary to be able to estimate elasticity of substitution and elasticity of transformation. Using the econometric estimations in transition economy imposes the necessity constantly to correct and update all historical data and, whenever possible, frequently to reconsider specification of the equations and an estimation of parameters, considering last updating of information base and having place or expected changes in structure of examined economy. Thus it is necessary to take into account the availability or presence of information in scope to specify adequately the regression equations at realisation the econometric estimations.

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# About a family of $C^2$ splines with one free generating function

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**Abstract**

The problem of interpolation of discrete set of data on the interval $[a, b]$ representing the function $f$ is investigated. A family of $C^2$ splines with one free generating function is introduced in order to solve this problem. Cubic $C^2$ splines belong to this family. The required conditions which must satisfy the generating function in order to obtain explicit interpolants are presented and examples of generating functions are given.

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**Keywords and phrases:** spline, interpolation, explicit interpolation

## 1 Introduction

Let us assume that the mesh $\Delta : a = x_0 < x_1 < ... < x_n = b$ is given on the interval $[a, b]$ and $f_i = f(x_i)$, $i = 0(1)n$, are the corresponding data points. The problem of the construction of a interpolation function $S$, such that interpolation conditions $S(x_i) = f_i$, $i = 0(1)n$, are hold and $S \in C^2[a, b]$ is considered.

It is well known (e.g. [1]-[3]) that $C^2$ cubic splines as well as different types of generalized cubic splines [5] may be used to solve this problem. In present work the family of splines, which includes many well known types of splines, is introduced. This family allows to generate new types of splines.

## 2 A family of $C^2$ splines with one free generating function

Let us define splines on the interval $[x_i, x_{i+1}]$ as follows:

$$\begin{aligned} S(x) &= f_i + (f_{i+1} - f_i)t + \\ &+ \{h_i^2 M_i[2(\nu(0,p_i) - \nu(t,p_i)) + 2(\nu(1,p_i) - \nu(0,p_i))t - \\ &- \nu^{"}(1,p_i)t(1-t)] + h_i^2 M_{i+1}[2(\nu(t,p_i) - \nu(0,p_i)) - \\ &- 2(\nu(1,p_i) - \nu(0,p_i))t + \nu^{"}(0,p_i)t(1-t)]\}/ \\ &/ (2(\nu^{"}(1,p_i) - \nu^{"}(0,p_i))) \end{aligned} \tag{1}$$

where

$$t = (x - x_i)/h_i, h_i = x_{i+1} - x_i, S"(x_i) = M_i.$$

The function $\nu$, which in the sequel will be called generating function for the spline (1), must satisfy the conditions

$$\nu(t,p) \in C^2[0,1], \nu^{"}(1,p) \neq \nu^{"}(0,p) \tag{2}$$

where $p$ is a vector of free parameters.

From (1) the following formulas for the first and second derivatives of the spline are obtained:

$$\begin{aligned} S^{'}(x) &= (f_{i+1} - f_i)/h_i + \{h_i M_i(-2\nu^{'}(t,p_i) + 2(\nu(1,p_i) - \nu(0,p_i)) - \\ &- \nu^{"}(1,p_i)(1-2t)) + h_i M_{i+1}(2\nu^{'}(t,p_i) - \\ &- 2(\nu(1,p_i) - \nu(0,p_i)) + \nu^{"}(0,p_i) - 2t\nu^{"}(0,p_i))\}/ \\ &/ (2(\nu^{"}(1,p_i) - \nu^{"}(0,p_i)) \end{aligned} \tag{3}$$

$$S^{"}(x) = \frac{[M_i(\nu^{"}(1,p_i) - \nu^{"}(t,p_i)) + M_{i+1}(\nu^{"}(t,p_i) - \nu^{"}(0,p_i))]}{(\nu^{"}(1,p_i) - \nu^{"}(0,p_i))} \tag{4}$$

From (1) and (3) taking into account (2) it follows immediately that the spline and the second derivative are continuous. From the requirement of the continuity of the first derivative of the spline at

the knots of mesh $\Delta$ we obtain the following system of linear algebraic equations:

$$c_i M_{i-1} + a_i M_i + b_i M_{i+1} = \delta_i^{(1)} - \delta_{i-1}^{(1)}, \quad i = 1(1)n-1, \tag{5}$$

where

$$c_i = \frac{h_{i-1}(-2\nu^{'}(1,p_{i-1}) + 2\nu(1,p_{i-1}) - 2\nu(0,p_{i-1}) + \nu^{"}(1,p_{i-1}))}{(2(\nu^{"}(1,p_{i-1}) - \nu^{"}(0,p_{i-1})))}$$

$$\begin{aligned} a_i &= \frac{h_{i-1}(2\nu^{'}(1,p_{i-1}) - 2\nu(1,p_{i-1}) + 2\nu(0,p_{i-1}) - \nu^{"}(0,p_{i-1}))}{(2(\nu^{"}(1,p_{i-1}) - \nu^{"}(0,p_{i-1})))} - \\ &- \frac{h_i(-2\nu^{'}(0,p_i) + 2\nu(1,p_i) - 2\nu(0,p_i) - \nu^{"}(1,p_i))}{(2(\nu^{"}(1,p_i) - \nu^{"}(0,p_i)))} \end{aligned}$$

$$b_i = \frac{h_i(-2\nu^{'}(0,p_1) + 2\nu(1,p_i) - 2\nu(0,p_i) + \nu^{"}(0,p_i))}{(2(\nu^{"}(1,p_i) - \nu^{"}(0,p_i)))}$$

$$\delta_i^{(1)} = (f_{i+1} - f_i)/h_i.$$

The system presented above is the undetermined one. Since (5) provides only $n-1$ linear equations in $n+1$ parameters $M_i$, it follows that two additional linearly independent conditions are always needed. In the present paper we shall consider that $f^{"}(a) = f_0^{"}$ and $f^{"}(b) = f_n^{"}$ are available, therefore $M_0 = f_0^{"}$ and $M_n = f_n^{"}$ is an obvious choice as end conditions.

It should be mentioned that in the case when $\nu(t,p) = t^3$ the splines (1) represent the well known $C^2$ cubic splines. As another examples of generating functions the following ones may be given: 1) $\nu(t,p) = t^n$, $n \geq 3$ and 2) $\nu(t,p) = t^3/(1+p(1-t))$, where $p$ is a free parameter.

## 3 A subfamily of $C^2$ explicit splines with one free generating function

In many practical problems the scheme of explicit interpolation represents a special interest. From family (1) we can obtain explicit interpolation schemes. In order to get an explicit interpolant the generating function has to satisfy the following conditions:

$$-2\nu^{'}(1,p)+2\nu(1,p)-2\nu(0,p)+\nu^{"}(1,p)=0 \tag{6}$$

$$-2\nu^{'}(0,p)+2\nu(1,p)-2\nu(0,p)+\nu^{"}(0,p)=0 \tag{7}$$

It is easy to prove that conditions (6) and (7) are satisfied, for example, by the following functions:

$$\nu(t)=t^2+6t^3-10t^4+4t^5,$$

$$\nu(t)=t^2/(2t^2-2t+1),$$

which were proposed in [4].

In this case $c_i=0$ and $b_i=0$ in the system of linear algebraic equations (5). As a result we get

$$M_i=(\delta_i^{(1)}-\delta_{i-1}^{(1)})/a_i, \quad i=1(1)n-1,$$

and (1) becomes the explicit one.

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# The comparative analysis of implementation practices of e-government services

Lyudmila Burtseva

**Abstract**

The general trends of e-government development are considered with special emphasis on e-government services. The analysis of current state of the governmental sites of the countries-leaders on the e-government development in comparison with the corresponding sites of Moldova are provided.

## 1 Introduction

The e-government conception is defined in many documents published by corresponding international ([1], [2]) and national organizations online and by paper. These definitions are being permanently complimented and developed. So the e-government is the complex conception, general aspects of which are: representation of new model of government; reformation of the relationship between state, citizens and business; information technologies application to whole infrastructure of government interactive cooperation with business and citizens. Considering these main aspects the conclusion can be made that satisfaction of citizens needs is determined by e-government services development.

## 2 E-government development trends

From the services point of view e-government can be formally represented as the implementation of schemes: G2C (Government to Citizen), G2B (Government to Business), G2G (Government to Government) [3]. The analysis of contemporary statuses of government sites of

states, leading in the e-government development, shows trends declared as achievements. Let us consider these trends.


Figure 1. E-debates at Denmark site

The government portals are still actual theme. After the level of government digitizing is passed, these portals are pointed to the implementation of e-democracy elements. At that stage it means to complete the portal functions with the possibilities for citizens participation in government decisions adoption. It can be implemented in the form of e-debates as at sites *www.hals.dk* (Denmark) presented at Fig.1 and *www.kleurrijkvlaanderen.be* (Belgium). Another way is the online petitions filling in as at the site *http* : *//epetitions.scottish.parliament.uk* (Scotland) presented at Fig.2. The most time consuming government transactions, such as licensing (fishing in Norway, gambling in New Zeland), taxes and customs dec-

laration, are represented ONLINE. This also includes citizens demands on receiving the identify documents (passport in UK), real estate information, even immigration papers (Norway).

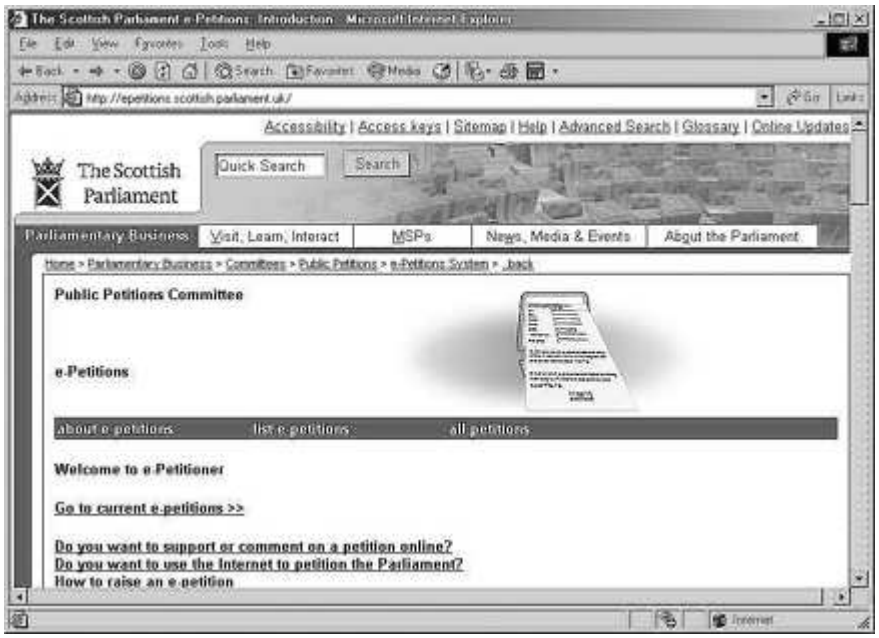


Figure 2. Online petitions filling at scotish site

Besides these transactions the e-elections become more popular theme. The possibility to implement the civil duty without leaving of home resolves the problem of electors presence and saves the finance. The consulting functions of government portals are also been extended by ONLINE consultation of lawyers and civil officials. The variety of internet databases is now complemented by topologic maps bases and real-time updated bases of road events. Although the main government functions belong to the politics the achievement of financial transparency is considered as break-out. In Germany the plans of this achievement are included into the program BundOnline2005.

E-portals are developed for governments of all levels but concept of e-city is considered separately as the unit. Among the winners of e-city competitions of last year there are Copenhagen (*www.kbhbase.kk.dk*) and Bremen (*www.bremen.de*). The analysis of these sites shows that all trends mentioned above are considered as positive at this initial level of e-government. The informative function is fully implemented: officials schedules; access to databases of laws, government acts, real estates, maps, etc; online consulting of officials, lawyers, customers rights defenders; police chronicles. All financial functions (at this level the municipal services payments are added) and the most part of permissive ones are represented online. The elements of e-democracy are also presented at these sites as forum where the citizens can discuss the government initiatives.

## 2.1 Implementation of G2C

In implementation of G2C e-government branch some achievements are also marked. Initially the main function of this direction was to supply the citizens by required information. More often this function is provided by downloading corresponding documents in PDF format. In particular many sites present tax form in this format (including Moldova). But this initial function is now extended. The materials are joined by theme on CD/DVD and sent according internet inquiry. For example the parliament site of Switzerland sends the bills selection on DVD. The site of ministry of education of the Federal State of North Rhine-Westphalia (Germany) also presents interesting extension of common function: all documents, for example for university entrants, are sent gratis by internet inquiry. Another interesting extension is preparing the personalizing document interactively. For example, the traders can select type of goods and their quantity in interactive form on site of German custom office www.internetzollanmeldung.de and then they will receive completed declaration in PDF format. The researcher can receive the table of statistical data from Norwegian site in format requested by him (excel, dbase, etc).

The second prevalent way to supply the citizens by informa-

tion is internet database. The legislation databases are broad presented. The database of Norwegian legislation translated into English is presented on site of Oslo University department of law (*www.ub.uio.no/ujur/ulov/english.html*) and is provided by keyword search and catalogue. Besides the legislation, internet databases cover the land and lodges prices (Germany), court practice cases (Germany), media and their owners (Norway), public WC with search system and clickable map (Australia).

The government of several countries becomes the customer and then the owner of some free downloadable software. For example the mentioned above site of ministry of education of the Federal State of North Rhine-Westphalia (Germany) presents the software PROGNOS - forecast of graduation emission, WinPlan half-automatic interactive creation of school schedules. The sites which provide customer rights defence should be emphasized. These sites are created by countries with developed e-government and contain corresponding legislation, forms for petition and lists of organizations dealing with theme. At the site of Norway, which is of such type, there is the list of assumption of government officials, traders, transport service people etc.

So, the task of information supply is implemented successfully. So, the main function is changed now to providing government services ONLINE. The most widespread services of this kind is the answer to the specific person question by ONLINE forums or just question form. Some government sites of Moldova also provide such service (Pagina Oficială a Republicii Moldova - *www.moldova.md*, Pagina Oficială a Preşedintelui RM - *www.prezident.md*, Ministerul Afacerilor Interne - *www.mai.md*, Departamentul Tehnologiilor Informaţionale - *www.registru.md*, Departamentul Dezvoltării Turismului - *www.turism.md*, Agenţia de Stat Pentru Protecţia Proprietăţii Industriale - *www.agepi.md*). Developing of telecommunication induces the popularity of internet bridges in which the officials answer the questions ONLINE.

In the financial domain the main barrier on the way of e-services is anonymity of Internet and vulnerability of user indentification. There are some methods: digital signature, encrypted data transferring pro-

tocols - which are used for confidentiality protection, but taking into consideration the activity of computer piracy these methods are not reliable enough. Moreover the position of several states concerning problem of digital signature is different. For example, Netherlands declares that the government must be independent of digital signature recognition software developers. On the contrary Norway state itself became customer and owner of such software. So in Norway, Austria, Germany the state services dealing with finances, mainly taxes, work ONLINE.

Among another ONLINE services the following ones have to be mentioned: the ministry of inner relation of UK allows the citizens to order the passport both the first and changed ones. The federal road service of Germany allows any car owner to use ONLINE the realtime updated database for finding traffic congestion.

## 2.2 Implementation of G2B

In implementation of G2B e-government branch the main function is licensing: requests for new enterprise foundation and obtaining, for example, permission of ecological organization. The German ministry of finances gets the expertise (auditing, ecological, etc.) requests ONLINE. Another services of this branch is the information ones. For example, site *www.kvk.nl* in Netherlands is about how to start business in that country.

Let us now examine the sites dealing with this branch in Moldova. Banca Naţională a Moldovei (*www.bnm.org*) presents financial markets reports. Agenţia de Stat pentru Protecţia Proprietaţii Industriale (*www.agepi.md*) presents ONLINE the library of patenting manuals (Fig.3), the samples of all documents required for invention registration, new about invention legislation and schedules of organizations dealing with patents. Serviciul Fiscal de Stat al Republicii Moldova (*www.fisk.md*) presents the samples of taxes declarations. Agenţia Naţională pentru Reglementare in Telecomunicaţii şi Informatica (*www.anrti.md*) sets the licenses of electronic and telecommunication domains, presents list and samples of documents required

for obtaining the license. Agenţia Naţională pentru Achiziţii Publice (*www.tender.md*) presents the information about government tenders the tender theme and information to address.

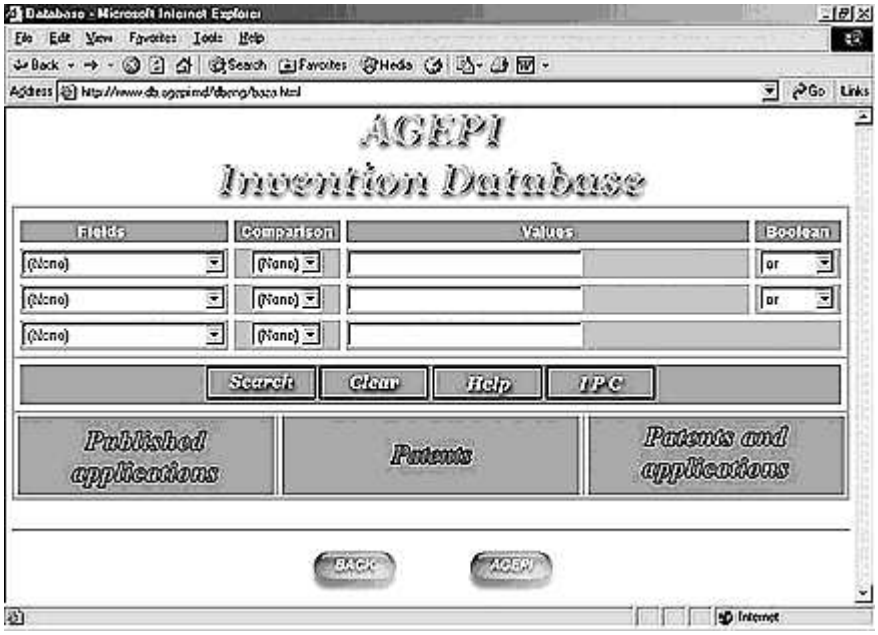


Figure 3. Online inventions database at AGEPI site

## 2.3 Implementation of G2G

The implementation of G2G e-government branch is usually hidden from ordinary internet user, so we can only rely on the announcements. The Spanish site *www.agenciatributaria.es* supplies the citizens by tax certificates arranging the coordination between corresponding government structures. The site of Pordenone Italian province provides the information for the businessmen of 51 cities. The EU program IDA ( *http* : *//europa.eu.int/ISPO/ida/* ) was started for development of pan-european sites for government services coordi-

nation. For example: EURES ($http : //europa.eu.int/eures$) allows the EU citizens to find job in any country of union; PLOTEUS ( $http : //europa.eu.int/ploteus/portal$ ) supports stuff training; SOLVIT ( $http : //europa.eu.int/solvit/site/index.htm$ ) presented at Fig.4 helps to resolve juridical problems and to break administrative barriers in business of EU counties.

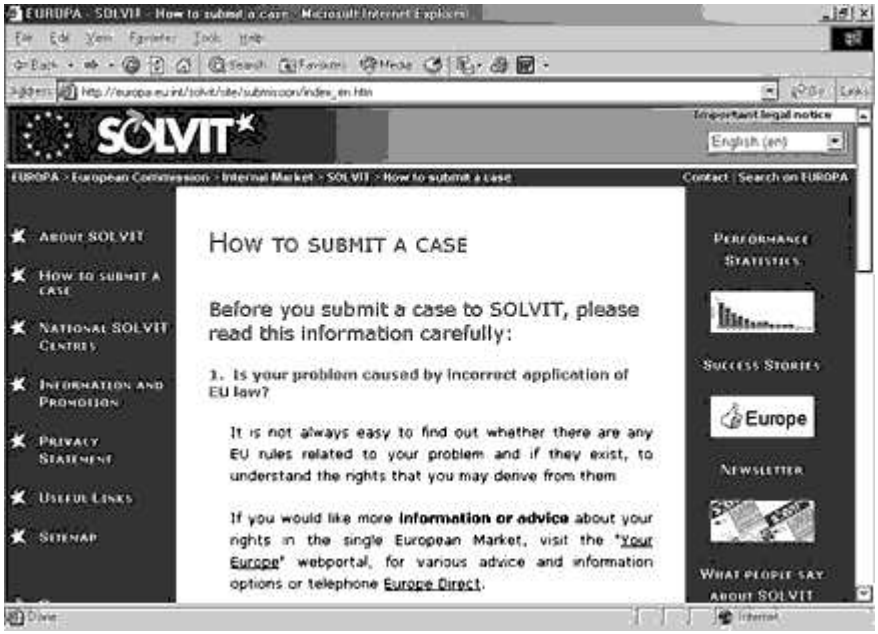


Figure 4. Pan-european site for juridical problem resolving

As was mentioned above the implementation of G2G e-government branch is not transparent for ordinary user, so the absence of due announcements in Moldova makes the analysis impossible.

## 3 Conclusions

Along with the positivity of trends some problems are also common for all countries. These problems can be practically divided into three

groups.

1. The first group contains the problems drowned by human factor. From one side the large part of officials and administrators do not want any changes in the methods of their functionality. From the other side the citizens themselves take the electronic transactions which are not supplied by hard paper copy distrustfully.

2. The second group relates to finance. This is not only the part of digital gap that is formed by expensiveness of physical carriers of new technologies. The implementation of e-government requires both the once-only financial support and further permanent expenses on its functioning supply.

3. The third group is law concerned. The paper-based office work has today strong enough support of law. To achieve both: the citizens trust in electronic transactions and the performance of new electronic function by government officials - it is necessary to develop and adopt the pack of laws.

The elements of e-government at the sites of Moldova can be analized summarizing currently announced worldwide achievements and problems. The conclusions can be made that some progressive trends are matched. Generally this refers to information supply services. The forum-like feedback services are also presented. The gate site *www.moldova.md* is set by design that is perspective for further development to meet best requirements.

From another side all main problems are reflected in e-government sites situation as well. While the legislation problems are beginning to be resolved, the financial services remain poor supplied by internet representation. The G2G branch implementation also has the slightly progress. It mainly consists in development of legislation that supplies information exchange between government structures. So the further development directions are to overcome the evident problems of internet representation of government services. The G2B branch is supplied better then others not only because of real financial support but also because of expectation of investment recoupment. To make G2C branch

working the public explanation must be done so, that e-government services can save the financial and time resources of corresponding state structures as well as of commercial ones.

Following the building of legislation base the citizens' trust in e-services can be formed by state insurance of reliability.

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# Electronic Services in Public Administration (e-government); Privacy and Freedom of Information. (review of study made for situation in Moldova)*

Lyudmila Burtseva, Svetlana Cojocaru, Constantin Gaindric, Galina Magariu, Tatiana Verlan

**Abstract**

This article is based on the analytical study [1] performed in the frames of the UNDP Project "Formulation of National Strategy in view of Information Society Technologies for Development" aimed for evaluation of e-readiness level in Moldova.

## 1 Introduction

Revolutionary development of Information and Communication Technologies (ICT) over the last 10 – 15 years and their penetration in all sectors makes it necessary to revise some customary concepts.

Use of ICT, on the one hand, requires growing investments, on the other hand, it allows settling some crucial problems at less cost and obtaining such results that could not be achieved by other methods. The new terms "Information Society (IS)", "electronic governance" speak about the domains where ICT is applicable.

The European regional conference (http://www.wsis-romania.ro) proposes "the vision of an Information Society, where all persons, without distinction of any kind, exercise their right to freedom of opinion and expression, including the freedom to hold opinions without interference, and to seek, receive and impart information and ideas through



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any media and regardless of frontiers. ... The Information Society is based on broad dissemination and sharing of information and genuine participation of all stakeholders - Governments, private sector and civil society."

The sphere of activity of governments and of public entities is a domain, which is expected to make a qualitative advance due to implementation of ICT. The term "electronic government" ("e-government") is analysed in numerous documents of the UN [2,3], during different international and regional forums [4,5], and also by some countries [6], both in the context of Information Society, and as a separate subject.

Its definitions are the same to a great extent; they are being complemented and, as time passes, completed with subtleties of meaning arising from experience of building "e-governments" in different countries. These interpretations present "e-government" only as *ordinary modernisation* of already existing structures and relations, rather than an independent idea of overall transformation of the very principles by which the state administration is organized. But it is necessary to make a distinction between a government with electronic interface and "e-government".

"E-government" should be treated as a complex and multifacet concept, which:

- reflects a model of state administration,
- is able to transform relations between the citizens and businesses with state authorities,
- includes the entire infrastructure of state administration and interaction of state, businesses and citizens via information technologies,
- supposes *orientation* of all public activities *towards satisfying citizen's needs.*

Implementation of electronic government means both transformation of the government and its activities towards achieving the following three objectives:

1. Improvement of efficiency of administrative agencies. To bring in order tax collection, drafting of laws and regulations, registration of information, circulation of documents.

2. Economic development through facilitation of procedures connected with production and commerce; increasing competitive capacity of the local producers on the world market.
3. Improvement of citizens' life standard by reforming the system of social services, healthcare, better life security, more training opportunities, more efficient protection of environment.

The White Book of Great Britain stipulates fundamental principles of electronic government as follows:

- it must create services ensuring possibilities to choose the form,
- the government and its services must become more accessible,
- it must contribute to inclusion in social sector,
- it must ensure responsibility to information content,
- government resources must be used rationally and efficiently.

The way the new techonological means are used in the existing administrative structures depend on adminstrative traditions, practice and culture proper to each country.

However, real transformations depend on the will of each leader and decision-makers at higher hierarchical levels.

## 2 E-governance

We shall consider governance not as a physical unit, and not as an act of management helped by some persons. More realistically, we should understand it rather as a process by which institutions and citizens "manage" themselves. Governance is an interaction between the public sector and the approaches to social organization for collective decision-making, facilitation of transparent mechanisms in order to enable monitoring of these decisions.

E-governance is also the method by which the public sector uses the most advanced ICT, including Internet, in offering all citizens good services, sustainable information, vast knowledge, to facilitate the access to management process and to encourage a more active participation of citizens.

The digital government is able to organize a union between citizens, some separate persons and decision-makers at such level that has never existed and could not have existed. It offers the citizen a wide access to information and knowledge, which will predetermine the state of personal freedom in the future.

The scheme of e-governance according [2] is presented at Fig.1:

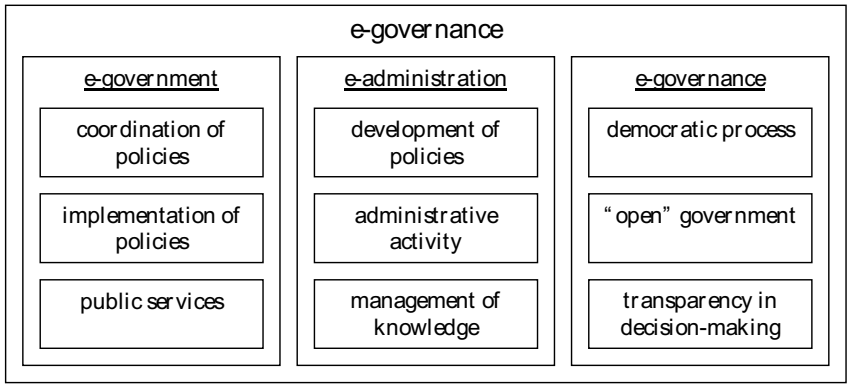


Figure 1. The scheme of e-governance

## 2.1 Stages in development of e-government

To determine the extent to which the government exploits its facilities to offer information and services, the following classification can be used (by stages) [2].

***Formation of presence (Emerging):*** Presence of government on world wide web is manifested through several independent official sites. Information is limited, incomplete and static.

***Growing presence (Enhanced):*** The content and the information is regularly updated.

***Interactive presence (Interactive***): Users can obtain application forms, can set up contracts with official agencies, can address interpellations and get job places.

***Transactional presence (Transactional***): Users can pay for services in real time and to carry out transactions on-line.

***Integrated presence (Seamless):*** Full integration of digital functions is finalized within one gate, independent of administrative and departmental frontiers.

Development of e-administration means looking for and finding an equilibrium between two components: external – citizens and consumers, and internal – the leadership and auxiliary administrative office.

These components are neither in conflict, nor they exclude each other. But experience of Internet use will require a more efficient cooperation. In many countries, both industrially developed and the developing and poorly developed ones, there is a trend within their e-government projects to allocate resources for citizens' services, without making sure, beforehand, that there are necessary possibilities to maintain these initiatives. It may appear that such a trend [4] consists in establishment of some stricter requirements, which will impose a more rapid development.

Implementation of progressive plans to create "e-government" faces a series of difficult problems:

1. All citizens, no matter whether they have access to Internet or not, whether they know how to operate computer or not, must have a possibility to use services of such government. Thus, a massive campaign against illiteracy will be required.
2. E-government must offer its services not only via Internet, but, for instance, via specially equipped television, mobile telephones, pocket computers and Internet public access points. This condition is of prior importance, because otherwise the society will have to face another dimension of divide, which is not desired. Currently, the European Commission studies ways to settle this problem.
3. Another difficulty is the necessity to use broadband channels to access Internet, which ensures rapid data transfer. According to the latest data, over 40% households and 90% of enterprises in EU countries are already connected to Internet. The growing

number of subscribers to these networks will require faster speed of access.

4. Equally important is the problem of security in Internet, because the e-government supposes transmission of some important and confidential information (like, for instance, citizen's annual income statement). One of the practiced methods can be the use of secured protocols and digital signature.
5. There are also difficulties caused by unwillingness of the current government employees and leaders to accept changes, which most directly will affect their working methods. The e-government imposes radical changes in perception of the way how a government employee should work.

Still, the most difficult problems are those that appear due to some structural changes, which are meant to meet the requirements of e-government, if they are not very well thought. The most efficient applications of e-government for auxiliary offices are not those that are used to improve the existing way of documents' circulation, but those that bring fundamental reorganization of government's operations (reengineering), integrating internal and external flow of government's documents, for the citizens to interact with it and see in it an entity that fully meets their requirements. Implementation of these reorganizational changes is difficult; however, potential advantages can justify the risks. The aim is not to minimize the risk, but rather to balance the risks and the benefits.

## 2.2 Calculation of e-government indices

The "e-government" index is a figure calculated on basis of the components, the hierarchical dependence of which is reflected at Fig.2.

E-government programs are influenced both by external and internal factors. Technologies and educated staff play an important role, although the available resources dictate measures to be undertaken by each government. Transition from one stage to another can be rapid and continuous for countries at the low level (emerging) and slower at

other stages. However, it is important to undertake permanent evaluation and measures that could guarantee improvement of the situation, no matter how slow it is, from one evaluation to another.

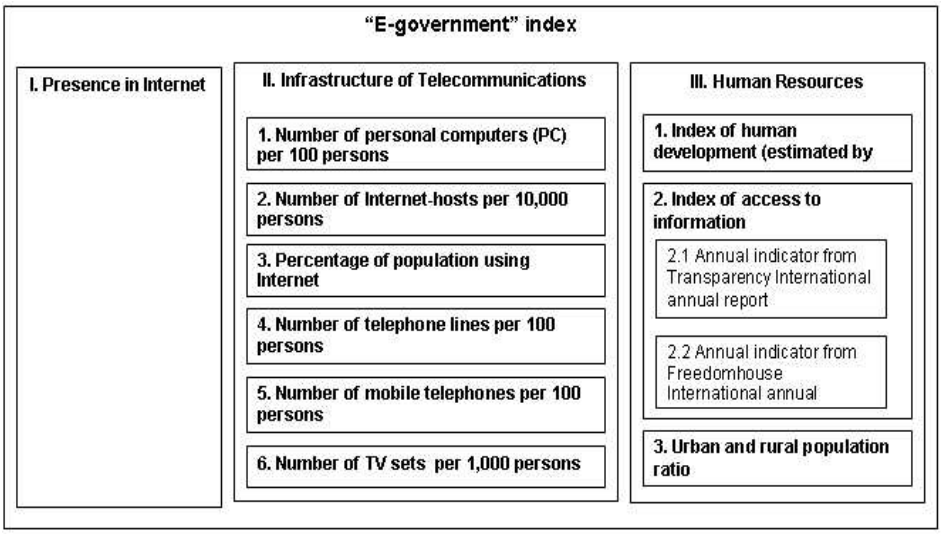


Figure 2. The hierarchical dependence of "e-government" index components

Fig. 3 presents graphs of e-government Index in 2001 (as per [2]) and e-government Readiness Index in 2003 (as per [3]) for a number of leading countries, for the countries with the lowest index, for Moldova, and for some countries with indices close to that of Moldova. In 2001 and 2003, calculations were made by using different methods, and therefore absolute values of these indices are incomparable. However, one may observe the ratios of values of each country to the values of others and how these ratios evolved. One may notice the progress of Australia, Norway, Great Britain, Holland, Denmark, Sweden, which came close to the US index. Estonia made a considerable progress. Slovakia, Slovenia, Romania also progressed. Moldova and Azerbaidjan remained at the same low level.

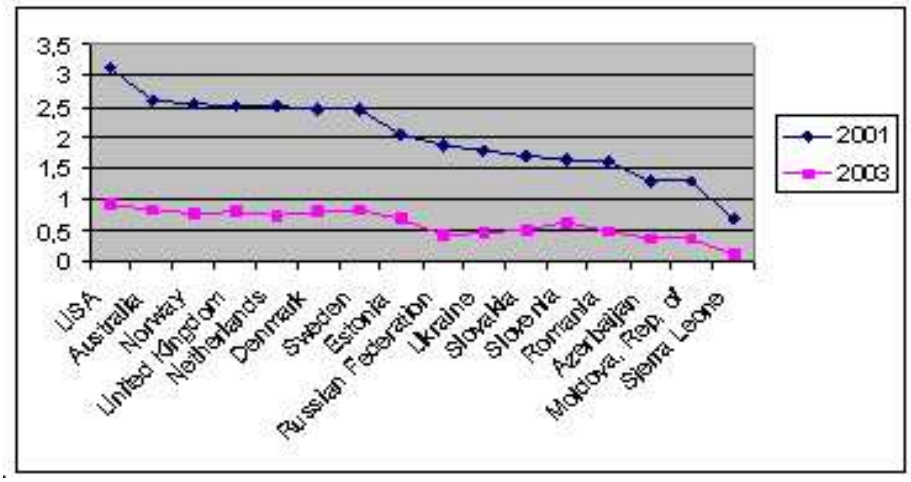


Figure 3. Comparison of e-government indices for the years 2001 and 2003

## 2.3 Structure of e-government

The digital government is not an experiment in administrative reform, but an element in the process of state administration. E-Government can be viewed (from a different perspective) as having three branches:

G2G –government to government;

G2B –government to business;

G2C –government to citizens.

It includes on-line services to business and citizens localized on the same gate, the circuit of documents in governmental and parliamentary structures, a database of common use to avoid duplication of information and additional costs for its obtaining and processing, and often, a specialized informational network (intranet) ensuring transactions between government structures, equipped with criptographic systems and other information protection facilities, including personal data, digital signatures, systems of authorized access to information and its process-

ing.

The comparative analysis of e-government implementation in different countries (including Moldova) according to the branches G2C, G2B and G2G is given in [7].

## 2.4 E-governance experience in countries–neighbors of Moldova

### 2.4.1 Romania

Among other neighboring countries, Romania, which as long as 12 – 13 years ago was not very successful in ICT, has made efforts that have brought about impressive results.

In conformity with the Romanian Government Decision on organization of the **Ministry of Communications and Technology of Information** (MCTI), and taking into consideration the Law on some measures to ensure transparence in exercise of public designations, public and business duties, prevention and prosecution of corruption, the MCTI ensures initiation, management, funding, monitoring, implementation and operation of **projects and programs aimed at informatization of government structures**, as well as funding for coherent development of the national informational infrastructure of the central public administration.

From early 2003, the MCTI issued 29 advisory notes on supply of payment instruments with remote access, of the type of Internet-banking or home-banking applications. Judging by the reports from banks the amount of such transactions reached approximately 25 billion EUR a year.

1. **National Electronic System** (www.e-guvernare.ro) represents **single access point** to public services and information of the central administrative institutions. It was awarded for "the best digital content" in e-Government, during the World Summit for Information Society, chosen together with other 40 projects from the whole world. It is the first step of a process of reforming of public administration so that it to be indeed at service of each citizen. The System includes:

- Over 160 forms made available for downloading;

- 465 involved public institutions;
- 5 online public services, available since September 2003:
  - VAT deduction;
  - Declaration on payment duties to the state budget;
  - Presentation of quarterly and annual accounting reports by large taxpayers;
  - Declaration on the record of insured persons and payment obligations towards the budget of state insurance;
  - System of collection of statistic data.

Approximately 50,000 visitors since the date of launching, hundreds of expressed opinions on how to improve the content of the gate.

2. **Center for Analysis and Response to Security Incidents** (CERIS, www.ceris.ro) is the place where IT officers in public administration will be able to ask for guidance and receive solutions regarding IT security problems they face. Services rendered by this center will focus on means of protection of systems against potential problems, and help to anticipate incidents and even resolve them. At the national level, in 2003 there were about 200 persons under examination, of whom 25 arrested, with prejudice of about $500,000 USD, of which app. $50,000 USD were recovered.

3. **Electronic system of public acquisitions** (www.e-licitatie.ro) is based on the idea of technology used for improvement of transparence in public acquisitions process. The system was launched on March 4, 2002. At the beginning of 2004 the system gave:

- Average saving percentage - 22.6%;
- Savings accumulated - app. 67 million EUR;
- Transactions through this system - app. 220 000;
- Subscription applications – 10 000;
- Authorities-contractors - 1 000 (being 159 on the date of launching);
- Categories of products traded through the system - over 80 (being 7 in 2002);
- National health programs carried out through the system – 10;
- Complex auctions for turnkey construction of sport halls:

- Stage I – 199 auctioned sport halls – Savings achieved compared to the estimated budget: app. 14 million EUR.
- Stage II – 172 auctions – Savings: 12 million EUR.

4. **Payment of taxes and fees by electronic means** (www.mcti.ro) - the system which has two elements:
a) informing the citizen by electronic means about the local fees and taxes to be paid, as well as about their balance;
b) payment of fees and taxes by using electronic payment instruments; the payer can choose paying via ATM, POS or via distant payment instruments of Internet banking, mobile banking or home banking types.

60% of municipalities implemented the system even if only to allow finding out the amount of taxes to be paid.

5. **Electronic system of authorizations in international cargo road transportation** (www.autorizatiiauto.ro) was launched in November 2003 and ensures transparence in electronic assignment of authorizations for transport:

- 1 400 operators of international cargo road transportation hold digital authentication certificates;
- approximately 5 700 authorizations had been allocated electronically before the first half of December;
- 83 000 authorizations taken from the ARR (Authority of Romanian Road-transport), of more than 392 000 administered in 2003 by the Authority;
- 1 800 companies carried out activities of international transportation from the beginning of the year until September 2003 (according to the data of Frontier Police).

6. **Driver's Licence** (http://permiseauto.e-licitatie.ro). This Project is meant to facilitate relation between the government and the citizens (G2C); the latter gain the advantage to contact the government either from their own PCs, or from any other Internet access point. The project changes the relation citizen-state by replacing repeated visits to different offices by placing an application online, by connecting to Internet (don't stand in line, get online).

So far, 31 licenses have been issued to banks authorizing them to carry out transactions via Internet.

### 2.4.2 Russia

The Russian Federation launched a program "Electronic Russia". Within it, feasibility studies were pursued (2003) and the Information-Telecommunications Center was set up within the Ministry of Economic Development and Trade. In 2002-2003, the hardware platform and software products were developed to operate applications within the Program.

The e-Government Project is part of a larger project "Global Gate of Development" supported by the World Bank through the Institute for Development of Information Society.

"E-Russia" provides for three stages:

I 2002 – analysis of the country's e-readiness in all spheres, audit of ICT legislation, launching first projects of electronic circulation of documents, modernization of educational system.

II 2003-2004. Formation of uniform infrastructure of telecommunications for public and informational Internet and facilities for training and retraining of specialists. Expansion to the world markets as a supplier of ICT services and solutions.

III 2005-2010. Penetration of ICT into economy. Preconditions will be enabled to ensure citizens' right to access to information. The program will ensure integral implementation of standard systems of information circulation.

By the year 2004, it was planned to elaborate a wide network of development gates in 7 regions and a "mother-gate" (http://www.rusia-gateway.ru), with support from 143 partner organizations.

A specialized informational system "Government Gate" was planned within the same dates, in order to ensure informational needs of small business.

Other projects: "Economy's Gate", "Statistic Gate", a model of electronic commerce system was elaborated to serve the regional and municipal needs; the system is now tested in the region of Chelyabinsk

and the Republic of Chuvashia. A package of documents for creation of the federal e-commerce center was drafted. Preliminary testing of e-commerce system was done to enable acquisitions for the federal needs of the state (2003).

Parallel to that it is elaborated: automated control system of cargo transportation on the customs territory of the Russian Federation, monitoring and analysis system of financial and economic activities of enterprises, the Project of unification of informational resources of Russian healthcare institutions and of obligatory health insurance foundations "Personalized record of medical services".

In the Russian Federation in 2002, 4% of population used Internet. This showed 39% growth compared to 2001. Eighty-three federal subjects opened access servers within post offices and 2 600 access points, including 800 in rural area.

The Republic of Chuvashia appears to be more advanced in terms of projects of Information Society.

Twenty-six Public Internet Access Points (PIAPs) were created on basis of municipal structures and 46 points in post-offices.

All regional (provincial) centers opened the PIAPs. In PIAPs citizens can have free access to the central gate of the Republic of Chuvashia.

All public authorities are connected to network. There is the system of electronic acquisitions, the system of monitoring and programs of economic and social development of the republic. A model provincial library is opened in the village Shemursha, for the facility of readers equipped with a computer lab connected to Internet, and there is an PIAP in the administrative building.

A decision from January 30, 2004 "On ensured access of citizens and organizations to information on activities of the authorities of the Republic of Chuvashia" appointed the central gate as the official resource providing information on the activity of the President, the Cabinet of Ministers and executive authorities of Chuvashia. This gate unites sites of over 70 central, provincial and village agencies, and provides multilateral information to citizens about government's activities. All decisions by the President and Cabinet of Ministers are operatively

published in Internet, as well as announcements about contest of national and municipal acquisitions, mass media publications, job vacancies, administrative plans of actions.

Rural population can obtain realistic information about prices on agricultural products in different regions, information about how to get a certificate of family composition, or how to get subsidies for purchase of pure-bred cattle or elite seeds, how to reimburse the profit, office hours of the local and regional administration, and many other socially demanded and important information.

Three medical telecenters using satellite communications have been set up. The telemedical network was set up on principles of co-financing from the federal and republican budget. It is planned that all regional hospitals will be connected to this network. Technologies of networked medicine are meant to solve two major problems: to ensure general access to medical assistance and a high quality of treatment.

For several years already the republican treasury has been supporting works of a system in which all financial documents are sent only via Internet and certified by digital signature.

In parallel the projects of formation of integrated medium of education are being financed. Through these projects the municipal and rural schools have received 3 447 computers. So if in 2000 there were 300 pupils per one computer, in October 2003 this number became 52.

## 2.5 "One desk" system for e-services implementation

**The essence of the idea of "one desk"** is the following: to receive the necessary document, the requester, either a citizen or an organization, is obliged to submit to the executive agencies just one's documents directly associated with the requester (application, a copy of identity card, school certificate, registration certificate, etc.). All other documents, which are not directly associated with the requester but which are necessary for issuance of the requested document, shall be collected by the executive agency itself, which is responsible for issuance of the document. So, it will be enough for the requester to contact only the executive agency that issues the final document.

**The principle of "one desk"** presumes existence of one single point of access for interaction with the authorities at any level. It is necessary to develop a system for the purpose, which will automatically direct the requests and the reports of the entitled agencies of state administration, regarding thus the state as service provider to citizens and businesses. A database of electronic documents issued in "one desk" mode is also necessary. When such base is set up, it will be open to all government entities, citizens and organizations. By entering the respective site on the government's gate, any citizen will be able to get the following information:

- what executive authority issues the requested document;
- what documents must be presented to the service of "one desk";
- the terms for preparation and issuance of the document;
- fee for preparation of the document (if applicable);
- how does the requested document look like (its format).

## 2.6 E-governance as instrument to combat corruption

E-governance is expected to improve operativeness of executive branch, especially in social services. It will make the executive more transparent and responsible to the citizens and organizations, it will change relations between the state and the citizens, involving the latter into the activities of democratic and public institutions.

In many cases, more transparence, predictibility of social procedures and introducing order, were just the primary objectives of establishment of e-government. When leaders of the government and all stakeholders in the society stick firmly to these objectives, e-governance can become an important instrument to combat corruption.

Three eminent examples below demonstrate us that more transparence and free access to information on decisions and activities of the executive employees, as well as more social responsibility, considerably reduce possibilities and wishes to obtain unjustified profit, as well as corruption among the responsible persons.

**Argentina: "Crystal Government" Initiative.** The goal of the program "Crystal Government", which was launched in September

1999, was to make all information concerning use of public resources accessible online (http://www.cristal.gov.ar). This refers not only to statistic data about the resources allocated for different government programs, but also to the data about management procedures of these resources.

The project's website was designed in such a way as to provide maximum comfort to the user and is formed around three main subjects:

1) explanation of the way in which the state resources are shared between the central government and the regions;

2) an information center for evaluation of the state policy;

3) short reports on combat of corruption in public and private sectors.

There is an external supervisory body – a committee of 15 NGOs (Non-Governmental Organizations) specializing in "transparence", which ensures annual audit of the site.

The "Crystal Government" Project was successfully "sold" to central agencies for popularization of their activities and possibility of feedback with the citizens. The result was that many administrative entities improved their methods of data collection by placing their questionnaires on the site.

**Gujarat, India : Monitoring check points.** Reloading of commercial trucks running along the roads of Gujarat became the main threat for security and the main premise to evade excise duties and taxes. These facts, as well as unprecedented scale of corruption among the frontier inspectors, many of whom were sued for robbing and abusing truck drivers, led in 1989 to reconstruction of the traditional system of check points in Gujarat. The state government installed computers and electronic devices in 10 points remote from the state borders and designed a computerized system of fining and collection of fines.

The control process includes use of video cameras installed in check points for identification of the vehicles' registration numbers, which are transmitted to the central database, so that electronic penalty provisions are issued on spot. The drivers are allowed to carry special pre-paid cards for instant payment of fines, to save them of the ne-

cessity to hold cash money on them, which deprives the inspectors of freedom of actions and prevents corruption among them.

Due to this the government of Gujarat, within two years, managed to reduce corruption in check points radically and to increase fiscal income three times by reducing the number of illegally reloaded trucks, not to talk about simpler process of collection of the funds. This program of e-government paid itself within six months and reduced waiting times from 30 minutes to 2 minutes for the drivers.

**Seoul, South Korea: the Anti-Corruption Gate "OPEN".** Over a decade of rapid economic development in Korea led to considerable growth of municipal bureaucracy in Seoul and led to opening numerous ways for corruption, especially bribes received by responsible officials for quicker processing of applications for business licenses and permissions. This problem gained such extent that in 1998 the mayor Goh Cun made a public declaration that he was going to start war against corruption. He supported creation of an online Internet Gate meant for improvement of processing of private applications (Online Procedure Enhancement for Civil Application -"OPEN"), which offered a wide range of information on permissions and licenses issued by municipal agencies. The web-site makes these procedures transparent for society and allows the applicants to follow their destiny online.

Over 5,000 officers in over 500 departments have been trained in use of the necessary programs to process the applications. Lately, a series of other government functions have been made public in Internet, including the functions of 20 civil departments. According to the latest data, app. 84% of 1100 respondents believes that the OPEN Project improved transparence of Seoul's government.

**Conclusions.** These three initiatives were successful due to the following considerations:

1. All were actively supported by political leaders at the local and central levels of the government.
2. All three increased the level of responsibility of officers to citizens and to ruling circles, increasing the level of transparence of current procedures and risks for those who look for illegal profit.

The main rules, procedures and punishments for their violation were formulated clearly and predictably.

3. In Argentina and Korea the society participated in e-governance processes.

E-governance can be seen as a support to reforms, but technologies are just an important help. They cannot bring about a result by themselves. Political responsibilities, social resources, participation of citizens are the necessary factors ensuring absolute success.

# 3 Liberty of access to information and secured confidentiality of private information

## 3.1 Liberty of access to information

### 3.1.1 General criteria and situation in the world

Bucharest Declaration [8] established the following fundamental principles to determine priorities for strategies of IS building.

**Principle 1.** *Protection of access to information and knowledge.*

Individuals and entities must benefit of access to information, knowledge and ideas. Access to information in social sector must be particularly facilitated. Information is basic to an operating and transparent process of decision-making and a precondition of any democracy.

Knowledge is the engine and key to transformation of global society and local communities. Social policy must extend possibilities for access to information to all citizens, including disabled persons, thus doing away with inequality.

**Principle 2.** *Contribution to universal access to information at accessible prices.*

**Principle 3.** *Stimulation of linguistic diversity and cultural identities.*

**Principle 4.** *Development of human potential through training and education.*

**Principle 5.** *Creation of a favorable environment, including legal, regulatory and political structures.*

**Principle 6.** *Ensuring confidentiality and security in use of the ICT.*

**Principle 7.** *Call to the global problems.*

The Article 19 of the Universal Declaration of Human Rights states: "Everyone has the right to freedom of opinion and expression; this right includes freedom to hold opinions without interference and to seek, receive and impart information and ideas through any media and regardless of frontiers."

The right to freedom of information and the demand for an efficient legislation to translate this right into practice has been recognized by a number of international organizations dealing with human rights protection: UN, European Council, African Union, British Commonwealth, Organization of American States.

The World Organization ARTICLE 19 (called after the article of this declaration) drafted standard principles both for the national and international law systems, according to which anybody can determine whether the country's laws guarantee access to official information or not.

**Principle 1.** *Maximum disclosure*

**Principle 2.** *Obligation to publish*

**Principle 3.** *Promotion of open government*

**Principle 4.** *Exhaustive list of exceptions from the principle of maximum disclosure.*

**Principle 5.** *Processes To Facilitate Access*

**Principle 6.** *Costs*

**Principle 7.** *Open meetings*

**Principle 8.** *Disclosure takes precedence*

**Principle 9.** *Protection for whistleblowers*

The fundamental right to freedom of information is guaranteed by 87.3% of constitutions of the world countries. This right was first consecrated in 18th century [9].

Constitutional norms insuring universal access to official information exist in Sweden, Finland, Denmark, Norway, Holland, Spain, Portugal, Austria, Hungary, Estonia, Belgium, Romania. Relevant laws declare the same right in France, Greece, Italy, Russia.

### 3.1.2 Freedom of access to information in Moldova

Article 34 of Moldova's Constitution stipulates the right of citizens to access to information "on social matters" and obligation of the authorities to offer this information. The law of the republic on access to information, adopted in May 2000 [10] and highly appreciated by the European Council for its compliance with the international standards, was largely discussed both nationally and internationally [11, 12].

International Organization Article 19 [13] mentions positive moments: anyone, in conditions of the law, has the right to look for, to receive and to learn official information (Art. 4). On the other hand, the article 7 lists a number of cases when access to information is prevented.

Article 10 stipulates that the solicitor of information is exempted from the obligation to justify one's "interest to the requested information".

Suppliers of information (article 11) are obliged to provide active and correct information in due time to citizens on matters of public interest and on matters of personal interest, and will publish at least once a year a guide with lists of orders, decisions, other official documents emitted by the respective institution.

A refuse to supply information will obligatorily contain a reference to a procedural act, which serves as foundation for the refuse, and a description of the appeal procedure. The refuse can be appealed against in a hierarchically superior institution, and also in the court, if the applicant considers that his/her right to information has been infringed (articles 21-23).

The Law on the state secret adopted in May 1994 establishes the foundations for restricted access to information.

Article 2 of this law defines the state secret as follows: "... is protected by the state in the fields like military, foreign policy, counterinformation and operative investigation, and diffusion, divulgation, loss, illegal receipt or elimination of which can endanger security of the Republic of Moldova" (underlined by us). This definition gets too many interpretations. The problem becomes even more acute with the wide

range of public agencies entitled to qualify information as secret: the Parliament, the President, the Government, central and local administration, legislative bodies (Article 4).

However, Article 28 renders control and supervisory functions to the Standing Bureau of the Parliament; it is this Bureau to which administrative bodies are obliged to present the required data. It seems that the most legal protection is enjoyed by commercial secrets, which are stipulated in the art. 344 of the Penal Code.

The most important obstacle in free circulation of information in Moldova is unawareness combined with abuses by officers. Very few citizens know about existence of these laws, and even fewer know their provisions, and only some of them have ever made reference or appealed to them. On the other hand, public officers ascertain that they have no resources to answer all requests for information operatively and efficiently, under the terms of law [12].

The report by ARTICLE 19 of October 2003 [13] mentions that from the time the law was adopted, public agencies have not become more transparent. From the few measures meant to improve the situation, there is an amendment of 2001 to the Administrative Code providing fines to the public officers for illegitimate refuses to present information. The Penal Code adopted in 2003, considers as the crime the infringement of legislation on access to information (article 188) and the concealment or fraudulent misrepresentation of data about environment pollution (article 225).

## 3.2 Insurance of Confidentiality of Private Information

### 3.2.1 General Notions and Situation in the World

Confidentiality as one of the human rights is periodically monitored, with digests "International Surveys of Privacy laws and Practice" published (in 1998 [14], in 2003 [15]).

Declaration of Human Rights (1948) promotes protection of personal and territorial confidentiality, as well as confidentiality of communications [16]. Article 8 of the Convention for protection of human rights and fundamental freedoms of 1950 [17] declares: "Everyone has

the right to respect for his private and family life, his home and his correspondence. There shall be no interference by a public authority with the exercise of this right except such as is in accordance with the law and is necessary in a democratic society in the interests of national security, public safety or the economic well-being of the country, for the prevention of disorder or crime, for the protection of health or morals, or for the protection of the rights and freedoms of others."

Almost all countries of the world constitutionally recognize the right to confidentiality. These constitutional norms include, as a minimum, the right to safety of home and confidentiality of communications. Some new constitutions, like those of the South Africa and Hungary, are very minute even about the rights of access to personal data.

Some countries are not specific in their constitutions about confidentiality (for example, the USA, Ireland, India), so that their courts rely on other laws when examining the cases of confidentiality. In some countries the national legislation is enhanced by regulations from the International Pact on civil and political rights [18], as well as from European Convention on Human Rights.

Early 70s was the time for adoption of the laws meant to defend confidentiality. Most of these laws were based on the model of the Organization for Economic Cooperation and Development of the Council of Europe. In 1995, having realized the deficiency of the national laws and the difference between the degrees of protection of confidentiality in different European countries, the European Union promoted the directives regarding "Protection of citizens' interests in personal data processing and free distribution of these data" [19]. They contained proposals for a more efficient protection of information and became a model for the national laws.

Even in such democratic states like the USA, infringements of human rights, connected with the control of communications, are widely spread. State Department in one of its annual reports affirms that over 90 countries exercise illegal control of information of political opposition, law-enforcement agents, journalists, trade union activists.

Even in countries where laws on protection of confidentiality are rather tough, police still keeps enormous files of citizens who are not to

blame for anything, not even suspected. These problems are examined today in Sweden and Norway, the two countries with one of the largest histories in what concerns insured confidentiality in relation to the police' files.

Businesses infringe the law regularly by collecting and distributing personal data. Thus, in the USA, although there is a law from time immemorial on protection of information about consumers, businesses continue to use it in marketing purposes.

As it follows from polls, people now are afraid of violation of private information more than ever in Modern history [20]. Whole groups of citizens from different countries express their concern with intrusion into their personal life, and this makes more and more countries to adopt laws with the specific purpose to protect intimacy and confidentiality. Right protection organizations mention that new technologies are frequently exported in developing countries, which do not have proper legislation able to protect citizens. Modern monitoring technologies create certain obstacles in the way of development of e-commerce.

As Privacy International[1] believes, "with efficient constitutional and legislative protection lacking, these technologies have a negative impact over democratic reforms. They can be used against anybody who may be of "interest" to the regime".

Possibility to use powerful computers in purposes of monitoring and control required adoption of some special rules in what concerns regulation of collection and processing of personal data. New constitutions adopted in many countries reflected this right. The upgrading process of specific legislation can be tracked back to the emergence of the first law on protection of data, adopted in Germany, Hessen land, in 1970. It was followed by the laws in Sweden (1973), United States (1974), Germany (1977) and France (1978) [21].

Definitions of what is data protection in different declarations and laws differ only in details. All acts stipulate that personal information must:

- be obtained fairly and legitimately;

[1] A human rights group formed in 1990 as a watchdog on surveillance and privacy invasions by governments and corporations (http://www.privacyinternational.org).

- be used only in conformity with the predetermined purposes;
- comply with the purpose for which it was collected;
- be precise and up-to-date, and then destroyed upon achievement of the purpose of its collection.

The Directive on data protection introduced in 1995 by the European Union and which became the model for national laws, brought about the effect that all these laws had to be harmonized for all EU countries [22].

There are several models of protection of confidentiality today. Some countries use several models simultaneously.

The model applied in Europe, Australia, Hong-Kong, New Zeeland and Canada supposes existence of a public officer responsible for all legislation on data protection. This officer is appointed a commissionaire or just an entitled person. He supervises law enforcement and carries out respective investigation. In some cases this officer acts against the delinquent. The Commissionaire is also responsible for promotion among the society and international relations in the field of data protection.

Data protection can be realized, theoretically, through autoregulation methods. Thus, commercial companies establish their own rules. However, observations show that this practice does not reach its goal: the rules thus established are not observed.

Lately, one has witnessed rapid development of technologies that can find applications in commercial activity. Protection of confidentiality happened to be in hands of citizens themselves. Internet users have access to a series of programs and systems offering diverse levels of protection of confidentiality and communications. The aspects of security and reliability of these systems are still debatable.

Some technologies present danger from the point of view of confidentiality. Many of them are implemented and used without any legislative restrictions.

Personal cards or files in this way or another exist in all countries of the world. Types of cards, their destination, as well as the volume of information are different. Most countries use obligatory official cards, made for all citizens in the same uniform national way and used in

diverse purposes. However, a number of developed countries have no such system of cards. Among them, the USA, Canada, New Zeeland, Australia, Great Britain, Ireland, Scandinavian countries. "The system of cards" is adopted in Germany, France, Belgium, Greece, Luxemburg, Portugal, Spain.

A new trend in identification process is biometrics, i.e. the process of collecting, processing and storing the data about personal physical characteristics for identification purposes. Among the most popular biometric systems – retina's scanning, hand geometry, dactiloscopy, voice recognition and digital photography, that are stored electronically. Biometrics managed to draw attention of governments, because unlike other methods (cards or other acts) it ensures complete and accurate identification.

Biometric systems are being implemented in different countries of the world. Spain started the process of general dactiloscopy to settle the healthcare-related and employment problems. The Russian government made public declarations about its intentions to apply dactiloscopy in banking. In Jamaica, the citizens are subjected to dactiloscopy of thumb to be admitted to voting. France and Germany are testing equipment that will allow placing finger-prints on bank cards. These technologies are applied in commercial enterprises, public agencies, children support centers, police and ATMs.

The most debatable is the ADN identification technology. In some countries (USA, Germany, Canada), police work on elaboration of national ADN databases.

It should be mentioned that use of personal information in purposes other than the ones initially declared in time of collection can cause enormous harm to person. Especially when we have to deal with sensitive data, i.e. such data that require especially cautious attitude in operating with them. This category includes, for instance, data about the state of health. Patients become victims of negligence, errors, and lack of responsibility of the medical staff. But there are other categories of professional secret as well: guaranteed by advocates, notaries, banks, commercial companies, reporters and, finally, the secret of religious confession.

### 3.2.2 Situation with Confidentiality in Moldova.

The analysis of the legislation of Moldova [10, 11] from the point of view of access to information and guaranty of confidentiality of private information brought to the following conclusion.

Moldova's effective legislation on access to information and protection of private information requires improvement. Government decision of March 18, 2002 is not a law, and new laws on protection of confidential information were not adopted. Majority of population does not know legislation and does not know world trends in the field, therefore, promotional measures are required. Besides, such measures are required that will make these laws to work for the benefit of citizens.

## 4 Proposals

Declaration of principles and the Plan of Actions adopted at Geneva Summit [23, 24] prescribe specific directions to enhance progress in order to achieve the goals accepted at international level, and namely: promotion of ICT products, networks, services and applications, support to the countries in overcoming the digital gap. The Information Society as foreseen in the Declaration of Principles, will be realized in cooperation and solidarity between the government and all other participants.

Particularly, the component **E-government** of the Plan of Actions provides:

a) Implementation of e-governance strategies targeted at applications meant to innovate and promote transparence in public administration and democratic processes, growth of efficiency and enhancement of relations with citizens;
b) Elaboration of national e-governance initiatives and services at all levels, adapting them to the citizens' and business' needs, to ensure a bigger efficiency of allocated resources and public goods;
c) Support to the initiatives of international cooperation in the field of e-governance for the purposes of growing transparence, responsibility and efficiency at all levels of governance.

Taking into consideration these stipulations and situation in Moldova it is necessary to note that the effective implementation of Electronic Services in Public Administration and of Inviolability of personal information and right to information require the coordinated solution of all programs connected with building of the IS in the framework of the unified state program. So the following actions are proposed for Moldova:

- Elaboration of a state program "Building Information Society in Moldova";
- Elaboration of official web-gate of the Republic of Moldova;
- Implementation of the "one desk" system;
- Bringing legislation on information in compliance with the European standards.

The state program is supposed to cover the realization of the following aspects:

- Interconnections government $\leftrightarrow$ citizen, government $\leftrightarrow$ business, government $\leftrightarrow$ ministries (departments), ministry $\leftrightarrow$ ministry;
- E-democracy – ensuring opportunities for citizens to participate in discussion of procedural and legislative acts, decisions and projects of government or other administrative entities;
- Development of information infrastructures;
- Connection of schools to Internet;
- Connection of local administration to Internet;
- Setting up Points of Public Access to Internet;
- Support to ICT activities;
- Continuous education;
- Elaboration of legislation proper to information society (electronic document, digital signature, e-commerce, mechanisms of efficient protection of confidential information, etc.);
- Elaboration of telemedicine network;
- Creation of digital museums and libraries;
- Standardization of government sites: from the point of view of content and friendly and reliable access;
- Elaboration of concept, legislative base and information instruments required for operation of "one desk";
- Restructuring of government's and ministries' activities.

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