

# Negative index materials based on rods with refractive index profile

**E. Foca, J. Carstensen, H. Föll**

*Chair for General Materials Science, Faculty of Engineering, Christian-Albrechts-University of Kiel, Kaiserstr. 2, Kiel-24143, Germany*  
[ef@tf.uni-kiel.de](mailto:ef@tf.uni-kiel.de), Tel. (+49)-431-8806180, Fax (+49)-431-8806178

**V.V. Sergentu, V.V. Ursaki, I.M. Tiginyanu**

*Institute of Applied Physics, Moldavian Academy of Sciences, Academiei. 5, Chisinau-2028, Moldova*

**F. Daschner, R. Knöchel**

*Microwave Laboratory, Faculty of Engineering, Christian-Albrechts-University of Kiel, Kaiserstr. 2, Kiel-24143, Germany*

**Abstract:** Negative index materials can be fabricated using cylinders with a special distribution of the dielectric constant. We present the theoretical approach for building such materials as well as a experimental proof of the corresponding principle.

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## 1. Introduction

The concept of left-handed electromagnetic media, which are also known as negative-index materials (NIMs), was introduced by Veselago [1] as a theoretical curiosity. Interest in these metamaterials was rejuvenated by Pendry [2] who noted that the growth of evanescent fields within a NIM provides the opportunity for building a “perfect lens” that can focus electromagnetic waves to a spot size much smaller than a wavelength. Negative refraction has nonetheless been confirmed in recent experiments [1 - 3]. NIM have recently been designed on the basis of composite wire and split ring resonator structures [4], backward-wave transmission lines [5], and photonic-band-gap crystals [6].

In our work we propose an approach based on using dielectric rods which themselves possess a negative refractive index at definite wavelengths [7]. It consists of the following steps: (i) the design of elementary units (rods) exhibiting properties of NIM; and (ii) different periodic or quasi-periodic structures assembled from these elementary units are considered, and their properties are calculated numerically and further experimentally tested with the goal of producing negative refraction and optimizing the focusing effect. A highly efficient and accurate multiple-scattering approach [16] is used to calculate propagation of electromagnetic waves through these structures.

## 2. Simulations of the NIM

The elementary building blocks of our design are dielectric rods with a changeable refractive index. Their gradient resembles a “fish-eye” profile [8] given by:

$$n(r) = \frac{n_0}{1 + \left(\frac{r}{r_0}\right)^2} \quad (1)$$

where  $r$  is the distance from the center of the rod and  $n_0$ ,  $r_0$  are constants. In such a material light propagates in circular (or spiral) trajectories with a radius comparable to the quantity  $r_0$ , i. e. a medium with the “fish-eye” dielectric constant profile behaves like a NIM from the point of view of light scattering. This approach is certainly valid for wavelengths much shorter than the diameter of the cylinder. To prove that this is still the case for longer wavelengths we consider the design of dielectric rods by analyzing the scattered light. We employ an approach based on the effective medium concept [9] to choose appropriate parameters for the dielectric profile of the rod. This

method relies on using a hypothetic background medium with variable index of refraction in which the dielectric rod is immersed [10]. The scattering cross section of the rod is calculated as a function of the refractive index of the background medium. It is obvious that the scattering cross section should exhibit a minimum when the refractive index of the background medium approaches that of the rod under investigation. For practical purposes, we approximate the fish-eye medium by use of several discrete layers of different refractive indices. We analyzed the scattering cross section for a rod which consists of three layers with the radii  $r_1 = 0.5a$ ,  $r_2 = 0.25a$ , and  $r_3 = 0.1a$ , and refractive indices  $n_1 = 1.5$ ,  $n_2 = 3$ , and  $n_3 = 4$ . In Fig. 1a are shown the result, one step forward, where basically the refractive index of each individual rod as a function of radiation wavelength is computed obtained from analyzing the scattering cores section of the rods for each wavelength. One can see that for peculiar wavelengths the  $n_{\text{eff}}$  approaches  $-1$ .

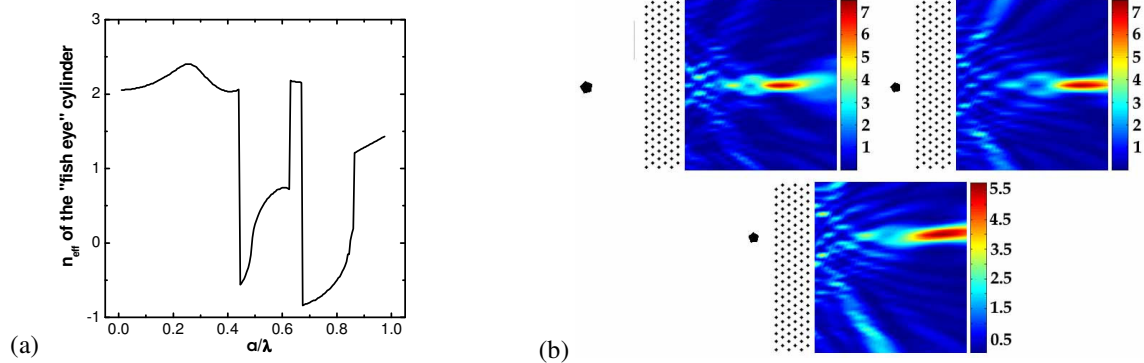


Figure 1 The variation of the  $n_{\text{eff}}$  of a single rod as a function of the inverse of the wavelength (a). Assembly of the fish-eye cylinders acting as a plane plate lens; simulations done for the TM polarization (b).

If the chosen approach is valid then an assembly of such cylinders would have to behave as a NIM with  $n_{\text{eff}} < 0$ . While filling a volume is somewhat impossible having as construction blocks only cylinders, one could instead arrange them into a triangular lattice with a lattice constant  $a = 2R$  where  $R$  is the radius of the cylinders and obtain merely the same effect. In order to check this approach we investigated if for the special wavelengths from Fig. 1a, the cylinders assembly does focus the radiation. The results are shown in Fig. 1b. The black dot on the images stands for the radiation source. The colorbar represents the normalization of the electromagnetic (EM) field intensity calculated with the lens to the one of the free space, i.e. a value bigger than one implies directly localized gain in intensity. From the calculations one can see that the fish-eye cylinders assembly indeed emulates the behaviour of a plane plate lens constructed from homogenous NIM. Any movement of the point source will be perfectly and correctly followed by the image spot. Thus, theoretically, we confirmed our approach.

### 3. Experimental implementation of the fish-eye approach

In order to experimentally confirm our approach we setup the experiment in the microwaves, due to easier handling of the objects. However, due to the scalability of the Maxwell equations the same effect would be possible to observe for higher frequencies, provided the right dimensions of the constituents are selected.

To approximate the fish eye profile we used a cylinder consisting of three layers. The following materials are used: in the middle, as high index material, is used  $\text{Al}_2\text{O}_3$ , followed by a layer of glass and for the final layer, the low index material is used the air. One has to mention that for the microwaves it is quite difficult to obtain easily handling materials with very high dielectric constants, i.e.  $n = 4$  as involved in the theory. The alumina cylinder has a diameter of 4mm. The glass tube which is put over the alumina cylinder has the diameter of 9mm with the wall thickness of 2mm. Finally, an assembly of 195 cylinders is arranged in a triangular lattice with the small lattice constant  $a = 12\text{mm}$  which then results in the thickness of the last layer, i.e. air, to be almost 1.5mm. The experiments are conducted in a specially aligned room with absorbing plates for excluding reflections from the walls and formation of the standing waves which could induce artifacts in the measurements. As a point source a dipole is used and measurements are done for two polarizations, i.e. TE and TM – oscillation of the electric field perpendicular and along the axis of the cylinders. More details on the measuring setup can be found in [11]. The wavelength is selected for which the  $n_{\text{eff}} = -0.8$ . This is the lowest value that could be achieved in the framework of this construction, hence one has to account with some efficiency losses due to the imperfect impedance matching.

The measurements results are shown in Fig. 2. In order to save the space the measurements are shown only for the wavelength where the focusing is the best, i.e. where  $n_{\text{eff}} = -0.8$ . Fig. 2a-c shows the focusing for the TE polarization. Fig. 2a shows the picture of the EM scanned behind the lens when the source is too far off the lens and

the focusing occurs merely inside the structure. However as soon as the source is shifted closer to the lens edge, Fig. 2b, the image point can be seen and it will further follow the source when it is shifted to the left, Fig. 2c.

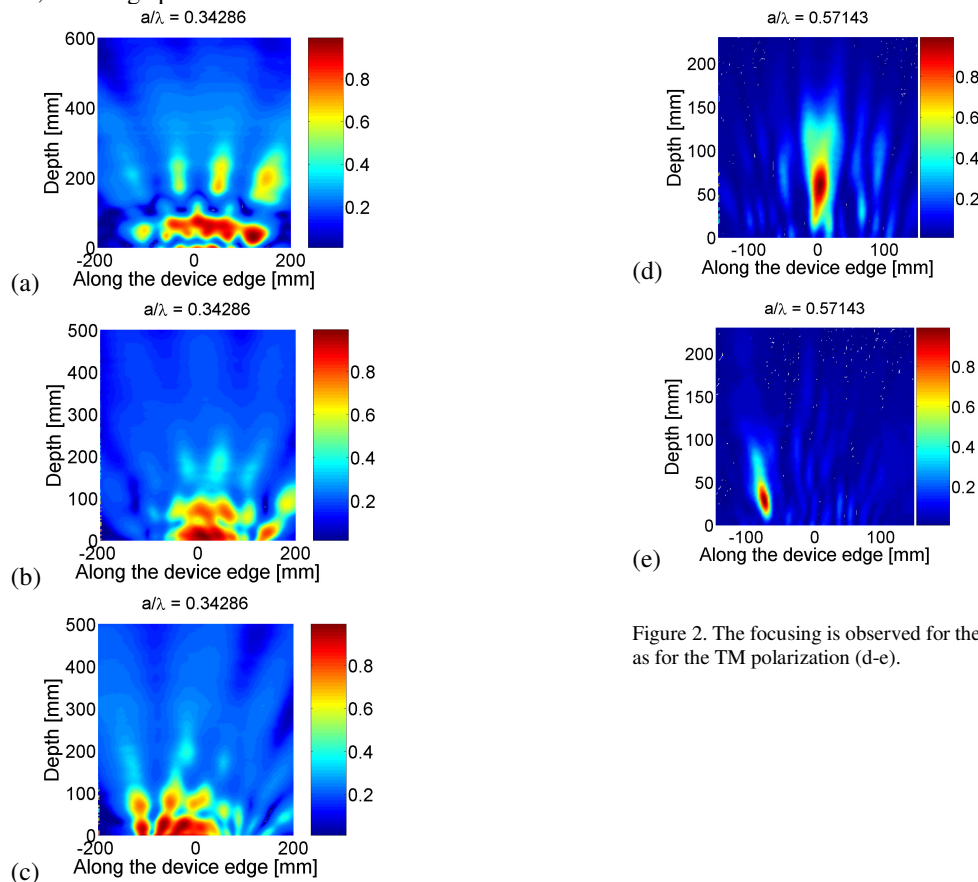


Figure 2. The focusing is observed for the TE polarization (a-c) as well as for the TM polarization (d-e).

Even better focusing could be obtained for the TM polarization, Fig. 2e-d. Very well defined focal point is observed and, similarly to the previous case, it is shifts as well as soon the source is shifted to the left. Actually, the lens is merely optimized for the TM polarization and it can also be seen that indeed the focus for the TE polarization is somewhat spoiled by contrast to the clean focus for the TM polarization. Figure 2 shows that also experimentally our theoretical approach could be well proved.

#### 4. Conclusions

In conclusion, the results of our calculations show that dielectric rods with the “fish-eye” dielectric constant profile are promising for designing and manufacturing negative index materials. The NIM designed in this work on the basis of multilayer dielectric rods is suitable for focusing the EM radiation in a relatively large wavelength range. The experiment prove the validity of our approach.

#### 5. References

- [1] C.G. Parazzoli, R.B. Greegor, K. Li, B.E.C. Koltenbah, and M. Tanielian, *Phys. Rev. Lett.* 90(10), 107401-1 (2003).
- [2] A.A. Houck, J.B. Brock, and I.L. Chuang, *Phys. Rev. L.* 90(13), 137401-1 (2003).
- [3] P.V. Parimi, W.T. Lu, P. Vodo, and S. Sridhar, *Nature* 426, 404 (2003).
- [4] D.R. Smith, W.J. Padilla, D.C. Vier, S.C. Nemat-Nasser, and S. Schultz, *Phys. Rev. Lett.* 84, 4184 (2000).
- [5] A.K. Iyer and G.V. Eleftheriades, in *IEEE MIT-S International Microwave Symposium Digest 2002*, 1067 (2002).
- [6] M. Notomi, *Phys. Rev. B* 62, 10696 (2000).
- [7] V.V. Sergentu, V.V. Ursaki, I.M. Tiginyanu, E. Foca, H. Föll, and R.W. Boyd, *Phys. Stat. Sol. (a)* 203(6), R48-R50 (2006).
- [8] C.T. Tai, *Nature* 182, 1600 (1958).
- [9] V.V. Sergentu, E. Foca, S. Langa, J. Carstensen, H. Föll, and I.M. Tiginyanu, *Phys. Stat. Sol. (a)* A 201, R31 (2004).
- [10] E. Foca et al., in *Materials, Integration and Technology for Monolithic Instruments*, eds. J.A. Theil, M. Böhm, D.S. Gardner, and T. Blalock, D4.4, Mater. Res. Soc. Symp. Proc. 869, Warrendale, PA (2005).
- [11] F. Daschner, R. Knöchel, E. Foca, J. Carstensen, V.V. Sergentu, H. Föll, and I.M. Tiginyanu, *Adv. Radio Sci.* 4, 1 (2006).