MOLDOVA STATE UNIVERSITY
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

MATHEMATICAL SOCIETY OF THE REPUBLIC OF MOLDOVA

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MATHEMATICS & IT: RESEARCH AND EDUCATION (MITRE-2021)

dedicated to the 75th anniversary of Moldova State University

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A B S T R A C T S

Chişinău, 2021

INTERNATIONAL CONFERENCE MITRE-2021

The International (virtual) Conference "*Mathematics & IT: Research and Education (MITRE-2021)*" is organized by the Faculty of Mathematics and Computer Science, Moldova State University, and the Mathematical Society of the Republic of Moldova.

The Conference MITRE in 2021 is at its 8th edition. It is dedicated to the 75th Anniversary of the Moldova State University, which was founded in 1946. The Faculty of Physics and Mathematics was one of the first faculties of the university.

The main goal of the Conferences MITRE is to provide a forum for specialists to discuss different aspects of the integration of research and education in Mathematics and Computer Science. The discussion is planed to be axed on the achievements in scientific research and advanced training of high qualification specialists in these areas, according to real needs of economy, on ways of involving young talents in research.

The Conference is organized in five sections:

1. Algebra, Geometry and Topology
2. Analysis, Differential Equations and Dynamical Systems
3. Applied Mathematics
4. Computer Science and IT
5. Didactics of Mathematics and Informatics

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This book contains the abstracts of communications, presented at the Conference MITRE-2021 (some communications in the Section Didactics of Mathematics and Informatics are presented in Romanian). The authors are listed in the alphabetical order by the last name of the first author within four Sections: Plenary Lectures, Mathematics, Computer Science and IT, Didactics.

The Organizing Committee MITRE-2021 thanks the authors for contributing their abstracts.

I. PLENARY LECTURES

NEXT GENERATION COMPUTING TECHNOLOGIES. AI HARDWARE AND NEW AI REGULATIONS

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Even our computers have evolved and changed a lot our lives in the last 50 years they still work on the same concept as very beginning. Our computers are based on silicon and have fundamental physical limits that address to economics and reliability issues. The current conventional computing requires revolutionary rethinking. New computing paradigms must address to unresolved challenges. The search for next-generation computing paradigms started.

Relaying on the latest discoveries, the presentation explains the high reliability and low power consumption in case of the brain comparing with actual computers. We will analyze the gated ion channels and the way they communicate, and we will introduce highly reliable communication using arrays of random devices. We will deal with reliability schemes and in particular with hammock networks. Inspired by the regular structure of the cytoskeleton inside axons we will present the classical (2D) hammock networks and extend to 3D. We will compare different such networks. The simulation results rely on results of our research team. Our goal is to understand the architectures of nano-devices brain-inspired. We will discuss also on AI hardware and some fresh regulations on AI.

PREDICTIVE ANALYTIC APPLICATIONS IN IOT

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ASYMPTOTIC BEHAVIOUR OF SOLUTIONS TO NONLINEAR FOKKER-PLANCK EQUATIONS

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This talk is a survey of a few recent results on long time behaviour of solutions to nonlinear Fokker-Planck equations obtained in a series or joint work with professor Mihachel Rockner from Bielefeld University, Germany.

GAUDUCHON METRICS AND STABILITY

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ADAPTIVE-ROBUST STOCHASTIC CONTROL WITH APPLICATIONS TO FINANCE

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Motivated by various real-world problems, we propose a new methodology, called adaptive-robust control, for solving a discrete-time stochastic control problem subject to model uncertainty, also known as Knightian uncertainty. The uncertainty comes from the fact that the controller does not know the true probability law of the underlying model but only that it belongs to a certain family of probability laws. We develop a learning algorithm that reduces the model uncertainty through progressive learning about the unknown system. One of the key components in the proposed methodology is the recursive construction of the confidence sets for the unknown parameters of a general ergodic Markov process. This, in particular, allows to establish the Bellman system of equations corresponding to the original stochastic control problem.

This general stochastic control framework will be applied to a classical finance problem of allocating wealth optimally across several risky assets, also known as optimal portfolio allocation problem. We will discuss both time-consistent and time-inconsistent terminal Markovian control problems.

Finally, we provide a machine learning algorithm in solving numerically some of these problems, such as the dynamic Markowitz mean-variance portfolio selection problem with the modern twist of model uncertainty.

From Moron to Intelligent Coach: How the Computer Can Contribute to Wellbeing

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Almost 55 years ago, Peter Drucker (1967) stated that "the computer makes no decisions; it only carries out orders. It's a total moron and therein lies its strength. It forces us to think to set the criteria. The stupider the tool, the brighter the master has to be — and this is the dumbest tool we have ever had". In our days, the PDAs (personal digital assistants) can be frequently met and utilized. Moreover, there are predictions that "by 2055, nearly everyone has 100 cognitive assistants that work for them [....]" and "Cognitive systems can potentially progress from tools to assistants to collaborators to coaches" (Spohrer et al., 2017). The paper is meant to present several milestones of such a fast evolution of the computer with particular emphasis on the impact of the [digital] artifact exerted on our professional and private lives and its contribution to the progress of the human being.

The paper is organized as follows: in the first section, the digital wellbeing and human resilience are reviewed to set the stage for presenting the possible impact of information technology on individual people and society. The early views of influent cultural, technical, and business leaders (P. Drucker, U. Ecco, D. Engelbart and J. Licklider) are presented on the computer role and its development and usage prospects. The current developments in human-machine collaboration in modern [social] CPS (*cyber-physical systems*) are described together with the updated meaning of the concept of automation and several foreseen roles of the computer in private human life. The concept of *computer supported collaboration* between various entities, such as humans (Filip et al., 2017), humans and machines (Nof et al., 2015), and combination of various technologies, such as [computerized] OR (*operations research*) models and AI (*artificial intelligence*) based tools (Simon 1967), are viewed as the enabling factor for progress, eventually leading the [digital] *cognitive systems* within a *wise service-oriented* setting (Spohrer et al., 2017). Several cautious views about the prospects of the future role of the AI are articulated by well-known scientists and technology leaders (Stephan Hawking, Elon Musk, Bill Gates, and Steve Wozniak). In order to ensure a favorable evolution of the AI impact, a series of

public authorities and institutions, such as EC (*European Commission*) and the *US Nat. Research Council*, or companies, such as Deloitte, envisage creating a *Trustworthy AI ecosystem*, the main attributes of which being described. The paper concludes with a comparison of *digital humanism* and *dataism* and the unanswered question about the evolution of the human being and society under the influence of pacing and emerging information technologies and devices.

Selected references

Drucker, P. *The manager and the moron*, In: Drucker P, Technology, Management and Society: Essays by Peter F. Drucker, Harper & Row, New York: 1967, p. 166-177.

Filip, F G, Zamfirescu C B, Ciurea E, *Computer Supported Collaborative Decision-making*, Springer, Cham, 2017

Nof, S Y,, Ceroni, J., Jeong, W., Moghaddam, M, *Revolutionizing Collaboration through e-Work, e-Business, and e-Service*, Springer,Cham, 2015

Simon, H. *Two heads are better than one; the collaboration between AI and OR*, Interfaces, 17(4): 1987, 8-15.

Spohrer, J., Siddike M A, *Rebuilding evolution: a service science perspective*, In: Proceedings of the 50th Hawaii International Conference on System Sciences, 2017, pp.1663-1672

FAMILIES OF VECTOR FIELDS WITH AN ALGEBRAIC GEOMETRIC STRUCTURE

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In 1878 Darboux published a seminal paper on a geometric theory of integrability based on invariant algebraic curves of vector fields. This work was much admired by Poincaré who published two articles (1891 and 1897) on the subject and proposed a problem still open today "the problem of Poincaré". He called this work "admirable" and "masterly" but except for two articles by Painlevé and Autonne (1891) and one by Dulac (1908) for a long time there were no other developments. Only over a century after the publication of Darboux' paper, significant new work was published on this topic by Jouanolou (1979) and by Prelle and Singer (1983). The publication of these papers were followed by a flourishing period when new results extended the theory of Darboux turning it into a very active area of research, fully justifying Poincaré's enthusiasm.

In this lecture I shall present some of the new results obtained from the beginning of this century and up to most recently that point to a sort of a nascent algebraic geometry of polynomial vector fields in which both dynamical and algebraic geometric aspects intertwine.

II. MATHEMATICS

SOME PROPERTIES OF ANALOGUES OF GENERALIZED NILPOTENT ELEMENTS OF COMMUTATIVE PSEUDO-NORMED RINGS

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Definition 1. An element a of a pseudo-normed ring (R, ξ) is called:

topologically nilpotent if for any positive real number ε there exists a positive integer n such that $\xi\left(a^i\right) < \varepsilon$ for any positive integer $i \geq n$;

generalized nilpotent if $\lim_{k\to\infty}(\sqrt[k]{\xi(a^k)}) = 0$, i.e., if for any positive real number ε there exists a positive integer n such that $\xi\left(a^i\right) < \varepsilon^i$ for any positive integer $i \geq n$.

Definition 2. If (R, ξ) is a pseudo-normed ring and p is a nonnegative real number, then an element a of the pseudonormed ring (R, ξ) is called *generalized nilpotent of degree p* if for any positive real number ε there exists a positive integer n such that $\xi\left(a^i\right) < \varepsilon^{i^p}$ for any positive integer $i \geq n$.

Properties of generalized nilpotent elements of pseudo-normed commutative rings are studied in [1, 2].

Remark. Definitions 1 and 2 easily imply the following statements:

1. *An element a of a pseudo-normed ring (R, ξ) is generalized nilpotent of degree 0 if and only if the element a is a topologically nilpotent element*;

2. *If p and q are real numbers such that $0 \leq p < q$, then any element of a pseudo-normed ring (R, ξ) which is generalized nilpotent of degree q is generalized nilpotent of degree p.*

Theorem. *If (R, ξ) is a pseudo-normed ring then for any real number $0 \leq r < 1$ any topologically nilpotent element of the pseudo-normed ring (R, ξ) is a generalized nilpotent element of degree r.*

References:

1. M. M. Gelfand, D. A. Raikov, G. E. Shilov. *Commutative normed rings.* State Publishing House of Physics and Mathematics, Moscow 1960, 315 pp. (In Russian)

2. S. A. Aleschenko, V. I. Arnautov, S.T. Glavatsky. Properties of generalized nilpotent elements of pseudo-normed commutative rings. *Fundamentalinaia i prikladnaia matematika*, t.23, no, 4, 3–11. (In Russian)

GROUP REPRESENTATIONS AND PLANE CURVES INVARIANTS

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The aim of the talk is to present two ideas for constructing invariants of plane curves. The first one, due to Libgober, starts with representation of the braid groups as input. For the later, we shall use the quantum representation, which in knot theory, gives the coloured Jones polynomial and the general Lawrence representation. In the meantime, we shall overview also the notion of Zariski pairs, and a possible application of our method for detecting them. The second one uses representations of the fundamental group of the curve complement. We will consider also the relation between them.

REGIONAL CONTROL PROBLEM IN REACTION-DIFFUSION EQUATIONS. APPLICATION TO EPIDEMIOLOGY

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A two-component reaction-diffusion system to describe the spread of malaria is considered. The epidemiological model describes the dynamics of the infected mosquitoes and of the infected humans. The spread of the disease is controlled by three actions (controls) implemented in a subdomain of the habitat: killing mosquitoes, treating the infected humans and reducing the contact rate mosquitoes-humans. To start with, the problem of the eradicability of the disease is considered, while the cost of the controls is ignored. We prove that it is possible to decrease exponentially both the human and the vector infective population everywhere in the relevant habitat by acting only in a suitable subdomain. Later the regional control problem of reducing the total cost of the damages produced by the disease, of the controls and of the intervention in a certain subdomain is treated for the finite time horizon case. An iterative

algorithm to decrease the total cost is proposed; apart from the three controls considered above, the logistic structure of the habitat is taken into account. The level set method is used as a key ingredient for describing the subregion of intervention. In order to quantify the decreasing speed for the number of infected mosquitoes and humans populations, the corresponding decreasing rates along the iterations are also evaluated.

SOME SUFFICIENT CONDITIONS FOR THE EXISTENCE OF NON-DISCRETE HAUSDORFF TOPOLOGIES ON COUNTABLE SKEW FIELDS

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The study of possibility to set a non-discrete Hausdorff topology on infinite algebraic systems in which existing operations are continuous was begun in [1]. In this article for any countable group a method of constructing such group topologies is given.

For countable rings the problem of the possibility to set non-discrete Hausdorff ring topologies was studied in [2].

For infinite fields the problem of the possibility to set of non-discrete field topologies was studied in [2].

For countable skew field the problem of the possibility to set non-discrete Hausdorff topologies is not solved.

The present article is a continuation of research in this direction.

Theorem. *If for any finite subset S of a countable skew field R there exists an infinite subset $M \subseteq R$ such that $r \cdot s = s \cdot r$ for any elements $r \in M$ and $s \in S$ (in particular, if the center of the skew field R is an infinite set), then the following statements are true:*

1. *The skew field R permits a non discrete Hausdorff skew field topology;*

2. *The skew field R permits continuum of topologies such that any two of them are comparable to each other;*

3. *The skew field R permits two to the power of continuum of coatoms in the lattice of all topologies of the skew field R.*

References:

1. A. A. Markov. On absolutely closed sets. *Mat. Sb.*, **18** (1945), 3–28, (In Russian)
2. V. I. Arnautov, S. T. Glavatsky, A. V. Mikhalev. *Introduction to the topological rings and modules.* Marcel Dekker, inc., New York-Basel-Hong Kong, 1996.

TOWARDS A COMPLETE CLASSIFICATION OF PHASE PORTRAITS OF PLANAR QUADRATIC DIFFERENTIAL SYSTEMS

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In this talk I will present the results obtained up to now for a systematic classification of phase portraits of planar quadratic differential equations, and the available tools needed to complete this work. Up to now, this work is achievable only if we limit ourselves to phase portraits modulo limit cycles, that is, without considering how many limit cycles a portrait has. Even with this strong restriction, the number of different phase portraits seems to be over 1000 and even close to 2000.

A NEW WAY TO CLASSIFY GEODESICS ON HYPERBOLIC MANIFOLDS

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This paper is devoted to the classification of all geodesic curves on 2-dimensional hyperbolic manifolds (the global behavior) by using a new constructive method - generalized colored multilateral method. We start by considering this problem on hyperbolic pants in hopes of discovering a method which can be easily generalized to the problem of behavior of geodesic on any hyperbolic surface. For the behavior of the geodesics on the specified fragments (hyperbolic pants, etc.) it is used a certain figure, named in the text of the work *the multilateral.* To facilitate the understanding and further description, we agree to call the sides of the six-rectangle (hyperbolic right-angled hexagon) black, if they are obtained from boundary geodesic circles of pants, and the other three sides we agree to consider painted in different colors (for example, red, blue and

green straight). The study in this paper is being carried out gradually, in order of collecting the surface, the reverse order of cutting the surface into fragments (i.e. pants). The surface is cut into typical pieces (for example, on pants or their generalized pants, on right hexagons, etc.) and the behavior of geodesics for each piece is solved on it, and then the result of the investigation returns (by gluing) onto the original surface. To summarize what has been said, we can conclude that a concrete method of investigating the behavior of geodesics on hyperbolic 2-manifolds is based on the idea of preliminary research on these pieces (on the set of hyperbolic pants and their generalized), in the subsequent consolidation of research results using the method of generalized colored multilateral.

In more details, the following main results of the study were obtained. On hyperbolic pants, we classified all possible type of behavior of geodesic lines, based on the algorithm for constructing the corresponding system of colored angles, and by the sides parallel to the considered side of the generalized multilateral obtained from a hyperbolic hexagon. Further, the concept of the category of angles is introduced, and with the help of these categories an algorithm for recognizing the type of a geodesic is given. The main purpose of this paper is to give a new constructive method for solving the problem of the behavior of geodesic on an arbitrary hyperbolic surface of signature (g, n, k). Such a compressed formulated result can be disclosed as follows. For this purpose, with the help of proposed practical approach at first: 1) we obtain a complete classification of all possible geodesic curves on the simplest hyperbolic 2-manifolds (hyperbolic horn; hyperbolic cylinder; parabolic horn (cusp)); 2) we describe the behavior of geodesics in the following cases: on a surface of genus 2; on an compact closed hyperbolic surface without boundary (general case); on hyperbolic surface of genus g and with n geodesic boundary components; on hyperbolic 1-punctured torus; on generalized hyperbolic pants; in general case: for any (oriented) punctured hyperbolic surface M of genus g and k punctures; in the most general case: for any hyperbolic surface of signature (g, n, k) (with genus g, n boundary components and k cusps).

DETERMINATION ON SOME SOLUTIONS TO THE STATIONARY 2D NAVIER-STOKES EQUATION

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Consider the following system of partial differential equations:

$$\begin{cases} \frac{P_x}{\mu} + uu_x + vu_y = a(u_{xx} + u_{yy}) + F_x \\ \frac{P_y}{\mu} + uv_x + vv_y = a(v_{xx} + v_{yy}) + F_y \\ u_x + v_y = 0 \end{cases} \tag{1}$$

$$P = P(x, y); \quad u = u(x, y); \quad v = v(x, y); \quad x, y \in \mathbb{R},$$

where $P, u, v, F : D \to \mathbb{R}^2$.

The system (1) describes the process of stationary fluid flow or gas on a flat surface. The function P represents the pressure of the liquid, and functions u, v represent the flow of the liquid (gas). The constants $a > 0$ and $\mu > 0$ are determined by the parameters of the liquids (of the gas), which are viscosity and liquid's density. The function F represents the exterior forces.

Theorem. *Suppose that $u, v \in C^2(D)$ admit the bounded derivatives up to including order 2 in D.*

If $f(z)$, $z = x + iy$, is an analytical function in D, then (u, v, P), with $u = Imf$, $v = Ref$, $P = [F - 0,5(u^2 + v^2) + c]\mu$ are solutions to the system (1).

If $W(x, y)$ is a harmonic function in D, then (u, v, P), with

$$u = W_y + c_1 y + c_2, \quad v = -W_x + c_3 x + c_4,$$

$$P = [F - 0,5(u^2 + v^2) + (c_1 - c_3)W + 0,5(c_1 y^2 - c_3 x^2) + c_2 y - c_4 x + c]\mu,$$

and the arbitrary constants c, c_1, c_2, c_3, c_4 are solutions to the system (1).

In addition, various special cases were studied, and particular and exact solutions of the system (1) were found in these cases.

MULTIFREQUENCY SYSTEMS WITH DELAY AND LOCAL-INTEGRAL CONDITIONS

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A system of differential equations with linearly transformed arguments of the form

$$\begin{aligned} \frac{da}{d\tau} &= X(\tau, a_\Lambda, \varphi_\Theta), \\ \frac{d\varphi}{d\tau} &= \frac{\omega(\tau)}{\varepsilon} + Y(\tau, a_\Lambda, \varphi_\Theta), \end{aligned} \tag{1}$$

where $\tau \in [0, L]$, $a \in D$ – limited area in $\mathbb{R}^n$, $\varphi \in \mathbb{R}^m$, small parameter $\varepsilon \in (0, \varepsilon_0]$, $\varepsilon_0 \ll 1$, $a_\Lambda = (a_{\lambda_1}, \ldots, a_{\lambda_p})$, $\varphi_\Theta = (\varphi_{\theta_1}, \ldots, \varphi_{\theta_q})$, $0 < \lambda_1 < \cdots < \lambda_p \leq 1$, $0 < \theta_1 < \cdots < \theta_q \leq 1$, $a_{\lambda_i}(\tau) = a(\lambda_i \tau)$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j \tau)$ is investigated. Vector-functions X and Y are defined and smooth enough for all variables in the area $G = [0, \tau] \times D^p \times R^{qm}$ and are 2π-periodic by vector components φ_Θ.

For the system (1) the following conditions are set (1)

$$\begin{aligned} \sum_{\nu=1}^{r} \alpha_\nu a(x_\nu) &= \int_{\tau_1}^{\tau_2} f(\tau, a_\Lambda, \varphi_\Theta) d\tau, \\ \sum_{\nu=1}^{r} \beta_\nu \varphi(x_\nu) &= \int_{\tau_1}^{\tau_2} g(\tau, a_\Lambda, \varphi_\Theta) d\tau, \end{aligned} \tag{2}$$

where $0 \leq x_1 < x_2 < \cdots < x_r \leq L$, $0 \leq \tau_1 < \tau_2 \leq L$.

In [1], for the system (1) under conditions (2), a much simpler problem is constructed by averaging [1] the vector functions X, Y, f and g over fast variables φ_{θ_ν} on the cube of periods $[0, 2\pi]^{mq}$. The existence and uniqueness of the solution of the problem (1)-(2) in the class $C^1[0, L]$ is proved. On the time interval $[0, L]$, some estimates of the deviations of the order $\sqrt[mq]{\varepsilon}$ between the solutions of the problem (1)-(2) and the solutions of the averaged system are established.

The results of the work generalize the results obtained in [2].

References:

1. A. M. Samoilenko, R. I. Petryshyn. *Multifrequency Oscillations of Nonlinear Systems.* Kluwer, Dordrecht–Boston–London, Netherlands, 2004.

2. Ya. Bihun, R. Petryshyn, I. Krasnokutska. Averaging method in multifrequency systems with linearly transformed arguments and with point and integral conditions. *Acta et Commentationes, Exact and Natural Sciences*, **2(6)** (2018), 20–27.

MULTIPLE SOLUTIONS FOR ANIZOTROPIC ELLIPTIC SYSTEMS

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The variable exponent problems have various applications in different areas, hence the interest for such problems is rising. Here, we establish a multiplicity result for a general class of anisotropic elliptic systems involving variable exponents and Leray-Lions type operators. Moreover, we provide several examples of systems to illustrate the generality of this class of systems.

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A METHOD FOR GENERATING LOCALLY CONVEX TOPOLOGIES WITH CERTAIN DENSITY PROPERTIES

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Let X be a (real or complex) normed space, $\mathcal{T}$ the topology on X induced by the norm, D a nonempty subset of X which is not dense in X, and

$$\mathcal{M} := \{\mathcal{T}' \mid \mathcal{T}' \text{ is a locally convex topology on } X,\ \mathcal{T}' \subseteq \mathcal{T},\ \overline{D}^{\mathcal{T}'} = X\},$$

where $\overline{D}^{\mathcal{T}'}$ stands for the closure of D in the topological space $(X, \mathcal{T}')$. We first give a method for obtaining elements $\mathcal{T}'$ in $\mathcal{M}$ in the case D is a linear subset of X and the domain of a linear operator $A\colon D \to X$. Afterwards we present an application to m-accretive operators.

ONE CLASS OF QUADRATIC DIFFERENTIAL SYSTEMS WITH TWO INVARIANT LINES

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Consider the family of real planar polynomial differential systems

$$x' = p(x, y), \quad y' = q(x, y), \tag{1}$$

$p(x, y), q(x, y) \in \mathbb{R}[x, y]$, $n = \max\{\deg(p), \deg(q)\}$; $n = 2$ - quadratic systems.

At the beginning of this century prof. Dana Schlomiuk initiated an extensive project for determining all possible configurations of invariant lines for the whole family of quadratic systems.

Consider a real planar polynomial differential system (1). We call *configuration of invariant straight lines* of this system, the set of (complex) invariant straight lines (which may have real coefficients), including the line at infinity of the system, each endowed with its own multiplicity and together with all the real singular points of this system located on these invariant straight lines, each one endowed with its own multiplicity.

We denote by $\mathbf{QSL_i}$ the family of all non-degenerate quadratic differential systems possessing invariant straight lines (including the line at infinity) of total multiplicity i with $i \in \{1, 2, 3, 4, 5, 6\}$. For any quadratic system on the affine plane the line at infinity is invariant. If this is the only invariant line and if it is of multiplicity 1 and in addition the system is non-degenerate, then the system belongs to $\mathbf{QSL_1}$.

Up to now the families $\mathbf{QSL_i}$ with $i \in \{4, 5, 6\}$ were classified topologically using their algebraic geometric structures and they were all proven to be Liouvillian integrable. Another family of systems possessing invariant lines is formed by the Lotka-Volterra systems, that have systems with two real invariant lines intersecting at a finite point. This family was classified by using the previous classifications of $\mathbf{QSL_i}$ with $i \in \{4, 5, 6\}$ and clearly contains a part of $\mathbf{QSL_3}$.

Another part of $\mathbf{QSL_3}$ is $\mathbf{QSL_3^{2p}}$ formed by quadratic systems that have two lines intersecting at infinity (parallel lines) and have the total multiplicity of their invariant lines equal to 3. In this work we classify the family $\overline{\mathbf{QSL_3^{2p}}}$ of quadratic systems that have two parallel lines and possess invariant lines of total multiplicity at least 3. This family clearly contains $\mathbf{QSL_3^{2p}}$.

First we determine affine invariant criteria for a system (1) to belong to the class $\overline{\mathbf{QSL_3^{2p}}}$.

Theorem. *The class* $\overline{\mathbf{QSL}_3^{2p}}$ *has exactly 58 distinct configurations of invariant lines, 26 of which are with invariant lines of total multiplicity exactly 3. Moreover for each one of these configurations necessary and sufficient affine invariant conditions for its realization are given.*

WEAK REFLEXIVE SUBCATEGORY

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In the subcategory of topological vector locally convex spaces Hausdorff are built proper classes of weakly reflective and weakly coreflective subcategories, respectively. We proved that the right product of two subcategories often leads to a weakly reflective subcategory, which is not reflective.

A full subcategory $\mathcal{R}$ of category $\mathcal{C}$ is called *weakly reflective*, if for any $X \in |\mathcal{C}|$ there are an object $rX \in |\mathcal{R}|$ and a morphism $r^X : X \to rX$ with the property: for an object $A \in |\mathcal{R}|$ any morphism $f : X \to A$ extends through r^X : $f = g \cdot r^X$ for some g. If the extension is always unique, then $\mathcal{R}$ is called the *reflective subcategory.*

In universal algebra, reflective subcategories are realized as factorizations of objects, but extensions are also known, for example localizations in torsion theories. In the general topology, reflective subcategories are more common as extensions, but there are also some as factorizations. In the subcategory of topological vector locally convex spaces Hausdorff $\mathcal{C}_2\mathcal{V}$ are known proper classes of reflective, coreflective and bireflective subcategories. In $\mathcal{C}_2\mathcal{V}$ we construct weakly reflective subcategories and weakly coreflective subcategories and study their properties.

We denote by $\mathbb{R}$ (respectively $\mathbb{K}$) the class of non-zero subcategories of category $\mathcal{C}_2\mathcal{V}$. For $\mathcal{K} \in \mathbb{K}$ and $\mathcal{R} \in \mathbb{R}$ with the respective functors $k : \mathcal{C}_2\mathcal{V} \to \mathcal{K}$ and $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$, either $\mu\mathcal{K} = \{m \in \mathcal{M}ono | k(m) \in \mathcal{I}so\}$, $\varepsilon\mathcal{R} = \{e \in \mathcal{E}pi | r(e) \in \mathcal{I}so\}$. Both $\mu\mathcal{K}$ and $\varepsilon\mathcal{R}$ are classes of bimorphisms.

If $\mathcal{B}$ is a class of bimorphisms and $\mathcal{A}$ a subcategory, then $S_{\mathcal{B}}(\mathcal{A})$ (respectively $Q_{\mathcal{B}}(\mathcal{A})$) is the full subcategory of all $\mathcal{B}$-subobjects (respectively: $\mathcal{B}$-factorobjects) of the objects of subcategory $\mathcal{A}$. We examine the following two conditions: a) $\mathcal{R}$ contains the subcategory $\mathcal{S}$ of spaces with weak topology; b) $\mathcal{K}$ contains the subcategory $\tilde{\mathcal{M}}$ of spaces with Mackey topology.

Theorem 1. *Let be* $r^X = u^X \cdot v^X$ *is the* $((\mu\mathcal{K})^\top, \mu\mathcal{K})$*-factorization of morphism* r^X, $k^X = w^X \cdot t^X$ *is the* $((\varepsilon\mathcal{R}), \varepsilon\mathcal{R}^\perp)$*-factorization of morphism* k^X.
1. $S_{\mu\mathcal{K}}(\mathcal{R})$ *is a weak reflective subcategory of the category* $\mathcal{C}_2\mathcal{V}$ *and* $v^X : X \to$

vX is the weak replique of object X.
1^*. *$Q_{\varepsilon\mathcal{R}}(\mathcal{K})$ is a weak coreflective subcategory of the category $\mathcal{C}_2\mathcal{V}$ and $w^X : wX \to X$ is the weak coreplique of object X.*
2. *$S_{\mu\mathcal{K}}(\mathcal{R})$ is a reflective subcategory if it meets one of the conditions a) or b).*
2^*. *$Q_{\varepsilon\mathcal{R}}(\mathcal{K})$ is a coreflective subcategory if it meets one of conditions a) or b).*

Theorem 2. *Let $\mathcal{R} \in \mathbb{R}$ and let Σ be the coreflective subcategory of the spaces with the most powerful locally convex topology and $\sigma : \mathcal{C}_2\mathcal{V} \to \Sigma$. The following statements are equivalent:*

1. *$\mathcal{S}_{\mu\Sigma}(\mathcal{R})$ is a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$.*
2. *$\mathcal{S} \subset \mathcal{R}$.*

Theorem 3. *Let $\mathcal{K} \in \mathbb{K}$ and let Π be the reflective subcategory of the complete spaces with weak topology and $\pi : \mathcal{C}_2\mathcal{V} \to \Pi$. The following statements are equivalent:*

1. *$\mathcal{Q}_{\tilde{\varepsilon}\Pi}(\mathcal{K})$ is a coreflective subcategory of the category $\mathcal{C}_2\mathcal{V}$.*
2. *$\mathcal{M} \subset \mathcal{K}$.*

TWO REAL-LIFE APPLICATIONS OF GRAPH PARTITIONING

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Graph partitioning is a widely researched topic that has extensive applications in many areas, including scientific computing, data mining, VLSI design, image analysis, air traffic control, etc.

Let $G = (V, E)$ be a simple graph and $P_k = \{V_1, V_2, ..., V_k\}$ a set of subsets of V. P_k is said to be a partition of G if:

1. no element of P_k is empty: $\forall i \in \{1, 2, .., k\}, V_i \neq \varnothing$;
2. the elements of P_k are pairwise disjoint: $\forall (i, j) \in \{1, 2, .., k\}^2, i \neq j$, $V_i \cap V_j = \varnothing$;
3. the union of all the elements of P_k is equal to V: $\bigcup_{i=1}^{k} V_i = V$.

The elements V_i of P_k are called the parts of the partition. The number k is called the cardinality of the partition, or the number of parts of the partition.

Usually, graph partitioning problems seek to optimize some objective functions and satisfy various constraints imposed on the parts of the partition. Such optimization graph partitioning problems typically fall under the category of NP-hard problems. As a result, solutions to these problems are generally obtained using heuristics and approximation algorithms.

We present two examples of real-life applications of graph partitioning problems, where some special heuristic were used to make them tractable: optimization of administrative-territorial structure [1, 2], organization of electoral districts of a State [3].

References:

1. R. Buzatu, C. Rabei, M. Roşcovan. A mathematical model for optimization of administrative-territorial organization of the Republic of Moldova. *Studia Universitatis Moldaviae, Exact and Economic Science*, **112**(2) (2018), 164–170. (In Romanian)
2. I. Beschieru, D. Toma, E. Levinta-Perciun, O. Utica, R. Buzatu, A.F. Ghita, E. Cepoi. *Study on administrative-territorial reform scenarios.* Chişinău, 2018, 272 p. (In Romanian).
3. I. Mandric, I. Roşca, R. Buzatu. *Gerrymandering and fair districting in parallel voting systems.* arXiv:2002.06849v1, 2020, 23 p.

LOCALIZATION OF SINGULAR POINTS OF MEROMORPHIC FUNCTIONS BASED ON INTERPOLATION BY RATIONAL FUNCTIONS

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Let consider a function $f(z)$ of a complex variable z, which is meromorphic on a finite domain $\Omega \subset \mathbb{C}$ so that the point $0 \in \Omega$. The finite values $f_j := f(z_j)$ of the function $f(z)$ are known on the set $\{z_j\}$, where the points z_j belong to a simple closed contour $\Gamma \subset \Omega$, that contains the point $z = 0$. We admit that points z_j form a dense set on Γ. The function $f(z)$ has a finite number of singular points of polar type on the domain Ω, but their number and locations are not known. Also, on the contour Γ the function $f(z)$ can have both poles and jump discontinuity points (which can be considered as removable singularities). If the function $f(z)$ has poles on the contour Γ, in order to avoid the computation difficulties, we consider that the values of the function $f(z)$ at the points $z_j \in \Gamma^\rho$ are given (here Γ^ρ represents a small perturbation of the contour Γ). We aim to determine the locations of the singular points on Ω, in particular those on the contour Γ. In our earlier works we have already examined the Padé approximation with Laurent polynomials [1] and the Padé approximation with Faber polynomials [2]. In both cases we have considered the meromorphic functions on a finite domain of the complex plane with given values at the points of a simple closed contour from this domain.

In this paper we examine two algorithms for localizing the singular points of meromorphic function $f(z)$, both are based on the approximation by rational

functions. The first one is based on global interpolation and gives the possibility to determine the singular points of the function on a domain Ω. The second algorithm, based on piecewise interpolation, establishes the poles and the discontinuity points on the contour Γ.

References:

1. M. Capcelea, T. Capcelea. Laurent-Padé approximation for locating singularities of meromorphic functions with values given on simple closed contours. *Buletinul Academiei de Ştiinţe a Republicii Moldova. Matematica*, **93**(2) (2020), 76–87.
2. M. Capcelea, T. Capcelea. Algorithm for the localization of singularities of functions defined on closed contours. In: *Proceedings of the Fourth Conference of Mathematical Society Of Moldova, CMSM4, June 28 - July 2, Chişinău*, 2017.

LATTICE OF FACTORIZATION STRUCTURES $\mathcal{L}(\mathcal{R})$

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In the category $\mathcal{C}_2\mathcal{V}$ of locally convex topological vector spaces we examine a class of factorization structures for which the reflector functor transforms the class of projections or the class of injections, or both classes into themselves. The functors which preserve certain classes of morphisms are widespread: left exact functors, right exact functors, exact functors, mono and epifunctors [1]. In categories of topological algebra and functional analysis, where there exist many factorization structures, it appears the necessity to study functors which preserve one or both classes of factorization structures. In this paper, for any reflector functor $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$ we construct a lattice $\mathcal{L}_u(\mathcal{R})$ of factorization structure, so that r preserves the respective classes for each element of this lattice.

Theorem. *1. Let $(\mathcal{E}, \mathcal{M}) \in \mathcal{L}_u(\mathcal{R})$. Then $r : \mathcal{C}_2\mathcal{V} \to \mathcal{R}$ is a $(\mathcal{E}, \mathcal{M})$-functor: $r(E) \subset E$ and $r(M) \subset M$;*
2. $f \in P"(R) \Longleftrightarrow r(f) \in P"(R)$.

References:

1. D. Botnaru, O. Cerbu. Functor of Special Type. In: *Proceedings of the VIII International Workshop, Lie Theory and its Applications in Physics, Bulgaria Academy of Sciences, Institute for Nuclear Research and Nuclear Energy, Varna, Bulgaria, 15-21 June 2009*, 299–311.

ON CONTROLLABILITY FOR FRACTIONAL DIFFERENTIAL INCLUSIONS OF CAPUTO-FABRIZIO TYPE

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We consider the following fractional differential inclusion

$$D^{\sigma}_{CF}x(t) \in F(t, x(t)) \quad a.e.\ ([0,T]), \quad x(0) \in X_0, \quad x'(0) \in X_1, \tag{1}$$

where $F(.,.) : [0,T] \times \mathbf{R} \to \mathcal{P}(\mathbf{R})$ is a set-valued map, D^{σ}_{CF} denotes Caputo-Fabrizio's fractional derivative of order $\sigma \in (1,2)$ and $X_0, X_1 \subset \mathbf{R}$ are closed sets.

We prove that the reachable set of a certain variational fractional differential inclusion is a derived cone in the sense of Hestenes to the reachable set of the problem (1). In order to obtain the continuity property in the definition of a derived cone we shall use a continuous version of Filippov's theorem for solutions of fractional differential inclusions (1). As an application we obtain a sufficient condition for local controllability along a reference trajectory.

ON THE STRUCTURE OF LEVINSON CENTER OF MONOTONE ALMOST PERIODIC SYSTEMS

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The aim of this talk is studying the problem of existence of Levitan/Bohr almost periodic solutions for dissipative differential equation

$$x' = f(t,x), \quad (f \in C(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n)) \tag{1}$$

when the second right hand side is monotone with respect to spacial variable. The existence of at least two quasi periodic (respectively, Bohr/Levitan almost periodic) solutions of (1) is proved under the condition that every solution of equation (1) is positively uniformly Lyapunov stable. These results we establish in the framework of general non-autonomous (cocycle) dynamical systems.

Along with the equation (1) we consider its H-class, i.e., the family of the equations

$$v' = g(t,v), \tag{2}$$

where $g \in H(f) = \overline{\{f_\tau : \tau \in \mathbb{R}\}}$ and $f_\tau(t,u) = f(t+\tau, u)$, where the bar indicates the closure in the compact-open topology.

This study is a continuation of the author's work, which gives a positive answer to the I. U. Bronshtein's conjecture for monotone systems.

I. U. Bronshtein's conjecture [1]. If an equation (1) with right hand side (Bohr) almost periodic in t satisfies the conditions of uniform positive stability and positive dissipativity, then it has at least one (Bohr) almost periodic solution.

Below we will use the following conditions.

Condition (A1). The function $f \in C(\mathbb{R} \times W, \mathbb{R}^d)$ is Bohr/Levitan almost periodic [2,3] in $t \in \mathbb{R}$ uniformly in u on every compact subset $K \subset W$ and the equation (1) is monotone.

Condition (A2). Equation (1) with regular right hand side f admits a compact global attractor (Levinson center) and every solution of equation (2) is positively uniformly stable.

Theorem. Under conditions $(A1) - (A2)$ if the function $f \in C(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n)$ is quasi-periodic (respectively, Bohr/Levitan almost periodic) in $t \in \mathbb{R}$ uniformly with respect to u on every compact subset from $\mathbb{R}^n$, then equation (1) has at least two (lower and upper) quasi-periodic (respectively, Bohr/Levitan almost periodic) solutions.

References:

1. D. N. Cheban. I. U. Bronshtein's Conjecture for Monotone Nonautonomous Dynamical Systems. *Discrete and Continuous Dynamical Systems: series B*, **24** (2019), 1095–1113.
2. D. N. Cheban. *Nonautonomous Dynamics:* Nonlinear Oscillations and Global Attractors. Springer Nature Switzerland AG 2020, xxii+ 434 pp.
3. B. M. Levitan, V. V. Zhikov, *Almost Periodic Functions and Differential Equations.* Moscow State University Press, Moscow, 1978, 204 pp. (in Russian). [English translation: Almost Periodic Functions and Differential Equations. Cambridge Univ. Press, Cambridge, 1982, xi+217 p.]

POISSON GEOMETRY OF MODULI SPACES OF FLAT CONNECTIONS VIA UNIVERSAL FINITE-TYPE INVARIANTS

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Let G be an algebraic, reductive, Lie group, and Σ a compact oriented surface, possibly with boundary. By classical works of Atiyah, Bott, and Goldman,

the moduli space $\mathcal{M}(\Sigma, G)$ of flat connections on a principal G-bundle over Σ, or equivalently the G-character variety $Hom(\pi_1(\Sigma), G)/G$, where G acts by conjugation, admits an interesting Poisson structure. Understanding the Poisson geometry of these spaces is an open area of research, connected to several problems in mathematics and theoretical physics.

The idea, and a method to use finite-type invariants of knots/links and of 3-dimensional manifolds to study $\mathcal{M}(\Sigma, G)$ was initiated in 1996 by Andersen, Mattes, and Reshetikhin (AMR), who introduced the Poisson algebras $\mathcal{C}(\Sigma)$ and $\mathcal{C}(\Sigma, G)$ of colored chord diagrams, and for every symmetric Ad-invariant tensor $t \in \mathfrak{g} \otimes \mathfrak{g}$, where $\mathfrak{g} = Lie\, G$, a Poisson homomorphism $\psi_t : \mathcal{C}(\Sigma, G) \to \mathcal{O}(\mathcal{M}(\Sigma, G))$ to the algebra of regular functions on $\mathcal{M}(\Sigma, G)$, which they proved is onto for many important examples.

The completion of the space of *non-colored* chord diagrams is the value space of the universal finite-type invariant of links (due to Kontsevich), and Andersen, Mattes, and Reshtikhin have attempted to use the latter to define a deformation quantization of $\mathcal{O}(\mathcal{M}(\Sigma, G))$, but the construction is non-canonical (depends on the choice of a "partition" of Σ).

In 2017, Habiro, and Massuyeau have extended (in the sense of morphism extensions) the Kontsevich-LMO functor (constructed in 2008 by Habiro, Massuyeau, and myself) to a functor with values in $\overline{\mathcal{C}(\Sigma, G)}$.

Theorem. *Let φ be a Drinfeld associator. Then the Habiro-Massuyeau extension Z^{φ} of the Kontsevich-LMO functor induces a $\star$-product (i.e. a deformation quantization) of $\mathcal{C}(\Sigma)$, independent of the AMR-partition of the surface.*

This theorem completes Andersen-Mattes-Reshetikhin construction, and allows, by specifying various Lie groups G, to use skein theory to study the moduli spaces $\mathcal{M}(\Sigma, G)$, and their associated quantum moduli spaces.

GROUPOIDS OF ORDER THREE WITH BOL-MOUFANG IDENTITIES UP TO ISOMORPHISM

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The list of all classical Bol-Moufang identities is given in [1]. We continue the research of these groupoids of small order [2, 3].

There exist 61 non-isomorphic groupoids of order 3 with identity F_1 $(xy \cdot zx) = (xy \cdot z)x$ from possible 314 groupoids.

There exist 40 non-isomorphic groupoids of order 3 with identity F_2 $xy.zx = (x.yz)x$ (middle Moufang) from possible 196 groupoids.

There exist 61 non-isomorphic groupoids of order 3 with identity F_3 $(xy \cdot zx) = x(y \cdot zx)$ from possible 314 groupoids. Notice, identities F_1 and F_3 are dual.

There exist 41 non-isomorphic groupoids of order 3 with identity F_4 $xy \cdot zx = (x \cdot yz)x$ (middle Moufang) from possible 196 groupoids.

There exist 49 non-isomorphic groupoids of order 3 with identity F_6 $(xy \cdot z)x = x(y \cdot zx)$ (extra identity) from possible 239 groupoids.

References:

1. F. Fenyves. Extra loops. II. On loops with identities of Bol-Moufang type. *Publ. Math. Debrecen*, v. 16, 1969, 187–192.
2. V. Chernov, A. Moldovyan, V. Shcherbacov. On some groupoids of order three with Bol-Moufang type of identities. In: *Proceedings of the Conference on Mathematical Foundations of Informatics MFOI 2018, July 2-6, 2018, Chişinău, Republic of Moldova*, 2018, 17–20.
3. G. Horosh, V. Shcherbacov, A. Tcachenco, T. Yatsko. On groupoids with Bol-Moufang type identities. *Computer Science Journal of Moldova*, **28**, no.3(84) (2020), 314–327.

DOMINATION OF BLOCKS, FUSION SYSTEMS AND HYPERFOCAL SUBGROUPS

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In the context of modular representation theory of finite groups, considering a finite group G, an algebraically closed field k of characteristic p, a block b of kG and a maximal Brauer (D, e), the block b is inertial if b and e lie in a special type of Morita equivalence. A particular situation of this equivalence makes b into a nilpotent block. For a normal p-subgroup P of G, setting $\bar{G} := G/P$, the G-acted epimorphism of group algebras $\pi : kG \to k\bar{G}$ determines the connection between b and its dominating blocks. We investigate the connections between some properties of blocks and of their dominating blocks. We find conditions to verify that a block is inertial if and only if its dominating block is inertial. In some situations the equality of the factor fusion systems associated with a block and with its Brauer correspondent block give information about the hyperfocal subgroups.

THE STUDY OF A GENERAL EPIDEMIOLOGICAL MODEL

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A general model for the spread of contagious diseases is proposed. It takes into account susceptible, isolated, exposed, asymptomatic and symptomatic infected, hospitalized, deceased and recovered persons. The social interaction and some other aspects, as the psychological reaction of the population when the number of infected people increases, are incorporated in a nonlinear incidence rate.

The existence and stability of the equilibria is analyzed and the reproduction number is computed. Various types of bifurcations are analyzed and the results are interpreted from the epidemiological point of view. The theoretical study is completed by numerical simulations.

CENTER CONDITIONS FOR A CUBIC SYSTEM WITH AN INVARIANT CUBIC CURVE

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We consider the cubic system of differential equations

$$\dot{x} = y + p_2(x,y) + p_3(x,y),\ \dot{y} = -x + q_2(x,y) + q_3(x,y), \tag{1}$$

where $p_j(x,y)$, $q_j(x,y) \in \mathbb{R}[x,y]$ are homogeneous polynomials of degree j. The origin $O(0,0)$ is a singular point which is a center or a focus (fine focus) for (1). We study the problem of the center for cubic system (1) assuming that the system has irreducible invariant algebraic curves (algebraic solutions).

The problem of the center was solved: for cubic system (1) with four and three invariant straight lines [1-3]; for cubic system (1) with two invariant straight lines and one irreducible invariant conic [3]; for cubic system (1) with two invariant straight lines and one irreducible invariant cubic [4]; for a nine-parameter cubic system that can be reduced to a Liénard type system [5].

In this talk we consider the following problems:

(i) determine the subclass of cubic differential systems (1) which has a given number of invariant algebraic curves of degrees one and three;

(ii) for this subclass find the conditions under which the origin is a center.

References:

1. A. Şubă, S. Turuta. Solution of the center problem for cubic differential systems with one or two affine invariant straight lines of total algebraic multiplicity four. *ROMAI Journal*, **15** (2019), 101–116.
2. A. Şubă, D. Cozma. Solution of the problem of center for cubic differential systems with three invariant straight lines in generic position. *Qualitative Theory of Dynamical Systems*, **6** (2005), 45–58.
3. D. Cozma. *Integrability of cubic systems with invariant straight lines and invariant conics.* Ştiinţa, Chişinău, 2013.
4. D. Cozma, A. Dascalescu. Integrability conditions for a class of cubic differential systems with a bundle of two invariant straight lines and one invariant cubic. *Bul. Acad. de Şt. a Republicii Moldova. Matematica*, **86** (2018), 120–138.
5. A.P. Sadovskii, T.V. Scheglova. Solution of the center-focus problem for a nine-parameter cubic system. *Differ. Eq.*, **47** (2011), 208–223.

ON A GENERALIZATION OF EXPRESSIBILITY AND COMPLETENESS IN SUPER-INTUITIONISTIC LOGICS

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Chain logics are intermediary between classical logic and intuitionistic one. A superintuitionistic logic is said to be a chain logic if the formula $((p \supset q) \vee (p \supset q))$ is true in it.

Let us consider the pseudo-Boolean algebra $\langle M; \Omega \rangle$, were $\Omega = \{\&, \vee, \supset, \neg\}$; here $\supset$ is the relative pseudocompliment and $\neg$ is the pseudocompliment. We say that the function f of algebra $\mathfrak{A}$ can be parametrically expressed via a system of functions Σ of $\mathfrak{A}$, if there exist the functions $g_1, h_1, \ldots, g_r, h_r$ which are expressed explicitly via Σ using superposition such that the predicate $f(x_1, \ldots, x_n) = x_{n+1}$ is equivalent to the predicate $\exists\, t_1 \ldots \exists\, t_l \big((g_r = h_1) \& \ldots \& (g_r = h_r)\big)$ on the algebra $\mathfrak{A}$. The system Σ of pseudo-Boolean terms on the set of variables X is parametrically complete in $\langle M; \Omega \rangle$, if we can parametrically express the operations from Ω via functions expressed by terms over Σ.

Let us examine the 5-valued pseudo-Boolean algebra

$$Z_5 = \big\langle \{0, \rho, \tau, w, 1\};\ \&,\ \vee,\ \supset,\ \neg \big\rangle;$$

where ρ and τ are incomparable elements and $0 < w < \rho < 1$; $0 < w < \tau < 1$.

The algebra $Z_3 = \big\langle \{0, w, 1\};\ \&,\ \vee,\ \supset,\ \neg \big\rangle$ is a subalgebra of Z_5.

The functions $f(x_1, x_2, \ldots, x_n)$ and $g(x_1, x_2, \ldots, x_k)$ are called permutable if the identity

$$f\Big(g(x_{11}, x_{12}, \ldots, x_{1k}), g(x_{21}, x_{22}, \ldots, x_{2k}), \ldots, g(x_{n1}, x_{n2}, \ldots x_{nk})\Big) =$$

$$= g\Big(f(x_{11}, x_{21}, \ldots, x_{n1}), f(x_{12}, x_{22}, \ldots, x_{n2}), \ldots, f(x_{1k}, x_{2k}, \ldots, x_{nk})\Big)$$

holds.

The set of all functions from an algebra $\mathfrak{A}$ which are permutable with a given function f, is the centralizer of f (denoted by $-\langle f\rangle-$) on algebra $\mathfrak{A}$. Let us define the function $f(p)$ on Z_5 as follows:

$$f(0) = 0, f(\rho) = \tau, f(\tau) = \rho, f(w) = f(1) = 1.$$

The logic of the algebra Z_m we denote by C_m.

Theorem. *The cardinality of every parametrical basis in algebra Z_5 is not grater than 7 and for every $k = 1, 2, .., 7$, there exists a parametrical basis in Z_5, the cardinality of which is k.*

Example. *The formulas*

$$\{0, 1, p\&r, p \vee q\},$$

$$\{0, 1, \neg\neg(p\&q), \neg\neg(p \vee q), \neg\neg p\&(p \vee \neg q), \neg\neg p\&(p \vee q \vee \neg q)\},$$

$$\{0, 1, \neg\neg(p\&q), \neg\neg(p \vee q), \neg\neg p\&(p \vee \neg q), \neg\neg p\&(p \vee q \vee \neg q), (p \supset (q \vee \neg q)) \supset p\},$$

are parametrical basis in the chain logics C_2, C_3 and L, respectively, where $L \subseteq C_4$.

ON THE REGULAR STAR EQUIDISTANT POLYTOPES IN HYPERBOLIC SPACE

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We have shown that a regular equidistant polytope of type {2m+1, 3} always can be "stellated" using method proposed by Coxeter. Obviously, the factorization in the base plane of equidistant polytope can transform the star regular unbounded polytopes into regular bounded star equidistant polytopes. We will discuss one of the most simple and interesting cases on the Klein surface of genus 3 with the regular map {7, 3}.

ON MAXIMUM PRINCIPLE FOR STRINGS, BEAMS, MEMBRANES AND THIN PLATES

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The maximum principle is one of the most useful and best known tools employed in the study of partial differential equations. It enables us to obtain information about the uniqueness, approximation, boundedness and symmetry of the solution, the bounds for the first eigenvalue, for quantities of physical interest (maximum stress, the torsional stiffness, electrostatic capacity, charge density etc), the necessary conditions of solvability for some boundary value problems, etc.

All problems mentioned in the title seem to have at least one thing in common. Whenever a force is applied in one direction the object moves in that direction, allowing a maximum principle to be valid. The corresponding boundary value problems that model such problems are of second order for strings and membranes, while beams and plates are modeled through fourth order equations. We' ll see that the major obstacle when moving from second order to fourth order equations is the lack of the maximum principle.

A significant part of the present talk will be devoted to discussing the following question:

Does downwards pushing of a string / membrane or beam / plate imply downwards bending ?

In mathematical terms:

Does a maximum principle hold for the solution u that describes the deflection of the string / membrane or beam / plate?

i.e., is the relation $\max_{\overline{\Omega}} u = \max_{\partial\Omega} u$ satisfied?

In other words: *Does the solution u attain its maximum on the boundary?*

ON THE UNIQUE SOLVABILITY OF THE CAUCHY BOUNDARY VALUE PROBLEM FOR A HYPERBOLIC EQUATION IN THE ABSENCE OF INITIAL DATA

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Calculation of the average coefficient of magnetic induction in an Ω medium with a fine-grained metal-dielectric structure is encountered in many applied problems. Including in those cases when at some points in time it is possible to measure the change in the electromagnetic field and the gradients of its components at the boundary of the Ω domain, but it is impossible to measure this field inside Ω even at some point in time. In this case, the problem of calculating the averaged magnetic induction coefficient leads to the search for a solution to a nonclassical boundary value problem for a hyperbolic equation in the absence of initial data, but with the measured values of the electromagnetic field and the gradients of its components at the boundary or part of the boundary of the Ω domain. If the Ω domain is one-dimensional, the solution to such a nonclassical problem is the solution of some integral equation of the second kind, which is uniquely solvable in the class of smooth functions.

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STRONG EDGE COLORINGS OF HAMMING GRAPHS

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An *edge coloring* of a graph G is a mapping $\phi : E(G) \to \mathbb{N}$. The edge coloring ϕ is called *strong* if $\phi(e) \neq \phi(e')$ for any two edges e and e' that are distance at most one apart. The minimum number of colors needed for a strong edge coloring of a graph G is called strong chromatic index of G and denoted by $\chi'_s(G)$. Strong edge colorings of graphs were introduced by Fouquet and Jolivet [1]. Clearly, $\chi'_s(G) \geq \Delta(G)$ for any graph G. In 1985, during a seminar in Prague, Erdős and Nešetřil put forward the following conjecture.

Conjecture 1. *For every graph G with maximum degree Δ,*

$$\chi'_s(G) \leq \begin{cases} \frac{5}{4}\Delta^2, & \text{if } \Delta \text{ is even,} \\ \frac{1}{4}(5\Delta^2 - 2\Delta + 1), & \text{if } \Delta \text{ is odd} \end{cases}$$

Erdős and Nešetřil provided a construction showing that Conjecture 1 is tight if it's true. In 1997, using probabilistic method, Molloy and Reed [3] showed that $\chi'_s(G) \leq 1.9982\Delta$ for a graph G with sufficiently large Δ. The currently best known upper bound for a graph G is 1.932Δ, due to Bruhn and Joos [2].

The Hamming graph $H(n, m)$ is the Cartesian product of n copies of the complete graph K_m $(n, m \in \mathbb{N})$.

Theorem 1. *Let $H(n, m)$ be a Hamming graph with $m \geq 2$. Then*

$$\frac{(2n-1)m(m-1)}{2} \leq \chi'_s(H(n, m)) \leq \frac{nm^2(m-1)}{2} \tag{1}$$

and the upper bound is sharp for $m = 2$.

Theorem 2. *Let $H(n, 3)$ be a Hamming graph. Then*

$$6n - 3 \leq \chi'_s(H(n, 3)) \leq \frac{15n}{2} - 6, \text{if } n \text{ is even,} \tag{2}$$

and

$$6n - 3 \leq \chi'_s(H(n, 3)) \leq \frac{15(n-1)}{2} + 3, \text{if } n \text{ is odd.} \tag{3}$$

and the upper bound is sharp for $n = 3$.

References:

1. J.L. Fouquet, J.L. Jolivet. Strong edge-colourings of graphs and applications to multi-k-gons. *Ars Comb.* 16 (A) (1983), 141–150.

2. H. Bruhn, F. Joos. A stronger bound for the strong chromatic index. *Electronic Notes Discrete Math.* 49 (2015), 277–284.

3. M. Molloy, B. Reed, A bound on the strong chromatic index of a graph, *J. Comb. Theory Ser.*, B 69 (1997), 103–109.

THE RIEMANN SPACES RELATED WITH NAVIER-STOKES EQUATIONS

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For solving Navier-Stokes equations

$$\frac{\partial}{\partial t}\vec{V}+\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}=\mu\Delta\vec{V}+\vec{\nabla}P(\vec{x},t),\quad \vec{\nabla}\cdot\vec{V}=0, \tag{1}$$

where $\vec{V}(\vec{x},t)$ is the fluid velocity, $P(\vec{x},t)$ is the pressure and μ is the viscosity of liquid, are used the conditions of their compatibility in the form of conservation laws $\frac{\partial}{\partial y}H(\vec{x},t)-\frac{\partial}{\partial x}E(\vec{x},t)=0$, $\frac{\partial}{\partial z}H(\vec{x},t)-\frac{\partial}{\partial x}B(\vec{x},t)=0$, $\frac{\partial}{\partial z}E(\vec{x},t)-\frac{\partial}{\partial y}B(\vec{x},t)=0$.

The $6D$-space with the metric

$$ds^2=-2\,B(t,x,y,z)dt\,dv+2\,E(t,x,y,z)dt\,dw+2\,H(t,x,y,z)dv\,dw-$$

$$-2\int\frac{\partial}{\partial y}H(t,x,y,z)dz\,dw^2+dt\,dx+dv\,dy+dw\,dz. \tag{2}$$

is applied to construct an example of solutions of the NS-equations.

In particular case $B(\vec{x},t)=\frac{\partial^2}{\partial z^2}Q(t,x,y,z)$,$H(t,x,y,z)=\frac{\partial^2}{\partial x\partial z}Q(t,x,y,z)$, $E(t,x,y,z)=\frac{\partial^2}{\partial y\partial z}Q(t,x,y,z)$, the metric (2) is simplified and solutions of the NS-equations are expressed in terms of the function $P(x)$ that satisfies the Monge-Ampere equation.

A more general approach to obtain the solutions of the NS-equations is connected by using the $14D$ space with condition on the Ricci curvature $R_{ik}=0$ on solutions of the NS system.

Theorem. *The metrics*

$$ds^2=2\,dxdu+2\,dydv+2\,dzdw+(-W(\vec{x},t)w-V(\vec{x},t)v-U(\vec{x},t)u)\,dt^2+2\,dtdp+$$

$$\left(-U(\vec{x},t)p-u\,(U(\vec{x},t))^2-uP(\vec{x},t)+w\mu\,\frac{\partial}{\partial z}U(\vec{x},t)-wU(\vec{x},t)W(\vec{x},t)\right)d\eta^2+$$

$$\left(v\mu\,\frac{\partial}{\partial y}U(\vec{x},t)-vU(\vec{x},t)V(\vec{x},t)+u\mu\,\frac{\partial}{\partial x}U(\vec{x},t)\right)d\eta^2+2\,\eta d\xi+$$

$$\left(-V(\vec{x},t)p-vP(\vec{x},t)-v\,(\vec{x},t))^2-V(\vec{x},t)W(\vec{x},t)w+v\mu\,\frac{\partial}{\partial y}V(\vec{x},t)\right)d\rho^2+$$

$$\left(u\mu\,\frac{\partial}{\partial x}V(\vec{x},t)-uU(\vec{x},t)V(\vec{x},t)\right)d\rho^2+2\,d\rho d\chi+$$

$$\left(-uU(\vec{x},t)W(\vec{x},t)-w\,(W(\vec{x},t))^2-wP(\vec{x},t)+w\mu\,\frac{\partial}{\partial z}W(\vec{x},t)\right)dm^2+$$

$$\left(v\mu\,\frac{\partial}{\partial y}W(\vec{x},t)-vV(\vec{x},t)W(\vec{x},t)+u\mu\,\frac{\partial}{\partial x}W(\vec{x},t)-W(\vec{x},t)p\right)dm^2+2\,dmdn$$

is the Ricci-flat on solutions of the equations (1).

The Cartan Invariants $K=R(a,j,b,^i)R(c,i,d,^j)A^aA^bA^cA^d$ of the metrics are used to construct solutions of the equations NS and their properties are discussed.

CONFORMAL η-RICCI SOLITONS WITHIN THE FRAMEWORK OF ϵ-KENMOTSU MANIFOLDS

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Scientists and mathematicians across many disciplines have always been fascinated to study indefinite structures on manifolds. The present paper is to deliberate the class of indefinite structure called ϵ-Kenmotsu manifolds which admits conformal η-Ricci soliton. In recent days geometric flows have emerged as significant tools to study various geometrical structures and also in general relativistic perfect fluid spacetime. Conformal Ricci flow is an evolutionary intrinsic geometric flow on smooth Riemannian manifold which deforms the underlying smooth Riemannian metric. Conformal Ricci solitons are self-similar solutions of the conformal Ricci flow. Here, we have studied some special types of Ricci tensor in connection with the conformal η-Ricci soliton of ϵ-Kenmotsu manifolds. We also have delineated some curvature conditions admitting conformal η-Ricci solitons on ϵ-Kenmotsu manifolds. Lastly we have developed an example for the existence of conformal η-Ricci soliton in ϵ-Kenmotsu manifold.

ON THE PALETTE INDEX OF SIERPIŃSKI-LIKE GRAPHS

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For a proper edge coloring of a graph G, the palette of a vertex $v \in V(G)$ is the set of colors appearing on the edges incident to the vertex v. The palette index $\check{s}(G)$ of G is the minimum number of palettes occurring in G among all proper edge colorings. In this paper we examine the palette index of Sierpński-like graphs. Namely, we determine the palette index of Sierpiński graphs $S(n,k)$ where k is even, $k = 3$, or $n = 2$ and $k \equiv 3 \pmod 4$ and we give an upper and a lower bound for the remaining cases. Additionally, we determine the palette index of graphs $S^+(n,k)$ when k is odd or $k \equiv 0 \pmod 4$ and we give an upper and a lower bound for the remaining cases. Moreover, we completely determine the palette index of graphs $S^{++}(n,k)$.

Theorem 1. *For every integer $n > 1$, we have $\check{s}(S(n,3)) = 3$.*

Theorem 2. *For integer $n > 1$ and $k > 1$, we have*

$$\check{s}(S^+(n,k)) = \begin{cases} 1 & \text{if } k \equiv 1 \pmod 2, \\ 3 & \text{if } k \equiv 0 \pmod 4, \end{cases}$$

and

$$3 \leq \check{s}(S^+(n,k)) \leq 4 \quad \text{if} \quad k \equiv 2 \pmod 4.$$

Proposition 3. *For integers $n > 1$ and $k > 1$, we have $\check{s}(S^{++}(n,k)) = 1$.*

LYAPUNOV FUNCTIONS AND CHAIN RECURRENT RELATIONS IN DISCRETE INCLUSIONS

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Charles Conley (1978) has introduced a very weak form of recurrence for flows, which he called "chain recurrence". Roughly speaking, a point is *chain recurrent* if it returns to itself by following the flow for an arbitrarily long time, making arbitrarily small jumps, or errors, along the way. He then proved the existence of what he termed as *complete Lyapunov function*, a real-valued function strictly decreasing everywhere except on components of the chain-recurrent set, where it is constant. This result nowadays is called "The Fundamental Theorem of Dynamical Systems" (D. Norton, 1995).

The Conley theory has been generalized for dispersive semiflows by I. U. Bronstein and A. Y. Kopanski (1984-85), and for relations by E. Akin (1993).

We are concerned with inclusions $x_{n+1} \in \mathcal{F}(x_n), n \in \mathbb{Z}$, with upper semicontinuous $\mathcal{F}$ in a complete metric space (X, d). A sequence $(x_n), n \in \mathbb{Z}$, which satisfies this inclusion is called a *chain*. If instead of inclusion, the inequality $\varrho(x_{n+1}, \mathcal{F}(x_n)) \leq \delta$ is satisfied by $x = x_0, x_1, \ldots, x_n = y$, then one speaks about a δ-*chain* connecting x and y (here $\varrho(a, B)$ denotes the Hausdorff semidistance).

The *chain-recurrent*, or *Conley's relation* $\mathcal{CF}$, is defined as follows: $y \in \mathcal{CF}(x) \iff \forall \varepsilon > 0$ there is an ε-chain beginning at x and ending at y. This relation is closed and transitive (E. Akin, 1993). The chain-recurrent set $|\mathcal{CF}|$ defined as the set of points x, such that $x \in \mathcal{CF}(x)$, is partitioned by the equivalence relation $\mathcal{CF} \cap (\mathcal{CF})^{-1}$ into equivalence classes, coined as basic sets.

Our aim is twofold: to generalize the Conley theory for discrete inclusions, and to adapt it to the subdynamics on weakly invariant (viable) subsets. A nonempty subset $\Lambda \subset X$ is said to be *viable on* $\mathbb{Z}$, if for every $x \in \Lambda$ there exists at least a chain $x_n, n \in \mathbb{Z}$, such that $x_0 = x$ and $x_n \in \Lambda$ for all $n \in \mathbb{Z}$.

In the particular case when $\mathcal{F}$ consists of a finite family of affine maps, whose linear parts satisfy the "generalized cone condition", the maximal compact viable set coincides with the closure of the subset of periodic points, and the dynamics on it is complex enough, say, it is mixing (GG, 2017).

A continuous real valued function L on X is called a *Lyapunov function* for the inclusion $x_{n+1} \in \mathcal{F}(x_n)$ $(n \in \mathbb{Z})$, if $y \in \mathcal{F}(x) \Rightarrow L(y) \leq L(x)$. If, in addition, $L(x) = L(y)$ if and only if x and y lie in the same basic set of the chain recurrent set $|\mathcal{CF}|$, then L is said to be *complete*.

Given a compact viable subset $\Lambda \subset X$, we prove that there exists a complete Lyapunov function with respect to Conley's relation $\mathcal{C}(\mathcal{F}|\Lambda)$ of the restriction of the initial relation $\mathcal{F}$ on the viable subset Λ.

In other words, the complete Lyapunov function decreases strictly beyond the chain recurrent subset of Λ and separates basic sets in $\mathcal{C}(\mathcal{F}|\Lambda)$. This means that the discrete inclusion behaves as a laminar flow behind the chain recurrent set, while the entire complexity is concentrated on the last set.

EXTENSION OF ORTHOSYMMETRIC MULTILINEAR OPERATORS

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Let E, F be vector lattices. We say that a multilinear operator

$$T : E^n = E \times E \times ... \times E \to F$$

is an orthosymmetric multilinear operator if $T(x_1, ..., x_n) = 0$ for all $x_1, x_2, ..., x_n \in E$ such that $|x_i| \wedge |x_j| = 0$ for some pair of indices $1 \leq i, j \leq n$.

Theorem. *n-th order adjoint of an orthosymmetric multilinear mapping on n-th product of a vector lattice is an orthosymmetric multilinear mapping.*

In this talk, we present n-th order adjoint of a multilinear mapping on product of vector lattices and by using this construction we prove the theorem.

TWO EIGENVALUE-TYPE PROBLEMS IN EXTERIOR DOMAINS

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Let $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ be a simply connected bounded domain, containing the origin, with C^2 boundary denoted by $\partial\Omega$. Denote by $\Omega^{\text{ext}} := \mathbb{R}^N \setminus \overline{\Omega}$ the exterior of Ω. We consider the two eigenvalue-type problems in Ω^{ext}.

Firstly, we present some properties of the set of parameters λ for which the following nontypical eigenvalue problem has nontrivial solutions

$$\begin{cases} -\Delta_p u = \lambda f(x, u(x)), & \text{in} \quad x \in \Omega^{\text{ext}}\,, \\ u(x) = 0, & \text{for} \quad x \in \partial\Omega\,, \\ u(x) \to 0 & \text{as} \quad |x| \to \infty\,, \end{cases} \tag{1}$$

where $p \in (1, N)$, $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ stands for the p-Laplace operator, and $f : \Omega^{\text{ext}} \times \mathbb{R} \to \mathbb{R}$, $f = f(x, t)$, is a function which for each $x \in \Omega^{\text{ext}}$ is given by

$$f(x,t) = \begin{cases} h(x,t), & \text{if} \quad t \geq 0, \\ K(x)|t|^{p-2}t, & \text{if} \quad t < 0\,, \end{cases}$$

with $K : \Omega^{\text{ext}} \to (0, \infty)$ a function having the property that $K \in L^\infty(\Omega^{\text{ext}}) \cap L^{N/p}(\Omega^{\text{ext}}) \cap L^1(\Omega^{\text{ext}})$. More precisely, we show that if the following conditions are satisfied

(C1) there exists a positive constant $C \in (0, 1)$ such that $|h(x, t)| \leq CK(x)t^{p-1}$, for any $t \geq 0$ and a.e. $x \in \Omega^{\text{ext}}$;

(C2) there exists $t_0 > 0$ such that $H(x, t_0) := \int_0^{t_0} h(x, s)\mathrm{d}s > 0$ for a.e. $x \in \Omega^{\text{ext}}$;

(C3) $\lim\limits_{t\to\infty} \dfrac{h(x,t)}{K(x)t^{p-1}} = 0$, uniformly in x;

then problem (1) possesses, on the one hand, a continuous family of parameters λ for which the problem has nontrivial solutions and, on the other hand, at least one more parameter λ which is isolated in the set of parameters for which problem (1) has nontrivial solutions.

Secondly, we consider the perturbed eigenvalue problem

$$\begin{cases} -\Delta_p u - \Delta_q u = \mu K(x)|u|^{p-2}u, & \text{for} \quad x \in \Omega^{\text{ext}} \\ u(x) = 0, & \text{for} \quad x \in \partial\Omega \\ u(x) \to 0, & \text{as} \quad |x| \to \infty, \end{cases} \tag{2}$$

where $p, q \in (1, N)$, $p \neq q$ and K is a positive weight function defined on Ω^{ext} having the property that $K \in L^\infty(\Omega^{\text{ext}}) \cap L^{N/p}(\Omega^{\text{ext}})$. We show that the set of parameters μ for which problem (2) possesses nontrivial solutions is exactly an unbounded open interval.

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THE PRECISE ESTIMATION OF LIMIT CYCLES NUMBER FOR PLANAR AUTONOMOUS SYSTEM

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For smooth autonomous systems

$$\frac{dx}{dt} = P(x, y), \ \frac{dy}{dt} = Q(x, y), \tag{1}$$

the problem of precise non-local estimation of the limit cycles number in a simply-connected domain Ω of a real phase plane containing one or three equilibrium points with a total Poincaré index $+1$ is considered.

To solve this problem, we present new approaches that are based on a sequential two-step construction of Dulac or Dulac-Cherkas or their modifications [1,2], which provide the closed transversal curves decomposing the simply-connected domain in simply-connected subdomains and doubly-connected subdomains.

Theorem 1. *Suppose that in a simply-connected domain Ω system (1) has the unique anti-saddle equilibrium point O, and Ψ is the Dulac-Cherkas function of system (1) with $k < 0$ in the domain Ω, where the set $W = \{(x, y) \in \Omega : \Psi(x, y) = 0\}$ consists of s embedded ovals ω_i surrounding the point O. Then, system (1) has exactly one limit cycle in each of the $s-1$ ring-shaped subdomains Ω_I that are bounded by neighboring ovals ω_i and ω_{i+1} and can have at most 1 limit cycle in the domain Ω_s.*

To obtain a precise estimate of the number of limit cycles, it is necessary to detect existence or absence of a limit cycle in the region Ω_s. Our idea is based on the following result.

Theorem 2. *Suppose that the assumptions of Theorem 1 are valid and system (1) has a closed transversal curve V that lies in a doubly connected subdomain Ω_s that surrounds the external oval of the curve W, two of them forming the boundary of a ring-shaped domain $\tilde{\Omega}_s \subset \Omega_s$. If the trajectories of system (1) enter, as t increases, the interior of the domain $\partial\tilde{\Omega}_s$ from outside (or vice versa) through the boundary $\tilde{\Omega}_s$, then there exists the unique stable (or unstable) limit cycle of system (1), in the subdomain $\tilde{\Omega}_s$ and system (1) has exactly s limit cycles in the domain Ω in total.*

Our approaches for the construction of the transversal curve V, satisfying to the requirements of Theorem 2, are presented in [1, 2].

The developed approaches are efficiently applied to several polynomial systems of Linard type, for which it is proved that there exist a limit cycle in each of the doubly-connected subdomains.

References:

1. A.A. Grin, A.V. Kuzmich. Dulac-Cherkas criterion for exact estimation of the number of limit cycles of autonomous systems on a plane. *Differential Equations*, **53** (2017), 171–179.
2. A.A. Grin. Transversal curves for finding the exact number of limit cycles. *Differential Equations*, **56** (2020), 415–425.

ON HYPERBOLIC 3-MANIFOLDS WITH RIGHT-ANGLED FUNDAMENTAL POLYHEDRON

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Our communication is devoted to the construction of a countable series of hyperbolic 3-manifolds for which a fundamental polyhedron P has right angles, is unbound, but has finite volume. Polyhedra P have both proper vertices and infinitely remote vertices. First we construct a countable series of polyhedra with all dihedral angles of $\pi/2$. Then we give motions that identify faces of these polyhedra and show that generated by these motions groups Γ are torsion-free. Factorizing H^3 by groups Γ we obtain non-compact 3-manifolds with finite volume.

The polyhedra P contain $4p$ faces which are hexagons with two infinitely remote and four proper vertices. There are two faces which are regular $4p-gons$ with all the vertices being on the absolute (upper and bottom bases). Also a polyhedron P has $4p$ faces which are triangles with one proper vertex and two improper vertices and they are adjacent to bottom base, and P has $4p$ analogous triangular faces adjacent to upper base of the polyhedron where $p = 1, 3, ...$ Denote by α_i hexagonal faces of the polyhedron P, by β_i triangular faces adjacent to upper base and by γ_i triangular faces adjacent to the bottom base of polyhedron P where $i = 1, 2, ..., 4p$. Let τ_1 denote the bottom base and τ_2 denote the upper base of the polyhedron P.

Indicate motions which identify faces of polyhedra. Every face α_i with odd number is identified, by ϕ_i a helical motion with rotation angle of π, with the opposite face α_{i+2p}. Every face α_i with even number is identified by ϕ_i a translation with the opposite face. Every face γ_i is identified by a translation φ_i with the face β_i, where $i = 1, 2..., 4p$. Finally face τ_1 is identified with the face τ_2 by a helical motion δ with rotation angle of π.

We show that under these conditions the group Γ generated the motions ϕ_i, φ_i and δ, $i = 1, 2, ..., 4p$, is torsion-free and, therefore, factorizing the hyperbolic space H^3 by the groups Γ we obtain a countable series of manifolds $M = H^3/\Gamma$. These manifolds are non-compact, but have finite volume, and all the dihedral angles of fundamental polyhedron for these manifolds are equal to $\pi/2$.

AN ALGORITHM TO CONSTRUCT SOME KINDS OF PLANAR FRACTALS

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A hyperbolic Iterated Function System (IFS) is a finite collection of contractions. M. Barnsley (1988) introduced the notion of Iterated Function System with condensation (IFSC), which consists of a hyperbolic IFS, completed with a constant compact-valued mapping (a "zero" set-valued contraction). This idea generated many new fractals, some of them becoming famous.

In [1] a question was formulated: *Is it possible to reduce the construction of the attractor of a hyperbolic IFS with condensation to the construction of the attractor of an appropriate standard hyperbolic IFS?*

We proved [2] that any finite union of convex compacta in $\mathbb{R}^n$ may be represented as the attractor of a standard hyperbolic IFS. Moreover, if such a union is added to a hyperbolic IFS as the condensation set, then the attractor of this IFS with condensation can be represented as the attractor of an appropriate hyperbolic IFS.

Earlier, we have presented [3] an algorithm to construct a hyperbolic IFS, whose attractor is an arbitrary convex compact set in the plane.

Based on these results we show here an algorithm to construct the corresponding hyperbolic IFS for such an IFS with condensation in the plane.

We use the "random" method to construct fractals, which is based on some results of set-valued dynamics, which provide conditions that a unique invariant set (attractor) of such dynamics exists. Moreover, the most of orbits are dense in the attractor, and this fact makes possible to construct the attractor with a sufficiently high approximation.

Although now there are many computer programmes for constructing fractals, this algorithm may be also of interest, due to a large class of IFSs with condensation (and not only), for which it can be applied. As a consequence, a large class of new fractals can be constructed using the computer.

References:

1. L. Gadomski, V. Glavan, V. Guţu. Playing Chaos Game on the Pythagoras Tree with CAS *Mathematica*. In: *Computer Algebra Systems in Teaching and Research. Differential Equations, Dynamical Systems and Celestial Mechanics. L. Gadomski et al. (Eds.).* Wydawnictwo Collegium Mazovia, Siedlce, 2011, 38–45.
2. V. Guţu. Sums of convex compacta as attractors of hyperbolic IFS's. *Topol. Methods Nonlinear Anal.*, **54**(2B) (2019), 967–978.
3. V. Guţu. How to construct compact convex spots on the plane using CAS *Mathematica. Computer Algebra Systems in Teaching and Research,* Siedlce, **IV**(1) (2013), 16–23.

A COMPARATIVE ANALYSIS OF THE CALCULATION TIME FOR THE DATA PARALLELIZATION IN BIMATRIX GAMES

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We consider the bimatrix game in the following strategic form $\Gamma = \langle I, J, A, B \rangle$, where $A = ||a_{ij}||_{i\in I}^{j\in J}$, $B = ||b_{ij}||_{i\in I}^{j\in J}$ are the payoff matrices of the players and denote by $NE[\Gamma]$ the set of all equilibrium profiles in the game.

Based on the definition, it is easy to develop the following algorithm for determining the Nash equilibrium profiles in bimatrix games.

1. For any fixed column $j \in J$, $i^*(j) = Arg \max\limits_{i\in I} a_{ij}$ is determined. Under algorithmic aspect it can be as follows: for any column j of the matrix A all maximum elements of this column are highlighted.

2. For any fixed row $i \in I$, $j^*(i) = Arg \max\limits_{j\in J} b_{ij}$ is determined. Under algorithmic aspect it can be as follows: for any row i of the matrix B all maximum elements on this row are highlighted.

3. The function graph of the application i^* from step 1) is built: $gr\_i^* = \{(i,j) : i = i^*(j), \forall j \in J\}$ and of the application j^* from step 2) is built as well: $gr\_j^* = \{(i,j) : j = j^*(i), \forall i \in I\}$. The equilibrium profiles are all the profiles belonging to the intersection of the two given function graphs: $NE = gr\_i^* \cap gr\_j^*$. From an algorithmic point of view it can be done as follows: we look for all highlighted elements in the matrices A and B and the indices of the elements whose positions coincide both in matrix A and in matrix B will be the equilibrium profiles.

We make an analysis of the possibilities of the symbolic calculation system Mathematics for the elaboration of parallel programs on the DMM type

parallel system. A parallel algorithm is developed for determining Nash equilibrium profiles in pure strategies for bimatrix games. For this algorithm, parallel programs are developed using the Wolfram Mathematica system and MPI programming models, in which different ways of distributing the calculations on cores (for Mathematica system), process (for MPI programing) and different ways of parallelization at the data level are performed. A comparative analysis of the calculation time for the developed programs is performed.

PARALLEL ALGORITHM FOR SOLVING 2D BLOCK-CYCLIC PARTITIONED BIMATRIX GAMES

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Contemporary decision-making problems are very complex and require the processing of a very large volume of data. In order to solve such problems in real time, parallel algorithms are built, and then implemented on various types of parallel computing systems. For parallel data processing we must use the ways of dividing, partitioning (sharing) and distributing data. According to the two dimensional block cyclic data distribution scheme, a $m \times n$ dense matrix is first decomposed into m_A by n_A blocks starting at its upper left corner. These blocks are then uniformly distributed in each dimension of the Process Grid. Thus, every process owns a collection of blocks, which are locally and contiguously stored in a two-dimensional column major array.

We consider the bimatrix game in the following strategic form $\Gamma = \langle I, J, A, B \rangle$, where $A = ||a_{ij}||_{i \in I}^{j \in J}$, $B = ||b_{ij}||_{i \in I}^{j \in J}$ are the payoff matrices of the players and denote by $NE[\Gamma]$ the set of all equilibrium profiles in the game Γ. The global matrices A and B are divided into submatrices and are distributed to the processes from the process grid. So we obtain the series of bimatrix subgames in complete and perfect information in the following strategic form $\Gamma_{(c,l)} = \langle I_{(c,l)}, J_{(c,l)}, A_{(c,l)}, B_{(c,l)} \rangle$ and denote by $NE\left[\Gamma_{(c,l)}\right]$ the set of all Nash equilibrium profiles of the bimatrix subgame $\Gamma_{(c,l)} = \langle I_{(c,l)}, J_{(c,l)}, A_{(c,l)}, B_{(c,l)} \rangle$.

Let's analyse the following problem: if in parallel for all processes (c, l) the Nash equilibrium profile $\left(i^*_{(l,c)}, j^*_{(l,c)}\right) \in NE\left[\left(A_{(l,c)} B_{(l,c)}\right)\right]$ was determined, then how can one construct the equilibrium profile $(i^*, j^*) \in NE[\Gamma]$ without further solving some optimization problems.

Let's analyse the case when there are (l, c) processes in the process grid, so that $\left(i^*_{(l,c)}, j^*_{(l,c)}\right) \in NE\left[\left(A_{(l,c)}, B_{(l,c)}\right)\right]$ but $\left(\varphi_{(l,c)}\left(i^*_{(l,c)}\right), \psi_{(l,c)}\left(j^*_{(l,c)}\right)\right) \notin$

$NE[\Gamma]$. Here $\varphi_{(l,c)}$ and $\psi_{(l,c)}$ are the applications which determine the correspondence between the "local indices" of the elements of the local matrices $A_{(l,c)}$, $B_{(l,c)}$ and the "global indices" of the elements of the global matrices A and B.

Theorem. *Supposing for a given $(l,c) \in L \times C$ process we found the strategy profile $\left(i^*_{(l,c)}, j^*_{(l,c)}\right) \in NE\left[\left(A_{(l,c)}, B_{(l,c)}\right)\right]$. Assume that*

1. *for any fixed c and any $\widetilde{l} \neq l$ such that $(\widetilde{l}, c) \in L \times C$ the conditions $a_{i^*_{(l,c)}, j^*_{(l,c)}} \geq a_{i^*_{(\widetilde{l},c)} j^*_{(l,c)}}$ are fulfilled;*
2. *for any fixed l and any $\widetilde{c} \neq c$ such that $(l, \widetilde{c}) \in L \times C$ the conditions $b_{i^*_{(l,c)}, j^*_{(l,c)}} \geq b_{i^*_{(l,c)} j^*_{(l,\widetilde{c})}}$ are fulfilled.*

Then $\left(\varphi_{(l,c)}\left(i^*_{(l,c)}\right), \psi_{(l,c)}\left(j^*_{(l,c)}\right)\right) \in NE[\Gamma]$.

Here $i^*_{(\widetilde{l},c)} \equiv i^*_{(\widetilde{l},c)}(j^*_{(l,c)}) = \arg \max\limits_{i_{(\widetilde{l},c)} \in I_{(\widetilde{l},c)}} a_{i_{(\widetilde{l},c)} j^*_{(l,c)}}$ and $j^*_{(l,\widetilde{c})} \equiv j^*_{(l,\widetilde{c})}(i^*_{(l,c)}) = \arg \max\limits_{j_{(l,\widetilde{c})} \in J_{(l,\widetilde{c})}} b_{i^*_{(l,c)} j_{(l,\widetilde{c})}}$.

MAIA'S FIXED POINT THEOREMS FOR DISCONTINUOUS MAPPINGS

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We establish new fixed point theorems for Maia's fixed point theorem in the setting of a space with a distance, more precisely when both metrics are replaced with two distance. We also indicate some particular cases of our main results and present some examples to illustrate the theoretical results and show that our generalizations are effective.

Theorem. *Let X be a nonempty set, d and ρ are two H-distance on X and $T : X \to X$ be a mapping. We suppose that:*

(i) $d(x,y) \leq \rho(x,y)$, $\forall x, y \in X$;

(ii) (X,d) is a complete H-distance space;

(iii) $T : (X,d) \to (X,d)$ is a Kannan contraction in a H-distance space;

(iv) $T : (X,\rho) \to (X,\rho)$ is a Kannan contraction in a H-distance space.

Then the Picard iteration of the point x is convergent in (X,d).

If, additionally the limit $\bar{x}$ of the Picard sequence is a fixed point of T, then $\bar{x}$ is the unique fixed point of T.

Working in the general setting of a complete H-distance space, we obtained significant generalizations of Maia's fixed point theorem in usual metric spaces.

ON THE COMPUTATIONAL APPROACH OF THE CHUA DYNAMICAL SYSTEM IN A MODIFIED VERSION

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The Chua circuit is the simplest electronic circuit which exhibits chaos, and few bifurcation phenomena as well, as verified from numerous laboratory experiments, computer simulations, and also rigorous mathematical analysis.

The behaviour of Chua's circuit system has been studied in thousands of studies and articles, and it is still surprising to find that it can generate a lot of topologically distinct attractors. Although this system is totally different and was obtained in another way, its chaotic behaviour reminds us of the Lorenz system. For these reasons, the topic of studying the stability of the Chua system remains actual and important both from the mathematical (theoretical) and engineering (practical) standpoint.

The present paper aims to analyse the Chua modified model. Namely, the cubic modified model is taken into account. Reformulating this model as a second order differential system, its approach from analytical and computational standpoint becomes gives very interesting results. The phase-portrait of the model is taken into account, for convenient parameter values according to recent literature analysis. The simulations obtained are further correlated to the analytical approach.

REDUCTION PRINCIPLE FOR INSTABILITY OF ORDINARY DIFFERENTIAL EQUATIONS

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As is well known [1-4], the aim of the reduction principle is to present conditions enabling one to replace the stability problem concerning the entire space by a stability problem with respect to a closed subspace. At first glance, it seems that the reduction principle is trivial for the instability property. However, here we can single out a class of systems for which the problem of reducing the property of instability to its consideration on a subspace has a nontrivial meaning.

Let a system of differential equations

$$\dot{x} = f(x,t),\ f(0,t) = 0 \in G \subset \mathbb{R}^n,\ t \geq 0, \tag{1}$$

be given, where G is an open neighborhood of the origin and $f : G \times \mathbb{R}^+ \to \mathbb{R}^n$ is a continuous function. Suppose that $f(x,t)$ satisfies the Lipschitz condition with respect to x and for any pair $(x_0, t_0) \in G \times \mathbb{R}^+,\ x(x_0, t_0, t)$ is a solution of (1) such that $x(x_0, t_0, t_0) = x_0$. Let $L^+(x_0, t_0)$ is positive limit set of $x(x_0, t_0, t)$, $B_\varepsilon = \{x \in \mathbb{R}^n : ||x|| < \varepsilon\},\ \varepsilon > 0$, and $d(x,y) = ||x - y||\ \forall x, y \in \mathbb{R}^n$.

We formulate now the reduction principle for equilibrium instability.

Theorem. *Let Y be a closed subset of G and U be a neighborhood of Y. Suppose that, for system* (1)*, there exists a function $V \in \mathbf{C^1}(U \times \mathbb{R}^+, \mathbb{R}^+)$, such that for all $(x,t) \in U \times \mathbb{R}^+$ the following conditions hold:*

1) $|V(x,t| \leq d(Y,x)$;

2) $\dot{V}(x,t) \geq 0$;

3) $\forall \alpha > 0\ \exists p \in B_\alpha \backslash Y$ *and* $\exists \tau > 0$ *such that* $V(p,\tau) > 0$;

4) $x = 0$ *is attracting with respect to the largest quasi-invariant subset from*

$$E = \{x_0 \in B_\alpha \backslash Y,\ t_0 \in \mathbb{R}^+ : L^+(x_0, t_0) \neq \emptyset\}.$$

Then the zero solution of the system (1) *is unstable.*

Here $\dot{V}(x,t)$ is the derivative of $V(x,t)$ for (1).

Some assertions about instability in the framework of Lyapunov's Direct Method for ordinary differential equations follow from the theorem (see [5]).

References:

1. A. M. Lyapunov. *General Problem of the Stability of Motion.* Taylor and Francis Ltd., London, 1992.

2. V.A. Pliss. The reduction principle in the theory of stability of motion. *Bulletin of the USSR Academy of Sciences. Ser. Mathematics.* **28**(6) (1964), 1297–1324.

3. P. Seibert, J.S. Florio. On the Reduction to a Subspace of Stability Properties of Systems in Metric Space. *Annali di Matematica pura ed applicata*, CLXIX (IV) (1995), 291–320.
4. P. Seibert. Reduction Theorems for Stability of Systems in General Spaces. *Nonlinear Analysis. Theory, Methods and Applications*, **30**(7) (1997), 4675–4681.
5. B.S. Kalitin. Instability of Closed Invariant Sets of Semidynamical Systems: Method of Sign-Constant Lyapunov Functions. *Mathematical Notes*. **85**(3) (2009), 382–394.

ON NONLINEAR INTEGRO-DIFFERENTIAL OSKOLKOV-STOKES SYSTEM WITH VARIABLE EXPONENT OF NONLINEARITY

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Let $n \in \mathbb{N}$ and $T > 0$ be fixed numbers, $\Omega \subset \mathbb{R}^n$ be a bounded domain with the boundary $\partial\Omega$, $Q_{0,T} := \Omega \times (0,T)$. For every $\varepsilon > 0$ we seek weak solution $\{u^\varepsilon, \pi^\varepsilon\}$ of the Oskolkov-Stokes system

$$u_t^\varepsilon - \Delta u^\varepsilon + g(x,t)\,|u^\varepsilon|^{q(x)-2}u^\varepsilon +$$

$$+\Phi\left(\int\limits_\Omega \mathrm{Z}(x,t,y)u^\varepsilon(y,t)\,dy\right) + \nabla\pi^\varepsilon = F(x,t) \quad \text{in} \quad Q_{0,T}, \tag{1}$$

$$\varepsilon(\pi_t^\varepsilon - \Delta\,\pi^\varepsilon) + \operatorname{div} u^\varepsilon = f(x,t) \quad \text{in} \quad Q_{0,T}, \tag{2}$$

$$u^\varepsilon|_{\partial\Omega\times(0,T)} = 0, \tag{3}$$

$$\pi^\varepsilon|_{\partial\Omega\times(0,T)} = 0, \tag{4}$$

$$u^\varepsilon|_{t=0} = u_0(x) \quad \text{in} \quad \Omega, \tag{5}$$

$$\pi^\varepsilon|_{t=0} = \pi_0(x) \quad \text{in} \quad \Omega. \tag{6}$$

Here $u^\varepsilon = (u_1^\varepsilon, \ldots, u_n^\varepsilon) : Q_{0,T} \to \mathbb{R}^n$ and $\pi^\varepsilon : Q_{0,T} \to \mathbb{R}$. Also we consider the Stokes system

$$u_t - \Delta u + g(x,t)\,|u|^{q(x)-2}u +$$

$$+\Phi\left(\int\limits_\Omega \mathrm{Z}(x,t,y)u(y,t)\,dy\right) + \nabla\pi = F(x,t) \quad \text{in} \quad Q_{0,T}, \tag{7}$$

$$\operatorname{div} u = 0 \quad \text{in} \quad Q_{0,T}, \tag{8}$$

$$u|_{\partial\Omega\times(0,T)} = 0, \tag{9}$$

$$u|_{t=0} = u_0(x) \quad \text{in} \quad \Omega. \tag{10}$$

Here $u = (u_1, \ldots, u_n) : Q_{0,T} \to \mathbb{R}^n$ and $\pi : Q_{0,T} \to \mathbb{R}$.

We have proved the existence of the solution $\{u^\varepsilon, \pi^\varepsilon\}$ to problem (1)-(6), and the solution $\{u, \pi\}$ to problem (7)-(10). Moreover, we have shown the convergence $\{u^\varepsilon, \pi^\varepsilon\} \underset{\varepsilon \to 0}{\longrightarrow} \{u, \pi\}$ if $f \equiv 0$.

ON RECURSIVELY DIFFERENTIABLE QUASIGROUPS

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Recursively differentiable quasigroups have been defined in [1], in connection with recursive MDS-codes. If $C(n, f)$ is a recursive MDS-code of length n, defined by a binary quasigroup (Q, f), with $|Q| = q$, then $n \leq r_{max} + 3$ and $r_{max} \leq q - 2$, where r_{max} is the maximum order of recursive differentiability of (Q, f). It is known that $r_{max} = q - 2$ when q is a power of a prime number, while in a general case to find r_{max} for a given q is an open problem [1,2,3].

Let (Q, f) be a binary quasigroup and $s \geq 0$. The operation $f^{(s)}$, defined recursively as follows: $f^{(0)}(x, y) = f(x, y)$, $f^{(1)}(x, y) = f(y, f(x, y))$, $f^{(s)}(x, y) = f((f^{(s-2)}(x, y), f^{(s-1)}(x, y)$, $\forall s \geq 2$, is called the recursive derivative of order s of (Q, f). A quasigroup (Q, f) is called recursively differentiable of order r if $(Q, f^{(s)})$ is a quasigroup for all $s \leq r$. It is known that there exist recursively 1-differentiable binary quasigroups of every order $q \neq 2, 6$ and, possibly, $q \neq 14, 18, 26, 42$.

We consider an extension of finite quasigroups using pairwise disjoint transversals and give necessary conditions when the prolongation of a recursively differentiable quasigroup is recursively 1-differentiable. Also we give an algorithm for the construction of finite linear recursively differentiable binary quasigroups of higher order, based on the following statements.

Let consider on the set of integers Z the operation $f(x, y) = (n - k)x + y$, where $n \geq 3$, $1 \leq k \leq n - 1$. Then:

1. There exist $u_s, v_s, b_s, c_s \in Z$, such that $f^{(s)}(x, y) = u_s x + v_s y$, $v_s = nb_s + c_s$, $\forall s \geq 1$, where $c_1 = -k + 1$, $c_2 = -2k + 1$, $c_i = -kc_{i-2} + c_{i-1}$, $\forall i \geq 3$.

2. (Z_n, f) is a recursively r-differentiable quasigroup if $((n - k)v_s, n) = 1 = (c_s, n)$, for every $1 \leq s \leq r$;

3. (Z_{p^t}, f), where p is a prime and $t \geq 1$, is a recursively r-differentiable quasigroup if $(c_s, n) = 1$, for every $1 \leq s \leq r$.

References:

1. E. Couselo, S. Gonzalez, V. Markov, A. Nechaev. Recursive MDS-codes and recursively differentiable quasigroups. *Discret. Mat.*, **10**(2) (1998), 3–29. (In Russian)

2. E. Couselo, S. Gonzalez, V. Markov, A. Nechaev. Parameters of recursive MDS-codes. *Discrete Mathematics and Applications*, **10**(5) (2020), 433–454.

3. V. Izbash, P. Syrbu. Recursively differentiable quasigroups and complete recursive codes. *Commentat. Math. Univ. Carol.*, **45**(2) (2004), 257–263.

ALGEBRAIC VIEW OVER HOMOGENEOUS LINEAR RECURRENT PROCESSES

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The main goal of this presentation is to study the algebraic properties of the deterministic processes with dynamic represented by a homogeneous linear recurrence over the field $\mathbb{C}$. We delve into the subsets of $\mathbb{C}$ to see if the dynamic of the given process is also a homogeneous linear recurrence over given subset in certain conditions. The challenge appears when we get out from comfort zone, given by field properties.

So, we start with an overview of homogeneous linear recurrent processes over $\mathbb{C}$ and its subsets. We remind the main definitions and properties from [5], like generating vector, characteristic polynomial and minimality. Also, we formulate the minimization method based on matrix rank definition, which was theoretically grounded in [6].

Next, we go deeper into homogeneous linear recurrent processes over numerical rings. We formulate and prove necessary and sufficient conditions for a homogeneous linear recurrence over $\mathbb{C}$ to be also a homogeneous linear recurrence over a subfield or subring, like $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$ or an extension field of $\mathbb{Q}$.

After that, we are interested in recurrence criteria over sign-based ring subsets. We split the ring into two subsets, one containing the positive elements and the second containing the negative ones. Based on results from [1] it is shown that the recurrence criteria other these subsets are based on the number of positive real roots of the minimal characteristic polynomial over that ring and, in the most complex case when it is a single one, they are also based on the relationship of that positive real root with the rest of the roots.

The last part is dedicated to deterministic processes with dynamic represented by a Littlewood, Newman or Borwein homogeneous linear recurrence. Mainly, these are homogeneous linear recurrences over subsets of $\{-1, 0, 1\}$.

Several results are presented, based on the properties of Littlewood, Newman and Borwein polynomials developed in [2-4] and [7].

References:

1. M. Avendaño. Descartes' rule of signs is exact! *Journal of Algebra*, **324** (2010), 2884–2892.
2. P. Drungilas, J. Jankauskas, J. Šiurys. On Littlewood and Newman polynomial multiples of Borwein polynomials. *Math. Comp.*, **87** (2018), 1523–1541.
3. A. Dubickas. The divisors of Newman polynomials. *Fizikos ir matematikos fakulteto, Seminaro darbai, Šiaulių universitetas*, **6** (2003), 25–28.
4. A. Dubickas, J. Jankauskas. On Newman polynomials which divide no Littlewood polynomial. *Mathematics of Computation*, **Vol. 78, 265** (2009), 327–344.
5. A. Lazari. Algorithms for Determining the Transient and Differential Matrices in Finite Markov Processes. *Bulletin of the Academy of Science of RM, Matematica*, **No. 2(63)** (2010), 84–99.
6. A. Lazari, D. Lozovanu, M. Capcelea. *Dynamical deterministic and stochastic systems: Evolution, optimization and discrete optimal control.* CEP USM, Chişinău, 2015, 310 p. (In Romanian)
7. A. Odlyzko, B. Poonen. Zeros of polynomials with 0, 1 coefficients. *Enseign. Math. (2)*, **39** (1993), 317–348.

RELIABILITY OF THE NETWORKS WITH RANDOM NUMBER OF THE UNITS IN EACH SUBNET

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Our work focuses on the reliability of networks for the serial-parallel type versus the parallel-serial type when the number of units in each subnet is a Power series distributed (PSD) random variable and the lifetimes of the units are independent, identically distributed random variables (i.i.d.r.v.). General formulas for calculating the survival/reliability functions of such networks were obtained. This formulas shows that solving the problem of identifying the best network in terms of its reliability does not matter the lifetime c.d.f. $F(x)$ of each units in each subnetwork, the answer depending only of the number M and the probability distribution of the number of units in each of M subnetworks.

Sufficient conditions have been formulated for the serial-parallel network to always be more reliable than the parallel-serial network. Some examples have been illustrated graphically.

SURVIVAL MAXIMIZATION FOR TIME SERIES

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We consider the controlled discrete-time, continuous state stochastic process $\{X_n, n = 0, 1, \ldots\}$ defined by

$$X_{n+1} = \mu + \alpha X_n + bu_n + \epsilon_{n+1} \tag{1}$$

for $n \in \{0, 1, \ldots\}$, where $\mu, \alpha \in \mathbb{R}$, $b > 0$, $u_n \in \{-1, 0, 1\}$ is the control variable, and $\{\epsilon_1, \epsilon_2, \ldots\}$ are independent and identically distributed continuous random variables with zero mean and finite variance σ^2. Moreover, ϵ_{n+1} is independent of $\{X_0, \ldots, X_n\}$. The process $\{X_n, n = 0, 1, \ldots\}$ is therefore a controlled autoregressive process. If $\alpha \neq 0$, it is an AR(1) process; when $\alpha = 0$, it reduces to an AR(0) process, that is, a white noise process.

In particular, ϵ_{n+1} can have a Gaussian distribution, so that $\{\epsilon_1, \epsilon_2, \ldots\}$ is Gaussian white noise, or be uniformly distributed on the interval $[-c, c]$. If $\alpha \in (0, 1)$, then $\{X_n, n = 0, 1, \ldots\}$ can be considered as a discrete version of a controlled Ornstein-Uhlenbeck process.

Assume that $X_0 = x \in [-d, d]$ and define the first-passage time

$$T(x) = \inf\{n \geq 0 : |X_n| \geq d \mid X_0 = x\}.$$

Our aim is to find the control u_n^* that minimizes the expected value of the cost function

$$J(x) = \sum_{n=0}^{T(x)-1} (qu_n^2 + \lambda),$$

where $q > 0$ and $\lambda \neq 0$ are constants. Hence, when $\lambda > 0$, the optimizer tries to minimize the (expected) time spent by the controlled process in the continuation region $C := (-d, d)$, while the aim is to maximize the expected survival time in C when $\lambda < 0$. In both cases, the quadratic control costs must of course be taken into account. This type of problem is known as *homing*.

To solve homing problems in continuous time, one can make use of dynamic programming to obtain the equation satisfied by the *value function*

$$F(x) := \inf_{u(t)} E[J(x)],$$

where the infimum is over all admissible values of the control variable in the time interval $[0, T(x))$. In discrete time, the control $u(t)$ is replaced by u_n, for $n \in \{0, \ldots, T(x) - 1\}$. It can be shown that in the continuous homing problem,

it is sometimes possible to transform the non-linear partial differential equation satisfied by $F(x)$ into a linear equation which is actually the Kolmogorov backward equation satisfied by a certain mathematical expectation for the corresponding uncontrolled process. However, in discrete time it is not possible to reduce the optimal control problem to a purely probabilistic problem.

In this paper, first the dynamic programming equation satisfied by the value function $F(x)$ will be given. Then, the cases when the parameter α in Eq. (1) is equal to 0, 1/2 or 1 will be considered. The optimal control will be computed explicitly, either exactly or approximately, in particular problems.

INTERNAL CONTROLLABILITY OF PARABOLIC SYSTEMS WITH STAR AND TREE LIKE COUPLINGS

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We consider systems of parabolic equations coupled in zero order terms in a star-like or a tree-like shape, with an internal control acting in only one of the equations. We obtain local exact controllability to the stationary solutions of the system under hypotheses concerning the supports of the coupling coefficients. The key point is establishing appropriate Carleman estimates for the adjoint to the linearized system.

C^1 INTEGRABILITY VIA PERIODIC ORBITS

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These last years the Ziglin's and the Morales-Ramis' theories has been used for studying the non-meromorphic integrability of an autonomous differential system. In some sense the Ziglin's theory is a continuation of Kovalevskaya's ideas used for studying the integrability of the rigid body because it relates the non integrability of the considered system with the behavior of some of its non-equilibrium solutions as function of the complex time using the monodromy group of their variational equations. Ziglin's theory was extended to the so-called Morales-Ramis' theory which replace the study of the monodromy group of the variational equations by the study of their Galois differential group, which

is easier to analyze (see [6] for more details and the references therein). But as Ziglin's theory the Morales-Ramis' theory only can study the non-existence of meromorphic first integrals.

Kovalevskaya's idea and consequently Ziglin's and Morales-Ramis' theory go back to Poincaré (see Arnold [1]), who used the multipliers of the monodromy group of the variational equations associated to periodic orbits for studying the non integrability of autonomous differential systems. The main difficulty for applying Poincaré's non integrability method to a given autonomous differential system is to find for such an equation periodic orbits having multipliers different from 1.

It seems that this result of Poincaré was forgotten in the mathematical community until that modern Russian mathematicians (specially Kozlov) have recently publish on it (see [1, 4]).

We shall apply Poincaré's results for studying the C^1 integrability of the Lorenz system, the Rossler system, the Michelson system, the Hénon–Heiles Hamiltonian system and the Yang-Mills Hamiltonian system (see [2, 3, 5]).

References:

1. V.I. Arnold, Forgotten and neglected theories of Poincaré, *Russian Math. Surveys* **61** No. 1 (2006), 1–18.
2. L. Jiménez and J. Llibre, Periodic orbits and non integrability of Henon-Heiles systems, *J. of Physics A: Math. Gen.* **44** (2011), 205103–14pp.
3. L. Jiménez and J. Llibre, Periodic orbits and non–integrability of generalized classical Yang–Mills Hamiltonian systems, *J. of Math. Phys.* **52** (2011), 032901, 9pp.
4. V.V. Kozlov, Integrability and non-integrability in Hamiltonian mechanics, Russian Math. Surveys **38** No. 1 (1983), 1–76.
5. J. Llibre and C. Valls, On the C^1 non-integrability of differential systems via periodic orbits, European J. of Applied Mathematics **22** (2011), 381–391.
6. J.J. Morales-Ruiz, *Differential Galois Theory and non-integrability of Hamiltonian systems*, Progress in Math. Vol. 178, Birkhauser, Verlag, Basel, 1999.

ON DETERMINING STATIONARY NASH EQUILIBRIA FOR AVERAGE SINGLE-CONTROLLER STOCHASTIC GAMES

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We consider the problem of the existence and determining stationary Nash equilibria for average single-controller stochastic games. Single-controller stochastic games with average payoffs represent a class of average stochastic games in

which the transition probabilities are controlled by one player only. The problem of determining stationary Nash equilibria in such games has been studied in [2–5]. In [2, 3] has been proposed a linear programming algorithm for computing stationary equilibria in the case of two-player zero-sum games. The problem of the existence and determining stationary equilibria for a more general case of single-controlled average stochastic games has been considered in [5, 6] . We propose an approach for determining stationary Nash equilibria for single-controller stochastic games with average payoffs in general case. We show that all stationary equilibria for a single controller stochastic game can be obtained from an auxiliary noncooperative static game in normal form where the payoffs are quasi-monotonic (quasi-convex and quasi-concave) with respect to the corresponding strategies of the players and graph-continuous in the sense of Dasgupta and Maskin [1, 4]. Based on this we present a proof of the existence of stationary equilibria in a single-controller average stochastic game and propose an approach for determining the optimal stationary strategies of the players.

References:

1. P. Dasgupta, E. Maskin. The existence of equilibrium in discontinuous economic games. *Rev. Econ. Stud.*, **53** (1986), 1–26.
2. J.A. Filar. On stationary equilibria of a single-controller stochastic game. *Math. Programming*, **30** (1984), 313–325.
3. J. Filar, T.E.S. Raghavan. A matrix game solution to a single-controller stochastic game. *Mathematics of Operations Research*, **9** (1984), 356–362.
4. D. Lozovanu. Stationary Nash equilibria for average stochastic positional games. In: *Petrosyan et al (eds). Frontiers of dynamic games, Static and Dynamic Games Theory: Foundation and Applications*, Springer, Birkhauser, 2018, 139–163,
5. T. Parthasarathy, T.E.S. Raghavan. An order field property for stochastic games when one player controls transition probabilities. *Journal of Optimization Theory and Applications*, **33** (1981), 375–392.
6. T.E.S. Raghavan, Z. Syed. Computing stationary Nash equilibria of undiscounted single-controller stochastic games. *Mathematics of Operations Research*, **27**(2) (2002), 384–400.

EXISTENCE AND MULTIPLICITY OF POSITIVE SOLUTIONS FOR A SINGULAR RIEMANN-LIOUVILLE FRACTIONAL DIFFERENTIAL PROBLEM

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We study the nonlinear fractional differential equation

$$D_{0+}^{\alpha}x(t) + f(t, x(t)) = 0, \;\; t \in (0, 1), \tag{1}$$

with the integral-differential boundary conditions

$$\begin{cases} x(0) = x'(0) = \cdots = x^{(n-2)}(0) = 0, \\ D_{0+}^{\beta_0} x(1) = \sum_{i=1}^{m} \int_0^1 a_i(t) D_{0+}^{\beta_i} x(t)\, dH_i(t), \end{cases} \tag{2}$$

where $\alpha \in \mathbb{R}$, $\alpha \in (n-1, n]$, n, $m \in \mathbb{N}$, $n \geq 3$, $\beta_i \in \mathbb{R}$ for all $i = 0, \ldots, m$, $0 \leq \beta_1 < \beta_2 < \cdots < \beta_m < \alpha - 1$, $1 \leq \beta_0 < \alpha - 1$, D_{0+}^k denotes the Riemann-Liouville derivative of order k (for $k = \alpha, \beta_0, \beta_1, \ldots, \beta_m$), the integrals from the boundary conditions (2) are Riemann-Stieltjes integrals with H_i, $i = 1, \ldots, m$, functions of bounded variation, the functions $a_i \in C(0,1) \cap L^1(0,1)$, $i = 1, \ldots, m$, and the nonlinearity f is nonnegative and it may be singular at the points $t = 0$, $t = 1$ and/or $x = 0$. We will present conditions for the data of problem (1),(2) connected to the spectral radii of some associated linear operators such that this problem has at least one or two positive solutions ($x(t) > 0$ for all $t \in (0,1]$). In the proof of the main existence theorems we use an application of the Krein-Rutman theorem in the space $C[0,1]$ and the fixed point index theory (see [1]).

References:

1. R. Luca. Existence and multiplicity of positive solutions for a singular Riemann-Liouville fractional differential problem. *Filomat*, **34** (2020), 3931–3942.

ON THE FINITE GROUPS OF RECENT GENERALIZATIONS OF THE CLASSICAL SYMMETRY

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One of the recent generalizations of classical symmetry is W_p-symmetry [1]. In a family with the same generating group G there are 8 different types groups of W_p-symmetry: the generating group, one major group and usually several middle, semi-major, semi-middle, semi-minor, pseudo-middle and pseudo-minor groups. Any group of W_p-symmetry is a subgroup of the major group of the same family. Any major group of W_p-symmetry with the finite groups G and P is constructed as a left standard direct wreath product of G with initial group P, accompanied by the fixed isomorphism $\varphi : G \to AutW$, $\varphi(g) = \overleftarrow{g}$, where $\overleftarrow{g} : w \mapsto w^g$.

Any group $G^{(W_p)}$ of W_p-symmetry with the finite group W can be derived from its finite generating group G and the group $W = \prod_{g_i \in G} P^{g_i}$ of multicomponent permutations by making the following steps:

1) we find in W all subgroups V and subsets W', which can be decomposed into left cosets of V, and in G we find all the subgroups H of index equal to the power of the set of all left cosets of W' of V and for which there is an isomorphism λ, which apply the quotient group G_1/H into W_1/V_1 by the rule $\lambda(Hg) = wV$, where $G_1 \leq G$, $W_1 \leq DiagW$ and $V_1 = V \cap DiagW \leq W_1$;

2) we construct a generalized exact natural left quasi-homomorphism $\tilde{\mu}$ of the group G onto the set of all left cosets of W' with respect to V by the rule $\tilde{\mu}(Hg) = wV$ and which preserves the correspondence between the elements of quotient groups G_1/H and W_1/V_1 received as a result of isomorphism λ;

3) we combine pairwise each g' of Hg with each w' of $wV = \tilde{\mu}(g')$;

4) we introduce on the set of all these pairs the rule of the composition

$$g_i w_i \circ g_j w_j = g_k w_k,$$

where $g_k = g_i g_j$, $w_k = w_i^{g_j} w_j$ and $w_i^{g_j}(g_s) = w_i(g_j g_s)$.

If $V = w_0$, where w_0 is the unit of the group W, then the mapping $\tilde{\mu}$ is an ordinary exact natural left quasi-homomorphism. In this case, the universal deduction method of the groups of W_p-symmetry becomes more simple and takes the form of the deduction method of semi-minor groups ($w_0 < W' < W$) or of the pseudo-minor groups, respectively, depending on W', where $w_0 \subset W' \subset W$, but W' is not a subgroup of W (see [2,3]).

References:

1. A. Lungu. Classification of groups of W-symmetry. In: *Studies in modern algebra and geometry*, Chişinău: Ştiinţa, 1983, 79–84. (In Russian)

2. A. Lungu. Methods for deriving semi-minor and pseudo-minor groups of W-symmetry. *Izvestia Akademii Nauk Respubliki Moldova. Matematica.* Chişinău, **2** (1994), 29–39. (In Russian)

3. A. Lungu. Universal method of deriving finite groups of W_p-symmetry. *Anale ştiinţifice ale Universităţii de Stat din Moldova. Seria "cStiinţe reale".* Chişinău, USM, 1997, 16–22. (In Russian)

NUMERICAL MODELING AND RESULTS OF PERFORMANCE CHARACTERISTICS FOR DD PRIORITY DISCIPLINE WITH SEMI-MARKOV SWITCHING

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Models from Queueing Theory, in particular, polling models can be found in different application fields, such as telecommunications systems [1], industry, economy, etc. Priority queueing systems are a large class of queueing systems where the requests that enter into the system are distinguished by their importance.

Priority Discretionary Discipline (DD) has its origin in the Jaiswal's monograph (1973), where it is studied regarding problem of service with one server and two flows of requests. This discipline is more flexible than classical disciplines of preemptive (absolute) and head-of-the-line (relative) priority disciplines, which are characterized by a high level of conservatives. For two flows of requests, the DD discipline, following [2], it is described as follows: if the service time of a request is less than set value θ, then it achieved the absolute priority, otherwise - relative.

Throughout time, new analytical and numerical methods have been developed, and important results have been achieved in this direction. We will mention only some scientific works devoted entirely or partially to mentioned models. These are the papers: M.I. Volkovinski and A.N. Kabalesky (1981), G.K. Mishkoy (1978), V.P. Dragalin and G.K. Mishkoy (1984), G.K. Mishkoy [3], where the discipline DD is analyzed more generally. Namely, it is supposed that the number of priority classes is arbitrary; it is assumed that service process of switching from one class of requests to another requires to spend some time for switching; the duration of switching is a random variable with an arbitrary distribution function; it allows to model and analyze different waiting times that objectively takes place in real systems.

Numerical algorithms for busy periods and auxiliary characteristics [4] are elaborated and numerical results for concrete distribution of service and switching are obtained. The elaborated algorithms can be applied at the determination others performance characteristics (probabilities of states, queue length, etc.), in whose expressions the busy period is involved.

References:

1. V. M. Vishnevsky, O. V. Semenova. *Polling Systems: The theory and applications in the broadband wireless networks.* Tehnosfera, Moscow, 2007. (In Russian)
2. N. K. Jaiswal. *Priority queues.* Moscow, Mir, Russia, 1973.
3. Gh. Mishkoy. *Generalized Priority Systems.* Ştiinţa, Chişinău: Academy of Sciences of Moldova, 2009. (In Russian)
4. Gh. Mishkoy, L. Mitev. Performance characteristics for DD priority discipline with semi-Markov switching. In: *Distributed Computer and Communication Networks*, Springer, 2014, 204–218.

ON SEMISTRONG EDGE COLORING OF GRAPHS

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Let M be a matching in graph G. A vertex covered by M is said to be *strong vertex* for M, if it has degree one in the graph induced in G by the vertex set covered by M. A *strong matching* M is a matching in which every vertex covered be M is strong for M. A *semistrong matching* M is defined by requiring that each edge of M has at least one strong vertex for M.

The minimal number of classes needed to partition $E(G)$ into matchings, strong matchings, and semistrong matchings are called *chromatic index* ($\chi'(G)$), the *strong chromatic index* ($\chi'_S(G)$), and the *semistrong chromatic index* ($\chi'_{SS}(G)$), respectively.

Semistrong edge coloring of a graph has been introduced by Andreás Gyárfás and Alis Hubenko [1]. In this work we show two tight lower bounds for semistrong chromatic indices of graphs, and various tight upper bounds for semistrong chromatic indices of some products of graphs.

In particular, we prove the following theorems (and a few others very similar to the ones about strong edge coloring shown by Oliver Togni [2]).

Theorem 1. *For any graph G with $\Delta(G) > 1$, $\chi'_{SS}(G) \geq \frac{3\delta(G)}{2}$.*

Theorem 2. *For any graph G, $\chi'_{SS}(G) \leq \chi'_{SS}(G)\chi'(G)$.*

Theorem 3. *For any graphs $G_1, G_2, \ldots, G_n$ with $\Delta(G_i) > 1$,*

$$\chi'_{SS}(\square_{i=1}^{n} G_i) \leq \sum_{i=1}^{n} \chi'(G_i) \left\lceil \frac{\chi'(\square_{j=1}^{n, i\neq j} G_j)}{2} \right\rceil .$$

References:

1. A. Gyárfás, A. Hubenko. Semistrong edge coloring of graphs. *Journal of Graph Theory*, **49** (2005), 39–47.

2. O. Togni. Strong chromatic index of products of graphs. *Discrete Mathematics and Theoretical Computer Science*, **9** (2007), 47–56.

ABOUT LINEAR AND JACOBI STABILITY OF TWO-DIMENSIONAL COMPETITIVE LOTKA-VOLTERRA SYSTEMS

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In this talk we will consider a community of two mutually competing species modeled by the Lotka–Volterra type system:

$$\begin{cases} \dot{x}_1 &= x_1\left(b_1 - a_{11}x_1 - a_{12}x_2\right), \\ \dot{x}_2 &= x_2\left(b_2 - a_{21}x_1 - a_{22}x_2\right), \end{cases}$$

where $x_i(t)$ is the population size of the i-th species at time t, $\dot{x}_i$ denote $\frac{dx_i}{dt}$ and all coefficients a_{ij}, b_i are strictly positive real numbers.

This kind of ordinary differential equations systems represent a class of Kolmogorov systems and they are widely used in the mathematical models for the dynamics of population, like predator-prey models or different models for the spread of diseases.

A qualitative analysis of this two dimensional Lotka–Volterra system based on dynamical systems theory will be performed, by studying the local behavior of the equilibria and obtaining local dynamics properties both from the linear (Lyapunov) stability and Jacobi stability point of view.

ON INTERVAL EDGE-COLORINGS OF COMPLETE MULTIPARTITE GRAPHS

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An edge-coloring of a graph G with colors $1, 2, \ldots, t$ is called an *interval $t-$coloring* if all colors are used and the colors of edges incident to each vertex of G are distinct and form an interval of integers. A graph G is *interval colorable* if it has an interval t-coloring for some positive integer t. The set of all interval colorable graphs is denoted by $\mathfrak{N}$. For an interval colorable graph G, the greatest

values of t for which G has an interval $t-$coloring is denoted by $W(G)$. The concept of interval edge-coloring of graphs was introduced by Asratian and Kamalian [1] in 1987. In [1,2], Asratian and Kamalian proved that if G is a triangle-free graph and $G \in \mathfrak{N}$, then $W(G) \leq |V(G)| - 1$. Generally, it is an NP-complete problem to determine whether a bipartite graph has an interval coloring [5]. In 1989, Kamalian proved [4] that the complete bipartite graph $K_{m,n}$ has an interval t-coloring if and only if $m+n-\gcd(m,n) \leq t \leq m+n-1$, where $\gcd(m,n)$ is the greatest common divisor of m and n. In [3], he also obtained a general upper bound on $W(G)$ for an interval colorable graph G depending on the number of vertices of G. In particular, he proved that if G is a graph with at least one edge and $G \in \mathfrak{N}$, then $W(G) \leq 2|V(G)| - 3$.

A graph G is called a complete r-partite ($r \geq 2$) graph if its vertices can be partitioned into r nonempty independent sets $V_1, \ldots, V_r$ such that each vertex in V_i is adjacent to all the other vertices in V_j for $1 \leq i < j \leq r$. Let $K_{n_1,n_2,\ldots,n_r}$ denote a complete r-partite graph with independent sets $V_1, V_2, \ldots, V_r$ of sizes $n_1, n_2, \ldots, n_r$. In this work we improved a general upper bound for some complete multipartite graphs. In particular, we proved the following result.

Theorem. *If $K_{n_1,n_2,\ldots,n_r}$ ($n_1 \geq n_2 \geq \ldots \geq n_r, r \geq 3$) is interval colorable, then*

$$W(K_{n_1,n_2,\ldots,n_r}) \leq 2\sum_{i=1}^{r} n_i - n_r - n_{r-1} - 1.$$

We also obtained some results on the existence and construction of interval colorings of complete multipartite graphs.

References:

1. A.S. Asratian, R.R. Kamalian. Interval colorings of edges of a multigraph. *Appl. Math.*, **5** (1987), 25–34. (In Russian)

2. A.S. Asratian, R.R. Kamalian. Investigation on interval edge-colorings of graphs, *J. Combin. Theory Ser.* **B 62** (1994), 34–43.

3. R.R. Kamalian. *Interval edge colorings of graphs*, Doctoral Thesis, Novosibirsk, 1990.

4. R.R. Kamalian. Interval colorings of complete bipartite graphsand trees, preprint, *Comp. Cen. of Acad. Sci. of Armenian SSR, Yerevan*, 1989. (In Russian)

5. S.V. Sevast'janov. Interval colorability of the edges of a bipartite graph, *Metody Diskret. Analiza* **50** (1990), 61–72. (In Russian)

ADMISSIBLE PERTURBATIONS OF THE GENERALIZED LANGFORD SYSTEM WITH ZERO–VALUED PARAMETERS

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Consider the generalized Langford system studied in [1]:

$$\begin{array}{rcl} \dot{x} & = & ax + by + xz, \\ \dot{y} & = & cx + dy + yz, \\ \dot{z} & = & ez - \left(x^2 + y^2 + z^2\right); \quad x, y, z, a, b, c, d, e \in \mathbb{R}, \end{array} \tag{1}$$

where a, b, c, d, e are model parameters.

If to the right-hand side of system (1) we add perturbations that do not change the reflecting function (see [2–3]) of the system (the so-called admissible perturbations), then many qualitative properties of solutions of the admissible perturbed (that is, preserving the reflecting function) systems will be preserved (see [4–5]).

Admissible perturbations were sought among perturbations of the form:

$$\Delta \cdot \alpha(t) = \left(\sum_{i+j+k=0}^{n} q_{ijk} x^i y^j z^k \quad \sum_{i+j+k=0}^{n} r_{ijk} x^i y^j z^k \quad \sum_{i+j+k=0}^{n} s_{ijk} x^i y^j z^k \right)^{\mathrm{T}} \alpha(t),$$

where $q_{ijk}, r_{ijk}, s_{ijk} \in \mathbb{R}$, $i, j, k, n \in \mathbb{N} \cup \{0\}$; $\alpha(t)$ is an arbitrary continuous scalar odd function.

Theorem. *For $a = b = c = d = e = 0$ the reflecting function of system (1) coincides with the reflecting function of the system*

$$\begin{array}{rcl} \dot{x} & = & xz\left(1 + \alpha_1\right) + y\left(\alpha_2 + \left(x^2 + y^2 + 2z^2\right)\left(x^2\alpha_3 + xy\alpha_4 - y^2\alpha_5\right)\right), \\ \dot{y} & = & yz\left(1 + \alpha_1\right) - x\left(\alpha_2 + \left(x^2 + y^2 + 2z^2\right)\left(x^2\alpha_3 + xy\alpha_4 - y^2\alpha_5\right)\right), \\ \dot{z} & = & -\left(x^2 + y^2 + z^2\right)\left(1 + \alpha_1\right), \end{array} \tag{2}$$

where $\alpha_i = \alpha_i(t)$, $i = \overline{1,5}$ are arbitrary scalar continuous odd functions.

The statement of the theorem is proved using Theorem 1 [6] by successive verification of the identity $\frac{\partial \Delta}{\partial t} + \frac{\partial \Delta}{\partial x} X(t,x) - \frac{\partial X(t,x)}{\partial x} \Delta = 0$ for each vector–factor Δ at $\alpha_i(t)$.

References:

1. Q. Yang, T. Yang. Complex dynamics in a generalized Langford system. *Nonlinear Dynamics*, **91** (2018), 2241–2270.

2. V. I. Mironenko. *Reflecting Function and Investigation of Multivariate Differential Systems.* Gomel University Press, Gomel, 2004.

3. E. V. Musafirov. *Time Symmetries of Differential Systems.* Lambert Academic Publishing, Saarbrucken, 2011.
4. E. V. Musafirov. Admissible perturbations of Langford system. *Probl. Phys. Math. Tech.*, **3** (2016), 47–51.
5. E. V. Musafirov. Perturbations of the Langford system which do not change the reflecting function. *Int. J. Bifurcation and Chaos*, **27** (2017), 1750154-1–5.
6. V. I. Mironenko, V. V. Mironenko. How to construct equivalent differential systems. *Appl. Math. Lett.*, **22** (2009), 1356–1359.

GROSS DOMESTIC PRODUCT EVOLUTION UNDER COVID-19 PERSISTENCE

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This note concerns optimal control problem formulation in a stochastic form. Equilibrium growth rate for the Republic of Moldova Gross Domestic Product evolution under COVID-19 was obtained. Basing on the stochastic maximum principle following approach [1], in which the stochastic dynamic programming formulation is transformed into formulation of the maximum principle.

Let‘s consider optimal control problem in stochastic formulation

$$\max_{(C,L_Y,L_R)} L \;=\; E\left[\int_0^T e^{(\beta_k-\rho)t}\frac{C^{1-\vartheta}}{1-\vartheta}dt\right] \qquad \text{supposed to}$$

$$\begin{aligned}\dot{K} = Y - C = K^{\alpha}AL_Y^{1-\alpha} - C,\; K(0) = 0,\\ L_R = b_1 L_R dt + gdz,\; L_R(T) = 0,\\ L_Y + L_R - L = 0,\end{aligned}$$

here E is an expectation operator, $x = (K, L_R)$ and $F = e^{(\beta_1-\rho)t}\frac{C^{1-\vartheta}}{1-\vartheta}$, utility function with constant elasticity of substitution ϑ, ρ is subjective rate of discount, β_k is subsidy for capital accumulation stimulation.

K is the capital stock attested in economic activity, L_Y is the labor force enrolled in the final goods production sector, L_R is the number of employers out of production activity as a result of pandemic evolution, C is the final consumption. dz is a stochastic Wiener process, $f = (Y-C, b_1L_R)$, b_1 is media of the unfit employers as COVID-19 consequence, $g = \sigma$ is a constant, while gdz is distributed normally with mean zero - $E[g\mathrm{dz}] = 0$, $Var(g\mathrm{dz}) = \sigma_z^2\, dt$. $dz = \sqrt{dt}$.

The respective optimality conditions now are:

$$0 \;=\; \max_{C,L_Y,L_R}\left[F+\frac{E(dL)}{dt}\right]$$

and the corresponding HJB (Hamilton-Jacoby-Bellman) equation becomes:

$$0 \;=\; \max_{C,L_Y,L_R}\left[F+\frac{\partial L}{\partial t}+\frac{\partial L}{\partial x}f+\frac{1}{2}\frac{g^2\partial^2 L}{(\partial x)^2}\right]$$

The Hamiltonian function H for this case is presented as:

$$H \;=\; F \;+\; L_x f + \tfrac{1}{2}g^2 L_{xx} = e^{(\beta_1-\rho)t}\tfrac{C^{1-\vartheta}}{1-\vartheta}+$$

$$L_{x_1}\left(K^{\alpha}AL_Y^{1-\alpha}-C\right)+L_{x_2}b_1L_R+\nu\left(L_Y+L_R-L\right)+\tfrac{1}{2}\sigma^2L_{xx}.$$

References:

1. E. Naval. Stochastic Modeling of Econpomic Growth. In: *Proceedings IMCS-50.The Third Conference of Mathematical Society of the Republic of Moldova*, Vol. 1, 2014, 402–406.

STABILITY CONDITIONS OF UNPERTURBED MOTION GOVERNED BY CRITICAL THREE-DIMENSIONAL DIFFERENTIAL SYSTEM OF DARBOUX TYPE WITH CUBIC NONLINEARITIES

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We examine the three-dimensional differential system with cubic nonlinearities

$$\dot{x}^j = a^j_{\alpha}x^{\alpha} + a^j_{\alpha\beta\gamma}x^{\alpha}x^{\beta}x^{\gamma} \quad (j,\alpha,\beta,\gamma=\overline{1,3}), \tag{1}$$

where $a^j_{\alpha\beta\gamma}$ is symmetric tensor in the lower indices, by which a total convolution is carried out here. By a center-affine transformation, the system (1) can be brought to the critical Lyapunov form [1] and in the center-affine condition $\eta_1 = a^{\alpha}_{\beta\gamma\delta}x^{\beta}x^{\gamma}x^{\delta}x^{\mu}y^{\nu}\varepsilon_{\alpha\mu\nu} \equiv 0$, from [2], the system (1) becomes a critical of Darboux type, of the form

$$\dot{x} = 3xD, \;\; \dot{y} = px+qy+rz+3yD, \;\; \dot{z} = sx+my+nz+3yD, \tag{2}$$

where $D = ax^2 + by^2 + cz^2 + 2dxy + 2exz + 2fyz, a_1^2 = p, a_2^2 = q, a_3^2 = r, a_1^3 = s$, $a_2^3 = m, a_3^3 = n$, and $a, b, c, d, e, f, m, n, p, q, r, s$ are real coefficients.

According to [3], a center-affine invariant condition which assures us that the system (1) is critical, is $L_{2,3} \equiv \frac{1}{2}(\theta_1^2 - \theta_2) = nq - mr > 0$, where $\theta_1 = a_\alpha^\alpha$, $\theta_2 = a_\beta^\alpha a_\alpha^\beta$, are center-affine comitants of the system (1), from [2].

We denote:

$$A_1 = (rs - np)L_{2,3}^{-1}, \quad B_1 = (mp - qs)L_{2,3}^{-1}. \tag{3}$$

Then, taking into account the Lyapunov Theorem [1, §32] and the expressions (3), we obtain the following result.

Theorem. *The stability of unperturbed motion, described by the critical system of Darboux type of perturbed motion* (2)*, includes all possible cases in the following three*:

I. $a + bA_1^2 + cB_1^2 + 2dA_1 + 2eB_1 + 2fA_1B_1 < 0$, *then unperturbed motion is* ***stable****;*

II. $a + bA_1^2 + cB_1^2 + 2dA_1 + 2eB_1 + 2fA_1B_1 > 0$, *then unperturbed motion is* ***unstable****;*

III. $a + bA_1^2 + cB_1^2 + 2dA_1 + 2eB_1 + 2fA_1B_1 = 0$, *then unperturbed motion is* ***stable****.*

In the last case, the unperturbed motion belongs to some continuous series of stabilized motions, and moreover this motion is asymptotically stable.

References:

1. A. M. Liapunov. *Obshchaia zadacha ob ustoichivosti dvijenia,* Sobranie sochinenii, tom. II, Izd. Academii Nauk SSSR, Moskva-Leningrad, 1956, 474 p. (In Russian)
2. N. Gerştega. *Lie algebras for the three-dimensional differential system and applications*, Synopsis of PhD thesis, Chişinău, 2006, 21 p.
3. I. G. Malkin. *Teoria ustoichivosti dvijenia.* Izd. Nauka, Moskva, 1966, 530 p. (In Russian)

ON THE EQUIVALENCE OF SOME ALGEBRAS OF INTEGRAL OPERATORS WITH SHIFT

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In this paper it is proved that some algebras of integral operators are equivalent to the algebra generated by singular integral operators with piecewise continuous coefficients.

Two Banach algebras (subalgebras) $\mathcal{A}_1(\subset L(\mathcal{B}_1)$ and $\mathcal{A}_2(\subset L(\mathcal{B}_2)$ will be called *equivalent* if there exists an invertible operator $M \in L(\mathcal{B}_1, \mathcal{B}_2)$ such that the set of operators of the form MAM^{-1} $(A \in \mathcal{A}_1)$ coincides with the algebra $\mathcal{A}_2$.

The notion of symbol is defined for the operators of these algebras by means of equivalence. It is established that the Noetherian conditions and the indices of the operators are expressed by determinant of their symbols.

In the study of algebras some fundamental results obtained by mathematicians I. Gohberg and N. Krupnik in [1–3] are used.

References:

1. I. Gohberg, N. Krupnik. Banach algebras generated by singular integral operators. *Colloquia mathematica societatis Janos Bolyai 5. Hilbert space operators*, 1970, 240–263.
2. N. Krupnik. *Banach algebras with symbol and singular integral operators.* Birkhäuse, Basel-Boston, 1987.
3. I. Gohberg, N. Krupnik. Extension theorems for invertibility symbols in Banach algebras. *Integral Eq. Operator Theory*, **15** (1992), 991–1010.

MAGNETIC CURVES IN $\mathbb{S}^3 \times \mathbb{S}^2$

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This presentation is based on the joint paper with M.I. Munteanu entitled "*Magnetic curves in quasi-Sasakian manifolds of product type*", which was accepted for publication in "*New Horizons in Differential Geometry and its Related Fields*", Eds. T. Adachi and H. Hashimoto, 2021.

The main result represents a positive answer to sustain our conjecture about the order of a magnetic curve in a quasi-Sasakian manifold. More precisely, we

show that the magnetic curves in quasi-Sasakian manifolds, obtained as the product of a Sasakian and a Kähler manifold, have maximum order 5.

In the following, we will focus on the study of the magnetic curves in $\mathbb{S}^3 \times \mathbb{S}^2$. First, we find the explicit parametrizations of such curves. Then, we find a necessary and sufficient condition for a magnetic curve in $\mathbb{S}^3 \times \mathbb{S}^2$ to be periodic. Finally, we conclude with some examples of magnetic curves in $\mathbb{S}^3 \times \mathbb{S}^2$.

MEASURES OF VALUE-AT-RISK

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Any financial activity can lead to success as well to fall, and it depends on many factors, but the most important is the management of the activities and the decisions we have to make. As an important problem for any investment is the analysis of the future possible risks. In economic terms "Value at risk" known as VaR is used to evaluate the possible risk for loss. This term "VaR" is used both for a risk measure and a risk metric. But sometimes this leads to confusion. Sources earlier than 1995 usually emphasize the risk measure, later sources are more likely to emphasize the metric. VaR did not emerge as a distinct concept until the late 1980s. The triggering event was the 1987 stock market crash.

To specify what is VaR, in simple terms, this is estimate of the loss from a given position over a fixed time period that will be equaled or exceeded with a given probability. Financial institutions use VaR to determine how much emergency cash they need to cover potential severe losses.

For VaR we can consider the two equivalent interpretations:
1) The Worst Case Loss: over one day, there is a 95% probability that we will not lose more than a fixed amount of money.
2) An unlikely event: on average, in one out of every 20 days, we should expect to incur a loss greater than or equal to a certain amount.

A VaR measurement has three components: a time frame, a confidence level and a loss amount. There are different types of VaR measures: Historical VaR, Analytic VaR (Variance Covariance, or parametric) and Monte Carlo VaR. Historical VaR is the simplest method to calculate VaR. The portfolio is valued with the help of full, non-linear pricing models for every scenario. Parametric method (also Variance-covariance method) requires the calculation of two things - standard deviation and expected return. In the case of Monte Carlo Simulation, the VaR can be calculated randomly creating scenarios for future stock price return, such a method is apt when we have complicated factors.

Var has either advantages as well as disadvantages. VaR is useful for banks and financial institutions, it pays a very important role in decision making for financial analysts. But also the calculation is full of assumptions, different calculation methods give different results.

As an important remark, we have to specify that the VaR is not a coherent risk measure since it violates the sub-additivity property. But it can be bounded by coherent risk measures like Conditional Value-at-Risk (CVaR) or entropic value at risk (EVaR). CVaR is defined by average of VaR values for confidence levels between 0 and α.

Conditional Value at Risk (CVaR), also known as the expected shortfall, is a risk assessment measure that quantifies the amount of tail risk an investment portfolio has. Conditional Value at Risk (CVaR) is derived by taking a weighted average of the "extreme" losses in the tail of the distribution of possible returns, beyond the value at risk (VaR) cutoff point. CVaR is used in portfolio optimization for effective risk management. Calculating CVaR is simple once VaR has been calculated. It is the average of the values that fall beyond the VaR.

FIRST-ORDER PERTURBATION FOR MULTI-PARAMETER CENTER FAMILIES

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In the weak 16th Hilbert problem, the Poincaré-Pontryagin-Melnikov function, $M_1(h)$, is used for obtaining isolated periodic orbits bifurcating from centers up to a first-order analysis. This problem becomes more difficult when a family of centers is considered. In this talk we shall present a compact expression for the first-order Taylor series of the function $M_1(h, a)$ with respect to a, being a the multi-parameter in the unperturbed center family. More concretely, when the center family has an explicit first integral or inverse integrating factor depending on a. We use this new bifurcation mechanism to increase the number of limit cycles appearing up to a first-order analysis without the difficulties that higher-order studies present. We show its effectiveness by applying it to some classical examples.

This is a joint work with Jackson Itikawa (UNIR, Brazil) and Joan Torregrosa (UAB, Spain).

DARBOUX' THEORY OF INTEGRABILITY FOR QUADRATIC SYSTEMS WITH INVARIANT HYPERBOLAS

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The goal of this work is to present the investigation in development about integrability of planar quadratic differential systems in the whole class of non-degenerate planar quadratic differential systems possessing at least one invariant hyperbola (QSH). Such class was investigated in [1] where the authors classify it according to its geometric properties encoded in the configurations of invariant hyperbolas and invariant straight lines which these systems possess.

In this talk we will present results about Darboux and Liouvillian integrability for QSH. For that, we will see some important results of Darboux' theory and investigate the existence of invariant algebraic curves, exponential factors, integrating factors and first integrals for some examples.

Our main motivation in this research is to study the relationship between integrability and the geometry of the systems as expressed in their configurations of invariant algebraic curves as well as their relations with the bifurcations of the phase portraits.

References:

1. R. D. S. Oliveira, A. C. Rezende, D. Schlomiuk, N. Vulpe. Geometric and Algebraic Classification of Quadratic Differential Systems with Invariant Hyperbolas. *Electronic Journal of Differential Equations*, **295** (2017), 1–112.

NUMERICAL MODELING AND CONTROL OF DYNAMIC PROCESSES IN A HEAT EXCHANGER

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The paper considers a pipe-in-pipe heat exchanger, the principle of operation of which is based on the constant contact of the coolant with the treated liquid. It is used in technological systems for heating or cooling a coolant with a small heat exchange surface in the gas, oil, petrochemical and chemical industries. Heat exchangers with such a design are also used in the food industry, for example, in winemaking and in dairy production.

The mathematical model of the dynamic process of transferring heat energy in devices of this type includes a system of three differential equations for the temperatures of cold water (heated), hot water (heating) and dividing walls. The formulated problem is solved numerically using the ideas of finite difference method. For this purpose a stable and converging difference scheme is constructed, that gives a possibility to find approximate solutions for discrete times for two equations of the system. In this case, the third equation (for temperature of dividing wall) becomes an ordinary differential equation, the solution of which can be obtained in an analytical form. As it follows from the structure of the initial equations, the model contains dissipative terms. This leads to the fact that the solution to the dynamic problem enters a stationary mode determined by the solution of the static problem. The static problem, being a special case of the original dynamic problem, is a system of two ordinary differential equations and one algebraic equation connecting unknown temperatures. The solution of such a system with given boundary conditions is obtained in the analytical form.

Using numerical solutions to these two problems we tried to develop an effective approach for establishing the control over maintaining the temperature of cold water at a constant level at the output of the device.

TWO PARAMETER SINGULAR PERTURBATION PROBLEMS FOR SINE-GORDON TYPE EQUATIONS

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Let $\Omega \subset \mathbb{R}^n$ be an open bounded set with smooth boundary $\partial\Omega$. Consider the real Hilbert space $L^2(\Omega)$, endowed with the usual inner product $(u,v)_{L^2(\Omega)} = \int_\Omega u(x)\, v(x)\, dx$, and the real Sobolev space $H_0^1(\Omega)$, endowed with the inner product $(u,v)_{H_0^1(\Omega)} = \int_\Omega \big(\nabla u(x),\, \nabla v(x)\big)_{\mathbb{R}^n}\, dx$.

We investigate the following boundary-value problem for sine-Gordon type equation

$$\begin{cases} \varepsilon\, \partial_t^2 u_{\varepsilon\delta}(x,t) + \delta\, \partial_t u_{\varepsilon\delta}(x,t) + A u_{\varepsilon\delta}(x,t) + b \sin u_{\varepsilon\delta}(x,t) = f(x,t), (x,t) \in Q_T, \\ u_{\varepsilon\delta}(x,0) = u_0(x),\ \partial_t\, u_{\varepsilon\delta}(x,0) = u_1(x), \quad x \in \Omega, \\ u_{\varepsilon\delta}|_{\partial\Omega} = 0, \quad t \geq 0, \end{cases} \tag{$P_{\varepsilon\delta}$}$$

where $T > 0$, $Q_T = \Omega \times (0,T)$, $f \in L^2(Q_T)$, $u_0 \in H_0^1(\Omega)$, $u_1 \in L^2(\Omega)$, $b \in \mathbb{R}$, $b \neq 0$, ε, δ are two small parameters and A is a strongly elliptic operator of second order.

The interest for the sine-Gordon equation is explained by the various applications in differential geometry and engineering, including junctions between two superconductors, the motion of rigid pendular attached to a stretched wire, dislocations in crystals, nonlinear optics.

Using similar specific techniques, the functional framework of the Sobolev space $H_0^1(\Omega)$ and the properties of the strongly elliptic operator, we investigate the behavior of solutions $u_{\varepsilon\delta}$ to the problem $(P_{\varepsilon\delta})$ in two different cases:
(i) $\varepsilon \to 0$ and $\delta \geq \delta_0 > 0$, relative to the solutions to the following unperturbed system:

$$\begin{cases} \delta\, \partial_t l_\delta(x,t) + A l_\delta(x,t) + b \sin l_\delta(x,t) = f(x,t), \quad (x,t) \in Q_T, \\ l_\delta(x,0) = u_0(x), \quad x \in \Omega, \\ l_\delta|_{\partial\Omega} = 0, \quad t \geq 0; \end{cases} \tag{P_δ}$$

(ii) $\varepsilon \to 0$ and $\delta \to 0$, relative to the solutions to the following unperturbed system:

$$\begin{cases} A v(x,t) + b \sin v(x,t) = f(x,t), \quad (x,t) \in Q_T, \\ v|_{\partial\Omega} = 0, \quad t \geq 0. \end{cases} \tag{P_0}$$

The problem $(P_{\varepsilon\delta})$ is the abstract model of singularly perturbed problems of hyperbolic-parabolic type in the case (i) and of the hyperbolic-parabolic-elliptic type in the case (ii).

VERTEX-DISTINGUISHING EDGE COLORINGS OF SOME COMPLETE MULTIPARTITE GRAPHS

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Let G be an undirected graph without multiple edges and loops, V(G) be the set of vertices of the graph G, $E(G)$ be the set of edges of the graph G. Denote by $K_n, K_{m,n}, K_{l,m,n}$, respectively, a complete graph with n vertices, a complete bipartite graph with m vertices in one partition and with n vertices in another, a complete tripartite graph with l vertices in one partition, m vertices in the other part, and n vertices in the third partition. Terminologies and notations not defined here can be found in [6].

A proper edge coloring f of a graph G is called vertex-distinguishing if for any different vertices $u, v \in V(G), S(u,f) \neq S(v,f)$. The minimum number of

colors required for a vertex-distinguishing proper edge coloring of a simple graph G is denoted by $\chi'_{vd}(G)$. The definition of vertex-distinguishing edge coloring of a graph was introduced in [1,2] and, independently, as the "observability" of a graph in [3-5].

In this work we obtain some results on vertex-distinguishing edge colorings of complete 3- and 4-partite graphs. In particular, the following results hold.

Theorem 1. *Let l,m and n be any natural numbers. Then*

$$\chi'_{vd}(K_{l,m,n}) \leqslant l + m + n.$$

Theorem 2. *Let l, m and n be natural numbers, such that inequalities $m > n + l - 2, n > l > 1$ hold. Then*

$$\chi'_{vd}(K_{m,n,l}) = m + n + 1$$

Theorem 3. *Let m and n be any natural numbers. Then*

$$\chi'_{vd}(K_{1,m,n}) = \begin{cases} m+n & for \quad n \geq m > 1 \\ m+n+1 & for \quad n \geq m = 1 \end{cases}$$

$$\chi'_{vd}(K_{2,m,n}) = \begin{cases} m+n+1 & for \quad n \geq m > 2 \ or \ n > m = 2 \\ 6 & for \quad n = m = 2 \end{cases}$$

Theorem 4. *Let m and n be any not equal natural numbers. Then*

$$max(m,n) + m + n + 1 \leqslant \chi'_{vd}(K_{m,m,n,n}) \leqslant max(m,n) + m + n + 3.$$

References:

1. A.C. Burris. *Vertex-distinguishing edge-colorings.* Ph.D. Dissertation, Memphis State University (1993).
2. A.C. Burris, R.H. Schelp. Vertex-distinguishing proper edge-colorings. *J. Graph Theory*, **26** (1997), 73–82.
3. J. Cerny, M. Hornak, R. Sotak. Observability of a graph. *Math. Slovaca*, **46** (1996), 21–31.
4. M. Hornak, R. Sotak. Observability of complete multipartite graphs with equipotent parts. *Ars Comb.*, **41** (1995), 289–301.
5. M. Hornak, R. Sotak. Asymptotic behavior of the observability of Q_n. *Discrete Math.*, **176** (1997), 139–148.
6. D.B. West. *Introduction to Graph Theory.* Prentice-Hall, New Jersey, 2001.

SOME NEW RESULTS IN METRIC FIXED POINT THEORY

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ORTHONORMALIZATION OF LIMIT LINEAL BASIS

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Let $\mathbb{B}^n$ be some homogeneous space [1] with specification $\{k_1, ..., k_n\}$, $k_p \in \{-1, 0, 1\}, p = \overline{1, n}$. Define:

$$x \odot_i y = \sum_{j=0}^{n} K_{ji} x_j y_j, \quad K_{pq} = \begin{cases} 1, & p = q, \\ \prod_{i=p+1}^{q} k_i, & p < q, \\ K_{qp}, & p > q. \end{cases}$$

For not null vector x define *index* as one of (smallest) index i so that $x \odot_i x > 0$. The notation $x \odot y = x \odot_{min(i,j)} y$, where i is index of x and j is index of y. If there is no such index, $x \odot_i x = 0, \forall i = \overline{0, n}$, then vector x is named *limit*.

As stated in [2], limit vector always can be presented as sum of indexed vectors $x = a + b$, so that $a \perp b, a \odot a = b \odot b$. Vectors a, b are named *decomposition vectors* of vector x. Lemmas 1 and 2 from [2] state for orthogonal vectors, from which at least one is limit, the possibility to orthogonalize also their decomposition vectors.

Lemma 1. *Indices of decomposition vectors do not depend on space basis choice.*

Define *lineal* as linear span of vectors (it may be congruent or not with some subspace of $\mathbb{B}^n$). Define *limit lineal* as lineal whose orthonormal basis contains limit vectors.

Lemma 2. *In limit lineal basis, the indices of decomposition vectors are not among indices of indexed vectors.*

Lemmas 1 and 2 permit to consider decomposition vectors indices as *double index* of a limit vector.

Remark 3. *If not collinear limit vectors x, y are not orthogonal, then vectors $x + y, x - y$ are not collinear and indexed.*

Proposition 4. *If limit vector x is not orthogonal with indexed vector y, it is possible to find decomposition vectors $x = a + b$ such that either a or b is collinear with y.*

It is possible to develop the **Algorithm** of *orthonormalization of vector family containing limit vectors*, where decomposition vectors are normalized and orthogonal to all other vectors.

Lemma 5. *Orthogonal complement do not exist for limit lineals.*

[1]The research is done for *GeomSpace* project https://sourceforge.net/projects/geomspace/.

Lemma 6. *Generally speaking, it is not possible to project a vector onto limit lineal.*

References:

1. A. Popa. *Analytic Geometry of Homogeneous Spaces*, https://www.researchgate.net/publication/326646690_Analytic_Geometry_of_Homogeneous_Spaces, 2017.

2. A.N. Popa. New methods in study of isotropic vectors. In: *XIII International Conference "Algebra, Number Theory and Discrete Geometry: Modern Problems and Applications", dedicated to 80 anniversary of professor S.S. Ryshkov*, 2015, 317–319. (In Russian)

ON THE STRUCTURE OF SOME LCA GROUPS WITH LOCAL RING OF CONTINUOUS ENDOMORPHISMS

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Let $\mathcal{L}$ be the class of locally compact abelian groups. For $X \in \mathcal{L}$, let $E(X)$ denote the ring of continuous endomorpgisms of X and $t(X)$ the torsion subgroup of X. Given a prime p and a positive integer n, we denote by $\mathbb{Z}(p^n)$ the cyclic group of order p^n, by $\mathbb{Z}(p^\infty)$ the quasi-cyclic group corresponding to p, and by $\mathbb{J}_p$ the group of p-adic integers, all taken discrete. Further, we denote by $\mathbb{J}_p^*$ the character group of $\mathbb{J}_p$, by $\mathbb{Z}_p$ the group of p-adic integers with its unique compact topology, and by $\mathbb{Q}_p$ the group of p-adic numbers with its usual locally compact topology.

Theorem 1. *Let X be a residual group in $\mathcal{L}$. The following conditions are equivalent:*

(i) $E(X)$ is a field.

(ii) $E(X)$ is local and $p1_X$ is invertible in $E(X)$ for all primes p.

(iii) $E(X)$ is local and X is densely divisible and torsionfree.

(iv) X is topologically isomorphic with $\mathbb{Q}_p$ for some prime p.

A group $X \in \mathcal{L}$ is said to be *purely topologically indecomposable* if every closed pure subgroup of X is topologically indecomposable.

Theorem 2. *Let X be a residual, purely topologically indecomposable group in $\mathcal{L}$ such that $t(X)$ is closed in X. The following conditions are equivalent:*

(i) $E(X)$ is local and commutative.

(ii) $E(X)$ is local.

(iii) X is topologically isomorphic either with a pure subgroup G of $\mathbb{J}_p$ satisfying $soc(G) = G$, or with one of the groups $\mathbb{Q}_p$, $\mathbb{Z}(p^\infty)$, or $\mathbb{Z}(p^n)$, where $p \in \mathbb{P}$ and $n \in \mathbb{N}$.

A group $X \in \mathcal{L}$ is said to be *co-purely topologically indecomposable* if for any closed pure subgroup C of X, the quotient group X/C is topologically indecomposable.

Theorem 3. *Let X be a residual, co-purely topologically indecomposable group in $\mathcal{L}$ such that $\overline{mX}/\cap_{n\in\mathbb{N}_+}\overline{nX}$ is compact for some $m \in \mathbb{N}_+$. The following conditions are equivalent:*

(i) $E(X)$ is local and commutative.

(ii) $E(X)$ is local.

(iii) X is topologically isomorphic either with a quotient group of $\mathbb{J}_p^*$ by a closed pure subgroup A satisfying $rad(A) = \{0\}$, or with one of the groups $\mathbb{Q}_p$, $\mathbb{Z}_p$, or $\mathbb{Z}(p^n)$, where $p \in \mathbb{P}$ and $n \in \mathbb{N}$.

THE CENTRAL PATH PROBLEM FOR THE CLASS OF A GRAPH

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The concept of central vertex in a graph as a vertex from which the maximum distance to any other vertex is minimized can be extended and applied to a path in a graph.

Consider a finite, connected, undirected graph $G = (V, E)$. We assume that each edge has length one. The distance in G between vertices u and v, denoted by $d(u, v)$, is the minimum number of edges in a u to v path. A shortest path between any two vertices is called a geodesic. The paths considered below are assumed to be geodesic. The distance between a vertex u and a set of vertices S, denoted by $d(u, S)$, is defined as the smallest distance from u to the vertices of S. The diameter $D = D(G)$ of a graph G is the length of a longest geodesic. The path with length D is called the diametral path. Let P be an arbitrary path in graph G. The eccentricity of a path P is defined by:

$$e(P) = max\{d(u, P) : u \in V\}.$$

A path P for which $e(P) \leq e(P')$ for any path P' in G, and such that there is no shorter path with the same property, is called a central path of G [1].

Let $G = (V, E)$ be a connected plane graph. Then G defines on the plane one closed unbounded domain F_0 and $\nu = |E| - |V| + 1$ closed bounded domains F_1, $F_2, \ldots, F_\nu$, homeomorphic to the disc. We denote by Φ the set of domains F_1, $F_2, \ldots, F_\nu$, which are called faces. On G, the following conditions are imposed:

1. for any two faces $F_i, F_j \in \Phi, i \neq j$:

$$F_i \cap F_j = \begin{cases} \varnothing, & \text{or} \\ v \in V, & \text{or} \\ e \in E; \end{cases}$$

2. each face $F_i \in \Phi$ is a quadrilateral;

3. if the vertex x of G does not belong to F_0, then at least four edges of the graph are incident to the vertex x.

We denote by K the class of all graphs satisfying conditions 1-3 [2]. We consider the central path problem for graphs from the class K. In connection to this, some properties were proved, among which:

1. each central vertex of the graph is contained in at least one diametral path;

2. each central path is contained in at least one diametral path;

3. each central path contains at least one central vertex.

References:

1. P.J. Slater. Locating central path in a network. *Transportation, Science*, **16** (1984), 1–18.

2. P.S. Soltan, D.K. Zambitskii, K.F. Prisakaru. *Extremal problems on graphs and algorithms for their solutions.* Shtiintsa, Kishinev, 1973. (In Russian)

AN OVERVIEW ON STRUCTURALLY UNSTABLE QUADRATIC DIFFERENTIAL SYSTEMS OF CODIMENSION TWO

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In this talk I present the state of the art of the topological classification of the phase portraits of some families of structurally unstable quadratic differential systems of codimension two. The phase portraits are obtained topologically and I present only the ones which can be realized by such differential systems. I am going to show the sets of such systems possessing either a cusp point, or two finite saddle-nodes, or a finite saddle-node and an infinite saddle-node (obtained by the coalescence of two infinite singular points), or a finite saddle-node and an infinite saddle-node (obtained by the coalescence of a finite singular point with an infinite singular point). This is a joint project with Joan C. Artés, Regilene D.S. Oliveira and Marcos C. Mota.

INTERVAL EDGE COLORINGS OF TREES WITH RESTRICTIONS ON THE EDGES

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An edge-coloring of a graph G with consecutive integers $c_1, \ldots, c_t$ is called an interval t-coloring if all colors are used, and the colors of edges incident to any vertex of G are distinct and form an interval of integers. A graph G is interval colorable if it has an interval t-coloring for some positive integer t. In this paper, we consider the case where there are restrictions on the edges of the tree and provide a polynomial algorithm for checking interval colorability that satisfies those restrictions.

Theorem. *If there is a polynomial algorithm that detects whether it is possible to have an interval edge coloring for any restrictions R on the edges then there is a polynomial algorithm that detects whether there is an interval t-coloring of the connected graph for the given restrictions.*

Problem. Given an arbitrary tree T with $N = |V(G)|$ vertices and given arbitrary restrictions R for every edge e with $1 \leq l(e) \leq r(e) \leq N$. Determine whether it's possible to have an interval coloring $\alpha : E(G) \to \{1, \ldots, N\}$ such that for each edge e, $l(e) \leq \alpha(e) \leq r(e)$ and for every vertex v, the colors of edges incident to the vertex v are distinct and form an interval of integers..

In [1] it was shown that every tree has interval-edge coloring. In [2] a solution for the simplified version of this problem was provided when the restrictions are on the spectrums, the restrictions are strict, and all the spectrums contain the color 1. The problem of finding an interval t-coloring on a tree for given restrictions on edges in polynomial time was first suggested by Kamalian and was an open problem since 1999.

References:

1. A.S. Asratian, R.R. Kamalian. Interval colorings of edges of a multigraph. *Appl. Math.*, **5** (1987), 25–34. (In Russian)

2. Y. Caro, J. Schönheim. Generalized 1-factorization of trees. *Discrete Mathematics*, **33** (1981), 319–321.

MULTIVALUED DIFFERENTIAL PROBLEMS UNDER POMPEIU EXCESS RESTRICTIONS

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The theory of differential equations driven by measures has lately gained increasing attention, due to the fact that it allows a unified study of classical differential problems, difference equations, impulsive differential equations or dynamic equations on time scales.

We focus on measure differential inclusions

$$\begin{aligned} &dx(t) \in F(t, x(t))d\mu_g(t) \\ &x(0) = x_0. \end{aligned}$$

The notation μ_g stands for the Stieltjes measure associated to a nondecreasing left-continuous function g and the multifunction on the right hand side has compact, possibly non-convex values in a real Euclidean space and satisfies bounded variation hypotheses with respect to the Pompeiu excess (and not to the Hausdorff-Pompeiu distance, as in related literature); the key element is a selection principle obtained by V.V. Chistyakov and D. Repovš [1].

We present an existence result and we analyze how the solution set changes when we allow perturbations of the function g generating the driving measure.

References:

1. V. V. Chistyakov, D. Repovš. Selections of bounded variation under the excess restrictions. *J. Math. Anal. Appl.*, **331** (2007), 873–885.

A MESHFREE METHOD FOR PDEs IN ENGINEERING PROBLEMS

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This report documents the development and application of a meshfree method for the numerical solution of the governing PDEs in engineering problems. The meshfree nature of this method gives the advantage of dealing with highly distorted domains and even fragmentation without the need of using remeshing approaches and with a very simple implementation. A description of the implementation of this method as well as the solutions of some test problems will be presented in order to demonstrate the potential of this formulation for dealing with different engineering problems and to introduce promising future fields of application.

APPROXIMATE ANALYTICAL APPROACH FOR REPRESENTING SOLUTIONS OF A SYSTEM OF ONE-DIMENSIONAL NAVIER-STOKES EQUATIONS

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A complete system of Navier-Stokes equations is considered, the solutions of which describe the motion of a viscous heat-conducting gas. The paper presents an approximate analytical approach for representing solutions of the Navier-Stokes equations describing one-dimensional unsteady flows arising in the problem of thermal interaction between a flow of a gas at rest and a heat-conducting stationary wall, the initial temperature of which differs from the initial temperature of the gas.

In the framework of the conjugate approach, the solution of the problem is complicated by the need to jointly solve the differential equations for gaseous medium and wall material. As a result of unsteady gas-wall interaction, gas-dynamic and thermal processes cause the appearance of a flow with complex internal structure. In order to describe the emerging flow, taking into account the influence of dissipative effects, leads to the need to use the complete system

of nonlinear differential Navier-Stokes equations in partial derivatives, which are based on the universal laws of conservation of mass, momentum and energy, and constitute a theoretical basis for describing and predicting a wide range of phenomena in gas dynamics [1, 2]. The temperature distribution inside the wall is modeled by a linear equation of heat conduction.

At moderate differences in gas-wall initial temperatures, perturbations of the parameters are small, and the flow field is studied based on the solution of a system of the Navier-Stokes equations, linearized around the initial state. Structure of the emerging flows is studied on the basis of the joint solution of a linearized system of the Navier-Stokes equations and the equation of heat conduction of the wall for given initial and boundary conditions[2]. Analytical solutions of the linearized problem and asymptotic expressions for gas-wall parameters, that describe formation of the continuous structure of the flow field and distribution of the wall temperature, are obtained by the Laplace transform method.

These solutions make it possible to study the influence of viscosity, heat conduction and other physical factors on formation of dissipative and ideal non-viscous and non-heat-conducting zones in gas flow field. It should be noted that linearized solutions, describing the continuous structure of the emerging flow, are also of interest as a test in the development and debugging of algorithms for numerical solution of the Navier-Stokes nonlinear equations.

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References:

1. L.G. Loitsyanskii. *Mechanics of Liquids and Gases.* Nauka, Moscow, 1973. (In Russian)
2. Yu.A. Demyanov, G.V. Secrieru et al. *One-dimensional unsteady flows of real gas.* Ştiinţa, Chişinău, 1980. (In Russian)

AN APPROACH FOR SOLVING THE MULTI-CRITERIA FRACTIONAL CONVEX PROGRAMMING PROBLEM

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This presentation is concerned with studying and solving the following generalized multi-criteria fractional convex programming problem [1]:

$$f(x) = \left\{ \frac{\varphi_1(x)}{\psi_1(x)},\ \frac{\varphi_2(x)}{\psi_2(x)} \ldots, \frac{\varphi_m(x)}{\psi_m(x)} \right\} \ \rightarrow \ min \tag{1}$$

$$h_k(x) \leq 0,\ k = \overline{1,p}, \tag{2}$$

where the functions $\{\varphi_i(x),\ i \in I\}$, $\{-\psi_i(x),\ i \in I\}$ and $\{h_k(x),\ k = \overline{1,p}\}$, are continuous, convex and differentiable in R^n.

We show that for this multi-criteria fractional convex programming problem three possible aggregation procedures can be applied that allow to obtain the following auxiliary generalized fractional-convex programming problems:

$$\min_{x \in S} \sum_{i \in I} \alpha_i \frac{\varphi_i(x)}{\psi_i(x)}; \quad \min_{x \in S} \max_{i \in I} \alpha_i \frac{\varphi_i(x)}{\psi_i(x)}; \quad \min_{x \in S} \prod_{i \in I} \alpha_i \frac{\varphi_i(x)}{\psi_i(x)},$$

where

$$S = \{x :\ f_k(x) \leq 0,\ k = \overline{1,p}\}; \quad \alpha_i > 0,\ i \in I; \quad \sum_{i \in I} \alpha_i = 1.$$

This means that the solutions of the considered multi-criteria fractional convex programming problem can be obtained by using the solutions of the auxiliary generalized fractional-convex programming problems above. The question which problem should be used for determining the solution of the multi-criteria problem is discussed in [1]. The proposed aggregation procedure for solving the multi-criteria problems (1),(2) can be applied also by using other equivalent fractional programming problems. In a detailed form how to determine the solutions of different classes of fractional programming problems can be found in [1].

References:

1. D. Solomon. *Fractional programming and non-differential optimization.* Chişinău, Evrica, 2010. (In Russian)

COMPARATIVE ANALYSIS OF LINEARIZATION TECHNIQUES WHEN MODELING NONLINEAR PROCESSES IN SEMICONDUCTOR DEVICES

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The mathematical model of investigated problem [1, 2] represents the system of nonlinear differential equations with respect to unknown functions that are electrostatic potential, electron concentration and holes concentration. The problem is significantly complicated by the fact that the boundary conditions are inhomogeneous: on a part of the boundary the Dirichlet conditions are specified, and on the other part – the Neumann conditions. One of the most important points in the numerical solution of this problem is the problem of choosing a suitable method for linearizing the given differential equations. For this purpose, we use two different approaches: Newton's and Gummel's algorithms [2]. Then the obtained linear system is solved numerically by applying the conjugate and bi-conjugate gradient methods. Numerical experiments were carried out on the basis of these two techniques. The obtained numerical solutions made it possible to compare the considered methods and to draw preliminary conclusions about the possibilities of their application.

References:

1. W.H.A. Schilders, E.J.W. Maten. *Numerical Methods in Electromagnetics. Special Volume of Handbook of Numerical Analysis.* ELSEVIER, Amsterdam. Guest, ed. 2005.
2. D. Vasileska, S.M. Goodnick, G. Klimeck. *Computational Electronics: Semiclassical and Quantum Device Modeling and Simulation.* Amazon, CRC Press, 2010.

THE CORE-WALRAS EQUIVALENCE IN NONADDITIVE ECONOMIES

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We present some results concerning the links between the set of walrasian equilibrium points $W(\mathcal{E})$ and the core $C(\mathcal{E})$ of an economy $\mathcal{E}$ with infinite dimensional space of agents and commodities where $\mathcal{E} = [(T, \tau, \mu), E, (X, >_t), e]$ ([1]). The following hypothesis will be considered.

(H_0) The submeasure μ satisfies the following condition:

$(\exists)$ $\{E_i,\ i = \overline{1,r}\}$ a partition of T such that, for every $A \in \mathcal{A}$, $\mu(A) = \sum_{i=1}^{r} \mu(A \cap E_i)$. ($E_i$ denotes the set of agents of type i).

(H_1) Each consumer $a \in E_i$ is characterized by its initial endowment $e(a) = e_i$.

(H_2) The preference relation $\{>_i\}_{i=\overline{1,r}}$ associated to the agents $a \in E_i$ is

(i) irreflexive and transitive,

(ii) monotone: $\forall x \in E_+,\ v \in E_+ \setminus \{0\},\ x + v >_i x,\ \forall i \leq r$,

(iii) continuous: the set $\{y \in E_+,\ y \geq_i x\}$ is closed in E, $\forall i \leq r$. In other words, in each coalition E_i agents share the same initial endowment and same preference criterions.

(H_3) The price system is $\mathcal{P} = \operatorname{conv}\operatorname{cone} \mathcal{G}$, where $\mathcal{G}$ is a family of functions $\mathcal{G} \subset \mathcal{F}(\mathcal{E}, \mathcal{R}_+)$ with the following properties:

(i) $g(0) = 0,\ \forall g \in \mathcal{G}$,

(ii) $g(c) \leq g(d),\ \forall g \in \mathcal{G} \Longrightarrow c < d$,

(iii) $d - c \in E_+ \setminus \{0\} \Longrightarrow g(c) < g(d), \forall g \in \mathcal{G} \setminus \{0\}$.

(H_4) The aggregate operator $A(f, \mu) = (\mathcal{G}) \int f\, d\mu$, where the $\mathcal{G}$-integral is given by

$$g((\mathcal{G}) \int f\, d\mu) = (Ch) \int g \circ f\, d\mu, \forall g \in \mathcal{G}.$$

Theorem. *Under assumptions $(H_0) - (H_4)$, we have $W(\mathcal{E}) \subset \mathcal{C}(\mathcal{E})$.*

References:

1. A. R. Sambucini. The Choquet integral with respect to fuzzy measures and applications. *Mathematica Slovaca*, **67**(6) (2017), 1427–1450.
2. C. Stamate. *Vector equilibrium problems and abstract economies.* 2017

ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO A CLASS OF INHOMOGENEOUS PROBLEMS

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The asymptotic behavior of the sequence of nonnegative solutions to a class of inhomogeneous problems settled in Orlicz-Sobolev spaces with prescribed Dirichlet data on the boundary of domain Ω is analysed. We show that this family converges uniformly in Ω o the distance function to the boundary of the domain. The report is based on joint works with Mihai Mihăilescu, Csaba Varga and Andrei Grecu. This presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-TE-2019-0456.

ON THE CALCULATION OF INTEGRALS BY USING FAMOUS INTEGRALS

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We will show how we can use some famous integral formulas, such as Gaussian integral, Dini integral, Fresnel integral to calculate other interesting integrals, using certain tricks (including derivation with respect to a parameter under the integral sign or development in series of powers).

CENTERS OF CUBIC DIFFERENTIAL SYSTEMS WITH TWO PARALLEL INVARIANT STRAIGHT LINES OF TOTAL MULTIPLICITY THREE

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We consider the real cubic system of differential equations

$$\dot{x} = y + P(x,y)\,,\ \dot{y} = -x - Q(x,y)\,,\ \gcd(y+P, x+Q) = 1, \tag{1}$$

and the vector field $\mathbb{X} = (y + P(x,y))\frac{\partial}{\partial x} - (x + Q(x,y))\frac{\partial}{\partial y}$ associated to this system. The critical point $(0,0)$ of system (1) is either a focus or a center. The problem of distinguishing between a center and a focus is called the *problem of the center.*

The straight line $\mathcal{L} \equiv \gamma x + \beta y - \alpha = 0$, $\alpha, \beta, \gamma \in \mathbb{C}$ is called *invariant* for (1) if there exists a polynomial $K \in \mathbb{C}[x,y]$ such that the identity $\gamma(y + P(x,y)) - \beta(x + Q(x,y)) \equiv \mathcal{L} \cdot K(x,y)$, $(x,y) \in \mathbb{R}^2$ holds. We say that $\mathcal{L}$ has multiplicity $m(\mathcal{L})$ if $m(\mathcal{L})$ is the greatest positive integer such that $\mathcal{L}^{m(\mathcal{L})}$ divides $(x+Q)\cdot\mathbb{X}(y+P) - (y+P)\cdot\mathbb{X}(x+Q)$.

Let the system (1) have the real invariant straight lines $\mathcal{L}_1 = 0, \mathcal{L}_2 = 0,\ \mathcal{L}_1 \parallel \mathcal{L}_2,\ m(\mathcal{L}_1) \geq 2$. Without loss of generality we consider that $\mathcal{L}_1 = x - 1$, $\mathcal{L}_2 = x - \alpha$, $\alpha \in \mathbb{R} \setminus \{0,1\}$. Then, the cubic system take the form

$$\begin{aligned} &\dot{x} = y(x-1)(x-\alpha)/\alpha \equiv \mathcal{P}(x,y)\,, \\ &\dot{y} = -(\alpha x + g\alpha x^2 + d\alpha xy + b\alpha y^2 - \alpha(1+g)x^3 + q\alpha x^2 y \\ &\qquad -(1-\alpha+b\alpha)xy^2)/\alpha \equiv \mathcal{Q}(x,y)\,,\ \gcd(\mathcal{P},\mathcal{Q}) = 1. \end{aligned} \tag{2}$$

Theorem 1. *The system* (2) *has at origin a center if and only if at least one of the following four sets of conditions is satisfied:*

$$\begin{array}{ll} (i) & b = 1,\ q = d(g+1); \\ (ii) & d = q = 0,\ b \neq 1; \\ (iii) & \alpha = 1/(1-b),\ g = -1,\ q = d(b-1),\ d \neq 0; \\ (iv) & \alpha = 1/(b-1),\ q = 0,\ g = -b,\ d \neq 0. \end{array} \tag{3}$$

In each of the cases $(i) - (iv)$ *the system* (2) *is Darboux integrable.*

Theorem 2. *The cubic system* (1) *with two real parallel invariant straight lines of total multiplicity three has at origin a center if and only if the first five Lyapunov quantities vanish* $L_j = 0, j = \overline{1,5}$.

Example.

$$\begin{aligned}\dot{x} &= y(x - 1(-5 + 12x + 2\sqrt{6}x)/5,\\ \dot{y} &= (-5x + 2(7 + 2\sqrt{6})x^2 - 5dxy - (8 + 3\sqrt{6})y^2 - (9 + 4\sqrt{6})x^3\\ &\quad +d(6 + \sqrt{6})x^2y + 5(3 + \sqrt{6})xy^2)/5.\end{aligned}$$

For this system the straight lines $x-1=0$ and $2(6+\sqrt{6})x-5=0$ are invariant. The line $x-1=0$ has multiplicity two. The first four Lyapunov quantities vanish: $L_1 = L_2 = L_3 = L_4 = 0$ and the five one no: $L_5 = 9(6+\sqrt{6})/2 \neq 0$. Therefore, the origin is a focus of multiplicity five.

SOME PERIODICITY AND DIFFERENTIABILITY PROPERTIES OF HAMILTONIAN SYSTEMS

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We consider the simplest Hamiltonian systems in the plane and iterated Hamiltonian systems in arbitrary dimension. The Hamiltonian is assumed to be of class C^1, that is the right-hand side of the differential systems is just continuous. The existence is a direct consequence of the Peano theorem, however the uniqueness is also valid via a special argument based on the Hamiltonian structure. Under the standard Poincaré-Bendixson assumptions (in dimension two), we show that the Hamiltonian structure does not allow the existence of limit cycles, that is the solution is periodic. The system in variations can be easily obtained and we also prove that the correspondence between the Hamiltonian and the associated period is differentiable and we derive the formula for the derivative of the period.

SEMISTABLE PAIRS AND THEIR MODULI IN HIGHER DIMENSIONS

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Let S be a nonsingular projective algebraic variety over a field $k = \overline{k}$ of zero characteristic, $\mathcal{O}_S$ its structure sheaf, E a coherent torsion-free $\mathcal{O}_S$-module. Let L be a very ample invertible sheaf on S; it is fixed and used as a polarization. The symbol $\chi(\cdot)$ denotes the Euler–Poincaré characteristic of a coherent sheaf. The following results generalizing the case when $\dim S = 2$ (see [1] and references therein) are to be discussed.

Theorem 1. [2] *Let T be an algebraic scheme of finite type over k and L be a (big enough) very ample invertible sheaf on a nonsingular projective algebraic variety S. Set $\Sigma = T \times S$. Let $\mathbb{E}$ be a T-flat coherent $\mathcal{O}_\Sigma$-module of rank r and such that $\mathbb{E}|_{t\times S}$ is a torsion-free coherent sheaf and the fiberwise Hilbert polynomial $\chi(\mathbb{E}|_{t\times S} \otimes L^n) = rp(n)$ is independent of the choice of $t \in T$. Then there are*

- *a flat family $\pi\colon \widetilde{\Sigma} \to T$ with a morphism $\boldsymbol{\sigma}\colon \widetilde{\Sigma} \to \Sigma$ of T-schemes,*
- *an invertible $\mathcal{O}_{\widetilde{\Sigma}}$-sheaf $\widetilde{\mathbb{L}}$,*

such that

- $\widetilde{\mathbb{E}} = \boldsymbol{\sigma}^*\mathbb{E}/\mathrm{tors}$ *is a locally free $\mathcal{O}_{\widetilde{\Sigma}}$-module,*
- $\chi(\widetilde{\mathbb{L}}^n|_{\pi^{-1}(t)})$ *is uniform over $t \in T$,*
- $\chi(\widetilde{\mathbb{E}} \otimes \widetilde{\mathbb{L}}^n|_{\pi^{-1}(t)}) = rp(n)$.

The transformation mentioned in Theorem 1 and described in [2] takes any coherent torsion-free $\mathcal{O}_S$-sheaf E on a polarized projective scheme S to an *admissible pair* of the form $((\widetilde{S}, \widetilde{L}), \widetilde{E})$, which is a generalization of the one in the dimension two (resp., homological dimension 1) case. In particular, $\widetilde{S}$ is an *admissible scheme*; its structure depends on the structure of the sheaf E under the resolution. We extend the notion of stability (semistability) for such pairs and provide a functorial approach to their moduli scheme. This gives rise to

Theorem 2. [3] *The (possibly, nonreduced) moduli scheme $\widetilde{M}$ for semistable admissible pairs with rank r and Hilbert polynomial $rp(n)$ is isomorphic to the (possibly, nonreduced) Gieseker–Maruyama moduli scheme $\overline{M}$ for semistable coherent $\mathcal{O}_S$-sheaves with the same rank and Hilbert polynomial.*

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References:

1. N. V. Timofeeva. Admissible pairs vs Gieseker–Maruyama. *Sb. Math.*, **210:5** (2019), 731–755.
2. N. V. Timofeeva. *Moduli of admissible pairs for arbitrary dimension,, I: Resolution.* arXiv:2012.11194 [math.AG] http://arxiv.org/abs/2012.11194, 2020.
3. N. V. Timofeeva. *Moduli of admissible pairs for arbitrary dimension, II: Functors and Moduli.* arXiv:2012.13660 [math.AG] http://arxiv.org/abs/2012.13660, 2020.

THE PARAMETRIC METHOD FOR SOLVING THE "BOTTLENECK" MULTI-CRITERIA FRACTIONAL TRANSPORTATION PROBLEM WITH "FUZZY" COST DATA

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In this paper we develop an algorithm for solving the multi-criteria fractional transportation problem with the same "bottleneck" denominators, including its separately and with "fuzzy" cost data. It generates for each (feasible) time value the best compromise multi criteria solution for each cost probabilistic parameter. So, finally, we will obtain one large and finite set of efficient solutions for solving the problem, which consists of several sets of efficient solutions, each corresponding to an allowable time value. The mathematical model of the proposed problem is the follows:

$$\min Z^k = \frac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \tilde{c}_{ij}^k x_{ij}}{\max\limits_{ij} t_{ij} \,|x_{ij} > 0\}} \tag{1}$$

$$\min Z^{k+1} = \max_{ij} t_{ij} \,|x_{ij} > 0\,\} \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1, 2, \ldots, m; \quad \sum_{i=1}^{n} x_{ij} = a_i, j = 1, 2, \ldots, n; \tag{3}$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, r. \tag{4}$$

In order to solve the model (1)-(4) we proposed an iterative algorithm [1]. It generates for every time possible value the corresponding set of the "best

compromise solution" of the first k criteria [2]. The algorithm was tested on several examples and was found to be quite effective.

References:

1. A.K. Bit, M.P. Biswal, S.S. Alam. Fuzzy programming approach to multicriteria decision making transportation problem. *Fuzzy Sets and Systems*, **50** (1992), 35–41.
2. A. I. Tkacenko. Multiple Criteria Fuzzy Cost Transportation Model of "Bottleneck" type. *Journal of Economic Computation and Economic Cybernetics Studies and Research*, Bucureşti, Romania, ISI Thomson Reuter Serv., **48**(2) (2014), 215–232.

THE SOLUTION OF THE DIOPHANTINE EQUATION $2^x + 33^y = z^2$

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In number theory the Diophantine equations are studied, equations in which only integer solutions are permitted. The famous general equation $a^x + b^y = z^2$ has many forms. The literature contains a very large number of articles on non-linear such individual equations [1–6].

In this paper, we solve the equation $2^x + 33^y = z^2$, where x, y, z are non-negative integer numbers.

Theorem. *The Diophantine equation* $2^x + 33^y = z^2$ *has exactly three non-negative integer solutions:* $(x, y, z) \in \{(3, 0, 3), (4, 1, 7), (8, 1, 17)\}$.

References:

1. D. Acu. On a Diophantine equation $2^x + 5^y = z^2$. *Gen. Math.*, **15** (2007), no. 1, 145–148.
2. S. Chotchaisthit. On the Diophantine equation of $2^x + 11^y = z^2$. *Maejo Int. J. Sci. Technol.*, **7** (2013), no. 2, 291–293.
3. C. E. Pumnea, A. D. Nicoară. On a Diophantine equation of $a^x + b^y = z^2$ type. *Gen. Math.*, **4** (2008), no. 1, 65–75.
4. J. Rabago. On the Diophantine equation of $2^x + 17^y = z^2$. *J. Indones. Math. Soc.*, **22** (2016), no. 2, 85–88.
5. B. Sroysang. More on the Diophantine equation $2^x + 19^y = z^2$. *Intern. J. Pure and Applied Mathematics*, **88** (2013), no. 1, 157–160.
6. A. Suvarnamani. Solutions of the Diophantine equation $2^x + p^y = z^2$. *Intern. J. Mathematical Sciences and Applications*, **1** (2011), no. 3, 1415–1419.

ON THE COMPLETE ENUMERATION OF 3-ISOHEDRAL SPHERICAL TILINGS FOR GROUP SERIES $n\times$

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As is known there are 7 infinite series and 7 sporadic discrete isometry groups of the sphere. In [1] we obtained all the fundamental Delone classes of 2-isohedral tilings of the sphere with disks, i.e. tilings with 2 transitivity classes of disks. Some splitting procedure applied to fundamental 2-isohedral tilings yields fundamental 3-isohedral tilings. All the fundamental 3-isohedral tilings have already been obtained for group series $*nn$, nn, $*22n$, and $n*$.

Now we turn to the series $n\times$ of isometry groups of the sphere, which corresponds to the series $\widetilde{2N}$ of 3-dimensional point groups of isometries. Earlier we obtained so-called proper 3-isohedral tilings of the sphere by disks with at least 3 vertices. Applying the splitting procedure to all the 20 series of Delone classes of fundamental 2-isohedral tilings of the sphere with disks, gives 293 series of Delone classes of fundamental 3-isohedral tilings of the sphere with disks.

The results coincide with the numerical results of [2], where the author developed some algorithms based on the theory of Delaney–Dress symbols and implemented algorithms using computer.

References:

1. E. A. Zamorzaeva. Classification of 2-isohedral tilings of the sphere. *Bul. Acad. Ştiinţe Rep. Moldova. Matematica*, **3** (1997), 74–85. (In Russian)

2. D. H. Huson. The generation and classification of tile-k-transitive tilings of the Euclidean plane, the sphere and the hyperbolic plane. *Geometriae Dedicata*, **47** (1993), 269–296.

III. COMPUTER SCIENCE AND IT

ANALYSIS OF MODERN AUTHENTICATION SCHEME TAILORED FOR IOT ENVIRONMENT

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The Internet of Things (IoT) technology brings numerous benefits and makes human life more convenient. Various modern systems are IoT-based, e.g., healthcare, monitoring, and transportation. However, the novel COVID-19 increases the need for IoT-based systems that intend to reduce human engagement to withstand the harmful consequences of physical distancing, also to increase systemsвЂ™ efficiency. The IoT sensors can sense and share data with other system entities. However, sharing relevant data over the public channel is subject to numerous security concerns. Fortunately, authentication is an important security service that can protect systems against potential attacks, e.g., reply, impersonation, DOS, man-in-the-middle, and lost/stolen smart card attacks. The authentication scheme should ensure mutual authentication among honest entities before gaining access and thus preventing unauthorized access. It is worth mentioning that the perfect authentication scheme should be secure against potential attacks. Additionally, due to the resource limitations, authentication scheme should be lightweight. In this context, many authentication schemes are proposed. Unfortunately, most of the existing authentication schemes are vulnerable to several attacks. Accordingly, IoT-based security is still an open research area that require further study. Hence, this abstract is about the analysis of the weaknesses associated with existing authentication schemes, which can help design new secure and lightweight authentication schemes.

The COVID-19 pandemic has forced many changes onto the way we live and work. Physical distancing, travel restrictions, and other health measures have bad consequences on the productivity of many organizations. Whenever it is possible, many organizations are changed to manage their activities via online communications. However, this huge shift toward remote work expands the need for security solutions to withstand potential risks. Unfortunately, most of the existing authentication schemes are suffering from several weaknesses that need to be addressed.

APPLICATION OF GRAY SCALE IMAGE PROCESSING ALGORITHMS TO COLOR IMAGE PROCESSING

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We have previously developed a pyramidal HWV color model as a discrete, one-to-one mapping into the cubic RGB hardware model. Its discrete coordinates from the very beginning correspond to the smallest, largest and intermediate between them components of the (r,g,b) pixel. Coordinates have strict unambiguous definition and at the same time they quantitatively and qualitatively correlate with the commonly used perceptually-based color attributes, such as whiteness (lightness), brightness and serial number of color. Their choice most closely corresponds to the principle of the human visual system functioning, because coordinates values are determined as a result of intercomparison, which is more consistent with the opponent theory of color.

The aim of this work consists in developing of the color image processing algorithms based on separate variation of the color chromatic (brightness v) and achromatic (lightness w) intensities defined in HWV color space. Basically all processing algorithms developed for processing gray scale images are applicable to these color components: γ-correction, expansion of the contrast range, histogram equalizing, etc. The problem, however, is that these components are interconnected with the hue component of color and their variation leads to distortion of the chromaticity characteristic. Proportionality relations, that compensate these distortions, are deduced in the paper. Specific examples show that the quality of a color image is really determined by the relative distribution of the chromatic and achromatic components in image.

POSSIBILITIES OF INCREASING THE OBJECTIVITY OF SUMMARY AND FINAL EVALUATIONS

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Critical situations bring correctives in daily activities. This is exactly the situation created as a result of the pandemic, when a lot of activities moved online. Thus, the pandemic also brought its corrections, moving many activities to the online version. Starting almost from the beginning of the pandemic, many teaching activities have been transferred onto the online. Although distance learning procedures are not something new in teaching, however, the massive transition to online development at all levels had been a little unusual for distance learning, because it still consisted of activities that used to take place in the classroom. These are particularly activities that require more rigorous control, including both summary and final evaluations. One of the most important criteria of evaluations is objectivity. This criterion belongs to both actors of the evaluation: teacher and student. The teacher must carefully weigh the load of the evaluation and correlate it with the time given and the curriculum of the discipline. The student on the other side, has to come with the required knowledge and correctness in carrying out the tasks. This is the only way to achieve the desired objectivity. Both conditions are quite difficult to achieve, but the student's role in disrupting the level of objectivity can be rather significant. By generating less "successful" tests, the teacher, as a rule, can raise or lower the level of the average grade of a group, as a result of obtaining a slightly higher or slightly lower grade for each student. On the other hand, the student in the testing process can act either correctly or incorrectly. His incorrect actions would lead to obtaining an increased grade, that would not be in correspondence to his real knowledge. In the conditions of conducting an online evaluation, the student is inclined to make use of incorrect actions. Although within the Faculty there has been set certain requirements regarding the conduct of online evaluations, that aimed to make the evaluation process more rigorous, there are still plenty of ways to cheat in the evaluation process, due to the multitude of freedoms difficult to control.

An idea of reducing the impact of existing freedoms would be that in the assessment process some restrictions would be imposed by the calculation system. First of all, a special application should be used, which will be called the evaluation environment. The given environment will have to be able to dictate a series of restrictions. Examples of possible restrictions could be:

- the possibility to open the necessary environment for the evaluation process;
- inability to open additional applications;
- the impossibility to minimize the evaluation environment;
- the impossibility to resize the evaluation environment;
- the impossibility to close the evaluation environment.

These restrictions are aimed to block the ability to get help from other applications and use the wallpaper as a source of information. At the same time, communication between students will be restricted through the computer system. Such measures will increase confidence in the objectivity of the online evaluation process.

The freedoms within the computer system are certainly not the only weaknesses of the online evaluation process. The use of a mobile phone as an aid is not excluded, but a number of rules can be formulated to keep the mobile phone visible.

A BRIEF HISTORY OF COMPUTER SCIENCE DEPARTMENT OF MOLDOVA STATE UNIVERSITY

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The history of Computer Science at Moldova State University is close related both to history of informatics teaching and research at Computer Science Department of the Faculty of Mathematics and Computer Science of Moldova State University, and to its history in our country, and the world. In this communication we present some interesting facts about the history of Computer Science Department, about scientific and didactic achievements, about its successes, difficulties, and failures, about the persons who has laid the base of information technologies in Moldova.

EMBEDDED VOICE PROCESSOR FOR EMERGENCY CONTROL

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New trends in development of the strategy Industry 4.0 are based on the nucleus of the cyber-physical system, this implying the integrated intelligent and communication systems aimed to increase the efficiency, performance and productivity [1]. The concept of Industry 4.0 foresees complex automation, with minimum engagement of human personnel, who can interfere only in case of emergencies (urgent situations, accidents or breakdowns). The majority of such emergencies may be detected through sounds, noises or voice expressions uttered by human beings. It is obvious that the process shall be either stopped, or switched to the maximum safety mode, in such situations.

It is suggested to use a specialized processor with cognitive features that will interfere in the technological process in emergencies and will either stop it, or will switch it to the maximum safety mode. The model of such a processor is defined by the following expression:

$$PV[T] = \{In[T], Kem[T], Kev[T], Out[T]\}$$

where $PV[T]$ is the configuration of the processor at a point of time T, $In[T]$ is the processor entry (sound, noise or voice signal), $Out[T]$ is the processor exit (signal of action), $Kem[T]$ is the knowledge model for deciding on taking actions (signal generation $Out[T]$), $Kev[T]$ is the knowledge model for knowledge evolution $Kem[T]$.

The evolution of knowledge models is based on the following expression:

$$PV[T+1] = \left\{ \begin{array}{c} Kem[T] \xrightarrow{Kev[T],Out[T]} Kem[T+1], \\ Kev[T] \xrightarrow{Kem[T],In[T]} Kev[T+1] \end{array} \right\}$$

where $PV[T+1]$ is the new state of knowledge model $Kem[T+1]$ and $Kev[T+1]$ obtained as a result of use of knowledge models $Kev[T]$, $Kem[T]$, of entry data $In[T]$ and of exit data $Out[T]$.

References:

1. V. Kumar, et al. *Ontologies for Industry 4.0. The Knowledge Engineering Review* Vol. 34, el17, 1-14, 2019. DOI: 10.1017/S0269888919000109.

INTELLIGENT SUPPORT FOR ASSESSING THE LEVEL OF MATURITY OF INFORMATION SECURITY

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Determining the level of maturity of information security (IS) is a mandatory requirement and/or good practice for the modern Internet and Web-based e-businesses.

Product architecture and used tools. During the doctoral research, a Web application was developed to identify the level of maturity of the IS, the gaps of the IS controls and the automatic generation of the risk report, further used to plan the necessary actions to improve the IS. Utilized tools for the back-end are PHP and MySQL, and for the front-end are HTML, CSS, JavaScript, without any frameworks or dependencies on external libraries, with multicriteria authentication (user + password + token).

The application is based on a Flexible and Extendable Maturity Metamodel, which meets the best known IS practices at the moment (e.g. ISO 27k family, NIST-SP 800, PCI DSS), the most widely accepted vulnerability dictionaries, solutions and products recommended by IS, intended for the subsequent intelligent generation of particular models.

Application domains. The application is made and approved for commercial banks, but it is possible to use it for any other institution.

Application scenario. In the first stage, the supervisor, auditor or regulator (SAR) configures the assessment areas and their content. After that, the institution approves and completes the questionnaire divided by areas and self-evaluates its maturity level according to the descriptors, with the attachment of evidence. In the second stage, the SAR verifies the self-assessment report against reality, confirming / refuting the level of maturity on descriptors with commenting on the detected divergences / gaps and proposing the recommended and/or mandatory areas for improvement. In the third stage, the institution analyzes the proposals and draws-up an improvement plan in accordance with a scale (e.g. if the obtained maturity level is lower than required). In the fourth stage, the application automatically generates a ratio with the risk and maturity levels (by areas), presented also in the form of dynamic radar graphics, on levels of descriptors, risks, controls: e.g. 0-19%, is not effective or is missing = 1; 20-39%, needs improvement = 2; 40-59% in the environment is effective = 3; 60-79%, efficient = 4; 80-100%, strong/tall = 5. All this provides the necessary support for the continuous monitoring and improvement of information security.

ON CLASSIFICATION OF 17TH CENTURY FONTS USING NEURAL NETWORKS

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This paper represents a solution to the problem of identifying the fonts in the books printed in the 17th century with characters from the Romanian Cyrillic alphabet. At the same time, this paper is an extension of the paper presented in [1].

Considering the fact that in Wallachia (Romanian: Ţara Românească) there was a diversification of printing styles (some printing houses borrowed the Slavonic printing, others created their own printing based on the Cyrillic alphabet), some documents from the 17th century require particular approaches to processing, namely optical character recognition (OCR).

The problem of identifying the font in a document printed in the 17th century can be formulated as follows: *Given a document X from 17th century printed in Cyrillic Romanian and a set of N OCR models trained on documents involved in this period. Choose the most appropriate model M from set N for document X.*

A trivial solution would be to recognize a sample (a page snippet) from document X using all models in set N and based on the results, choose the model that offers the highest accuracy (best result). This solution is easy to implement, but the time complexity is too big, as we have to load each model separately. Model upload time and sample recognition can exceed 2 minutes depending on page size. If we have 5 different models, we'll have to wait for approx. 10 minutes to find the right model.

The second and the proposed solution would be to train a neural network to classify snippets from document X based on samples from several Romanian documents printed in the years 1630-1680 at different printing houses in Wallachia. A neural network will learn a data set consisting of tuples *(image character, class (Font1, Font2, Font3, ..., FontN))* in order to be able to further classify new samples.

Therefore, we train a neural network to classify between 2 distinct fonts used by two printing houses: *1. Cetatea Belgradului din Ardeal; 2. Casa Sf. Mitropolii din Iaşi;* from the 17 century.

References:

1. T. Bumbu, S. Cojocaru, A. Colesnicov, L. Malahov, Ş. Ungur. User Interface to Access Old Romanian Documents. In: *Proceedings of the 4th Conference of Mathematical Society of Moldova CMSM4-2017, June 25-July 2, 2017*, 479–482.

HETEROGENEOUS DOCUMENT PROCESSING: CASE STUDY OF MATHEMATICAL TEXTS

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Most of historical documents have heterogeneous character containing, along with the text, elements of another nature. The aim of our research is to create a web-based platform to process such documents obtaining presentation of non-textual elements in scripting languages.

The general problem is to recognize the document layout and then to apply recognition for each type of content [1]. One of subproblems is that of mathematical formula recognition.

Text OCR systems can't adequately recognize formulas. The republishing of a monograph [2] showed that mathematical formulas makes a half of the book. Inclusion of formulas in the reissued version was done manually consuming much more time and efforts than the text processing.

Since the introduction of deep learning techniques, the significant progress in formula recognition was achieved. Modern systems recognize rather complex formulas, both printed and handwritten.

Open source systems that solve this problem are, for example, im2latex, image2latex, LaTeX-OCR. We tested them and selected LaTeX-OCR.

LaTeX-OCR is written in Python and supported by clear instructions of install and run. It is supplied by dataset that contains about 200,000 items and covers all LaTeX base macros. This dataset can be supplemented by sample images from documents, which user intend to recognize.

We tested also several commercial systems: SESHAT, INFTY, Mathpix, MT-Recognition. The best results were demonstrated by Mathpix. It recognizes complex formulas and texts in many languages. During the testing Mathpix on a page from [2], it made only two errors because of image quality.

Commercial systems offer only limited access for free. For example, Mathpix performs free recognition of no more than 50 images per month.

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References:

1. A. Colesnicov, L. Malahov, S. Cojocaru, L. Burtseva. Semi-automated workow for recognition of printed documents with heterogeneous content. *Computer Science Journal of Moldova*, **28**, no. 3(84) (2020), 223–240.

2. V. Andrunachievici, I. Chitoroaga. *Numere şi ideale. (Numbers and ideals.)* Chişinău, Lumina, 1979. 224 p. (In Romanian (the Cyrillic script)). – Russian edition: Chişinău, Lumina, 1980. – Reprinted in Romanian (the Latin script): 2017.

THE KNOWLEDGE BASE STRUCTURE OF THE RESEARCH FIELD "MENTAL AND BEHAVIORAL DISORDERS IN EPILEPSY"

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The structure of the knowledge base of the research field "Mental and behavioral disorders in epilepsy" (MBDE) was proposed. The knowledge base represents a relationship of three sets:

$$\langle 28\, diagnoses \rangle \leftrightarrow \langle 17\, syndromes \rangle \leftrightarrow \langle 162\, symptoms \rangle.$$

The MBDE basic knowledge is a set of 19 tables organized hierarchically in three levels, which integrated 5012 elements. Each element of the basic knowledge is an integer in the range [0, 10].

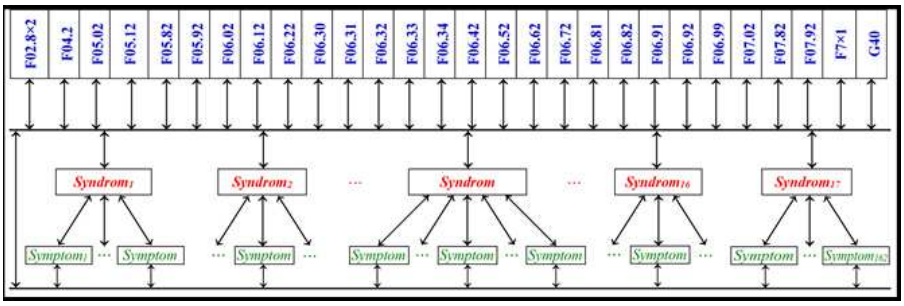

Figure 1: The structure of the MBDE basic knowledge

Based on this structure, the description by the numerical values of the MBDE research field was elaborated [1].

The MBDE knowledge base demonstrates the following advantages:

- A better understanding of the mechanisms and the obtained MBDE remissions.
- Transfer of experience obtained in MBDE remissions to other medical and research institutions.
- Development of intelligent support systems for MBDE.
- Training of specialists in medicine and computer science.

References:

1. Al. Popov, M. Butnaru, Gh. Căpăţână, A. Căpăţână. *Descrierea prin valori numerice a domeniului de cercetare "Tulburări psihice şi de comportament în epilepsie"*, Certificat AGEPI de înregistrare a obiectelor dreptului de autor şi drepturilor conexe. Seria OŞ (operă ştiinţifică) No. 5771 din 12.12.2017. USM, 2017. 24 p.

ASSESSING THE DISTANCES BETWEEN THE MENTAL AND BEHAVIORAL DISORDERS IN EPILEPSY

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A metric space was constructed to assess the distances between mental and behavioral disorders in epilepsy (MBDE) [1]. The table of distances between MBDE has been developed.

	F05.12	F02.82×2	F07.82	F07.92	F7×1	F05.52	F05.92	F07.02	F06.42	F06.91	F06.81	F06.62	F05.02	F06.12	F06.31	F05.82	F06.92	F06.30	F06.99	F05.32	F06.22	F06.34	F06.82	F06.72	F06.33	F04.2	F06.02	G40
F05.12	0.0	18.8	35.7	40.6	40.2	40.1	36.2	39.5	38.6	35.9	33.4	44.5	40.3	37.3	41.8	41.1	48.8	40.3	53.7	57.6	46.9	63.5	65.2	63.7	66.9	73.4	71.1	100
F02.82×2	18.8	0.0	25.2	31.8	32.4	33.3	35.2	35.8	35.8	36.9	37.4	38.6	39.5	40.1	40.1	40.3	41.0	43.6	45.9	50.3	50.5	54.5	56.4	57.3	58.5	66.8	69.1	94.5
F07.82	35.7	25.2	0.0	9.6	14.6	18.6	24.8	18.2	22.7	25.0	28.9	18.0	31.7	40.4	27.8	32.0	18.6	38.3	25.8	29.9	40.1	34.5	35.1	35.5	37.3	48.7	49.8	77.1
F07.92	40.6	31.8	9.6	0.0	12.2	19.9	26.9	16.0	22.3	24.8	30.0	15.3	35.2	42.2	27.9	35.9	16.4	39.3	22.7	26.7	38.9	31.6	32.0	32.3	33.7	47.0	45.5	76.2
F7×1	40.2	32.4	14.6	12.2	0.0	19.7	28.3	18.6	21.4	25.2	27.9	17.2	36.5	40.1	26.5	36.8	19.7	36.2	22.3	26.3	36.8	29.1	33.7	31.0	33.8	50.1	45.7	75.8
F05.52	40.1	33.3	18.6	19.9	19.7	0.0	18.4	19.2	11.3	17.8	22.4	12.2	25.1	35.1	20.3	23.8	15.0	31.7	18.2	27.0	35.3	31.4	29.5	28.5	29.9	41.2	40.5	65.9
F05.92	36.2	35.2	24.8	26.9	28.3	18.4	0.0	20.9	19.2	13.3	16.6	21.4	15.3	34.9	24.4	14.6	22.6	32.6	29.5	38.6	33.0	44.6	39.7	36.4	41.2	42.1	42.9	67.9
F07.02	39.6	35.8	18.2	16.0	18.6	19.2	20.9	0.0	17.2	15.3	21.4	14.8	28.5	38.0	25.8	28.9	16.8	32.8	20.4	29.7	31.5	33.8	32.0	32.7	33.7	46.1	44.0	74.4
F06.42	38.6	35.8	22.7	22.3	21.4	11.3	19.2	17.2	0.0	12.2	17.0	14.3	26.5	32.6	16.4	25.2	19.2	27.4	20.1	27.0	29.6	33.1	30.7	29.0	31.3	43.7	38.7	67.1
F06.91	35.9	36.9	25.0	24.8	25.2	17.8	13.3	15.3	12.2	0.0	10.0	18.4	21.3	32.5	19.7	21.1	22.1	26.7	26.1	33.2	27.5	39.6	36.9	33.7	37.9	44.1	40.3	70.2
F06.81	33.4	37.4	28.9	30.0	27.9	22.4	16.6	21.4	17.0	10.0	0.0	23.2	24.4	31.1	21.8	23.0	28.3	25.1	31.7	36.5	24.8	43.4	42.6	37.8	43.3	51.0	42.1	73.0
F06.62	44.5	38.6	18.0	15.3	17.2	12.2	21.4	14.8	14.3	18.4	23.2	0.0	29.6	39.0	22.4	28.5	9.6	34.6	14.6	21.6	32.9	27.4	25.0	23.5	25.9	40.8	34.4	65.5
F05.02	40.4	39.5	31.7	35.2	36.5	25.1	15.3	28.5	26.5	21.3	24.4	29.6	0.0	38.8	32.7	8.0	29.0	38.8	35.3	46.5	41.6	50.9	44.7	40.3	46.6	39.5	50.3	67.8
F06.12	37.3	40.1	40.4	42.2	40.1	35.1	34.9	38.0	32.6	32.5	31.1	39.0	38.8	0.0	28.2	38.2	43.2	20.9	44.6	50.0	26.9	55.0	54.7	52.6	55.0	63.0	58.0	84.5
F06.31	41.8	40.1	27.8	27.9	26.5	20.3	24.4	25.8	16.4	19.7	21.8	22.4	32.7	28.2	0.0	31.3	26.3	19.9	28.0	28.3	24.1	36.0	35.6	31.8	35.7	46.7	39.1	67.0
F05.82	41.1	40.3	32.0	35.9	36.8	23.8	14.6	28.9	25.2	21.1	23.0	28.5	8.0	38.2	31.3	0.0	28.4	36.9	34.6	44.9	39.3	49.4	43.0	39.2	45.0	41.2	47.4	65.1
F06.92	48.8	41.0	18.6	16.4	19.7	15.0	22.6	16.8	19.2	22.1	28.3	9.6	29.0	43.2	26.3	28.4	0.0	39.3	12.2	22.4	37.8	25.9	20.8	21.1	22.6	34.7	35.9	61.8
F06.30	40.3	43.6	38.3	39.3	36.2	31.7	32.6	32.8	27.4	26.7	25.1	34.6	38.8	20.9	19.9	36.9	39.3	0.0	40.1	41.6	20.3	46.8	49.4	45.7	49.1	60.9	50.6	78.7
F06.99	53.7	45.9	25.8	22.7	22.3	18.2	29.5	20.4	20.1	26.1	31.7	14.6	35.3	44.6	28.0	34.6	12.2	40.1	0.0	19.9	38.6	19.9	15.5	20.6	15.3	37.2	36.6	61.0
F05.32	57.6	50.3	29.9	26.7	26.3	27.0	38.6	29.7	27.0	33.2	36.5	21.6	46.5	50.0	28.3	44.9	22.4	41.6	19.9	0.0	40.3	13.6	20.3	21.9	19.7	45.8	34.0	62.9
F06.22	46.9	50.5	40.1	38.9	36.8	35.3	33.0	31.5	29.6	27.5	24.8	32.9	41.6	26.9	24.1	39.3	37.8	20.3	38.6	40.3	0.0	47.1	46.5	43.1	46.5	60.1	43.4	76.6
F06.34	63.5	54.5	34.5	31.6	29.1	31.4	44.6	33.8	33.1	39.6	43.4	27.4	50.9	55.0	36.0	49.4	25.9	46.8	19.9	13.6	47.1	0.0	19.2	24.4	17.0	47.4	40.1	63.6
F06.82	65.2	56.4	35.1	32.0	33.7	29.5	39.7	32.0	30.7	36.9	42.6	25.0	44.7	54.7	35.6	43.0	20.8	49.4	15.5	20.3	46.5	19.2	0.0	20.3	7.0	35.2	34.4	53.9
F06.72	63.7	57.3	35.5	32.3	31.0	28.5	36.4	32.7	29.0	33.7	37.8	23.5	40.3	52.6	31.8	39.2	21.1	45.7	20.6	21.9	43.1	24.4	20.3	0.0	19.0	32.0	27.0	48.4
F06.33	66.9	58.5	37.3	33.7	33.8	29.9	41.2	33.7	31.3	37.9	43.3	25.9	46.6	55.0	35.7	45.0	22.6	49.1	15.3	19.7	46.5	17.0	7.0	19.0	0.0	36.1	33.0	53.2
F04.2	73.4	66.8	48.7	47.0	50.1	41.2	42.1	46.1	43.7	44.1	51.0	40.8	39.5	63.0	46.7	41.2	34.7	60.9	37.2	45.8	60.1	47.4	35.2	32.0	36.1	0.0	42.4	45.3
F06.02	71.1	69.1	49.8	45.5	45.7	40.5	42.9	44.0	38.7	40.3	42.1	34.4	50.3	58.0	39.1	47.4	35.9	50.6	36.6	34.0	43.4	40.1	34.4	27.0	33.0	42.4	0.0	48.8
G40	100	94.5	77.1	76.2	75.8	65.9	67.9	74.4	67.1	70.2	73.0	65.5	67.8	84.5	67.0	65.1	61.8	78.7	61.0	62.9	76.6	63.6	53.9	48.4	53.2	45.3	48.8	0.0

Figure 2: The table of distances between MBDE

References:

1. Gh. Căpăţână, AL. Popov, M. Butnaru. *Tabelul distanţelor între tulburările psihice şi de comportament în epilepsie*, Certificat de înregistrare a Obiectelor Dreptului de Autor şi Drepturilor Conexe. AGEPI, 2021. 9 p. Seria O Nr. 6807 din 01.02.2021.

METHODOLOGICAL APPROACHES TO THE SECURITY OF VIRTUAL AND PHYSICAL SERVERS BASED ON OS LINUX

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Information security has become a topic of growing interest not only in the Linux community, but in all areas of information technology.

As the Internet develops, there is an increase of number of Linux servers, as well as Linux vulnerabilities that are made public. A Linux user has a wide variety of tools and techniques to protect against most types of intrusions. Unfortunately, there is no a secure system. Nevertheless, by improving network security measures as well as their proper implementation, the system has become a more difficult target to break its security.

Linux [1] has a series of features:

- Flexibility: Linux is flexible because it supports high-performance server applications, desktop applications, and embedded systems.

- Stability: In the Linux system, if a new program or software is installed, it does not require periodic restart. therefore, it maintains the performance level of the system.

- Performance: does not degrade the performance level of the system, even if it manages a large number of users simultaneously.

- Network compliance: Linux is an easy-to-use operating system in terms of network functionality because it can be easily configured.

Server security requirements [1] fall into one of the following categories:

- Data confidentiality;
- Data integrity;
- User authentication and access control;
- Availability of data and services.

As the cost of maintaining effective security is constantly rising, server administrators have decided to migrate private infrastructure to Linux operating systems. In order to maintain effective security we cannot claim that the information is in the security of a perfect defensive system regardless of all its facilities, therefore the ability of any security configuration must include the adaptability of the infrastructure according to new attacks and stress-tests to reduce detected vulnerabilities.

References:

1. R. Blum, C. Bresnahan. *Linux Command Line and Shell Scripting BIBLE*, Wiley, 2015.

WEB APPLICATION PERFORMANCE ANALYSIS

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Speed counts on mobile devices, but the perception of speed is just as important. A web application, like any other product, has features that directly or indirectly influence the business of owners. These characteristics are dictated primarily by the technological level of the tools available online, but also by the level of skill of the team carrying out the project. According to international studies, most people (75%) pay attention primarily to the loading speed of the web application. The next features in order of importance would be the accessibility of the information (66%), the adaptability of the interface (61%) and how attractive the design is (24%).

With the advent of high-speed internet connections, users have higher expectations from the performance of web applications. Search engines have taken this into account and have begun to develop algorithms that take into account the performance of search results. At the same time, the potential site can be the customer's first contact with the company, so a pleasant user experience is needed.

To meet market requirements, we have several optimization methods such as: minimized source code, optimized images by size, a clean source code, excluding unnecessary source code, gzip data compression, API with excellent response time, etc. All these practices are welcome, but without a correct choice of the type of rendering, their efficiency decreases dramatically.

Following the study, we highlighted the types of rendering and their characteristics to understand how web applications can be optimized at the highest level.

INFORMATION SECURITY IN IOT

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The Internet of Things or IoT refers to physical devices that are connected to the Internet, all collecting and sharing data, turning anything into an IoT component. For IoT security, fundamental algorithms are applied to find the physical location of a node in a network [1] that are based on the Delaunay triangulation and the Voronoi diagram. For S, a set consisting of a number n of nodes in the plane, any triangulation of S consists of $2n-2-k$ triangles and $3n-3-k$ edges, where n is the number of nodes in the set S and k is the number of nodes that are part of the convex cover. The Delaunay triangle of a set of data points S is defined by the following property: any circle circumscribed to one of the component triangles of the triangulation does not contain any other point of the set within it. A Delaunay triangulation is actually the dual graph structure associated with a Voronoi diagram.

The main properties of a Delaunay triangulation are [2]:

- The meeting of the triangles in the triangulation represents the convex coverage of the set S;

- Each triangle belonging to the triangulation corresponds to a vertex of the Voronoi diagram;

- The triangles in the Delaunay triangulation that have a side located on the boundary of the convex coverage of S correspond to an infinite (semi-straight) side of the Voronoi diagram.

References:

1. L. Lazos, R. Poovendran. SeRLoc: Secure range independent localization for wireless sensor networks. In: *ACM workshop on Wireless security (ACMWiSe 04), Philadelphia, PA, October 1, 2004.*

2. GE. Blelloch, GL. Miller, D. Talmor. *Developing a Practical projection-Based Parallel Delaunay algorithm.* Proc of the 12th Annual Symposium on Comput. Geometry, ACM, 1996.

DISTRIBUTED MIS ALGORITHMS IN COORDINATED GRAPHS

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Through this paper we want to provide an improvement of the bounds for computing maximal independent set (MIS) using distributed CONGEST model. Our research focuses on the number of rounds of distributed communication.

We study the problem in the case of coordinated graphs, which are undirected graphs constructed by adding a new vertex (called coordinator) and edges from it to all other vertices. The role of the coordinator is to assist in achieving a faster computation.

We assume that the communication occurs in synchronous rounds. Considering the fundamental constraint of the chosen model, which refer to the cost of sending messages, we restrict the number of bits that are processed and sent per round by each node to be polylogarithmic in n, the network size. Note that the computation performed by the nodes locally does not affect the number of rounds.

Firstly, we design two algorithms that compute a minimum cost spanning tree and a BFS tree from the root node in $O(\log n)$ rounds. Then, using these algorithms, we design other two algorithms that compute the MIS in $O(\sqrt{n})$, respectively $O(n^{\frac{1}{3}})$ rounds. Finally, we capitalize on the algorithms presented previously to design an algorithm that computes the MIS in ... rounds.

IMPLEMENTATION OF INTELLIGENT ALGORITHMS IN IMAGE PROCESSING

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Currently, image processing is a major field of research, which offers solutions to improve the quality of images made using digital technologies.

Image processing and analysis manage to gain ground by virtue of modern software development approaches, the evolution of innovative tools to support program development, as well as the peculiarities of state-of-the-art development environments. At present, in virtually any field of activity, there are many applications that require the use of image processing and analysis techniques, capable

of improving visual information. Digital image processing involves a succession of stages of hardware and software processing, as well as the implementation of theoretical methods.

Due to the visible successes of artificial intelligence, *machine learning methods* are becoming more widely used, offering solutions to many problems in various fields. Sophisticated algorithms of unsupervised learning methods are successfully implemented in the field of graphic processing [1]. These are used to classify images according to certain criteria, identify templates, monitoring of objects / processes in real time.

Therefore, the main idea of the paper is to present research for the development of a computer application for image processing, using algorithms and techniques of artificial intelligence, to contribute to: *improving pictorial information* to be interpreted by humans; and *processing image data for storage, transmission and representation* for the purpose of being perceived by autonomous machines.

The process of developing the computer application includes the use of new development tools: *Python programming language, OpenCV toolkit, keros libraries*. Functionalities allow the insertion of digital images and their processing with the help of machine learning algorithms. The obtained results can be used to detect the contours identified on the images in handwriting recognition.

References:

1. M. Cristei. Digital image processing by implementing unsupervised learning algorithms. *Studia Universitatis*, **7**(117) (2019), 43–49.

USE OF BIO-STATISTICS IN DATA ANALYSIS OF THE INFLUENCE OF BIOLOGICALLY ACTIVE SUBSTANCES ON PLANTS

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The paper presents ways to analyze data in the field of plant physiology grown on different nutritive environments and supplemented with compounds aimed to increase crop productivity and adaptation to various environmental conditions, as well as the analysis of influence of plant nutrition and the obtained results. The objectives of the research are to find non-trivial inter-dependencies between genetic data according to various factors such as nutrition, humidity, temperature, and their impact on agriculture in the Republic of Moldova.

In addition to the study of the association between variables, statistical techniques can also be used to determine the differences between groups: a

reference group (control) and independent groups, and allow the evaluation of the effects of an independent variable on a dependent variable [1].

In order to find out the degree of influence on the researched parameters and having a small sample, the t test is used.

Since the data sets are independent, having different variances, and the numbers of repetitions are equal, the difference is calculated [2].

1. Calculate the difference of means: $\overline{d} = x_1 - x_2$.
2. Calculate the error of differences, using the formula: $s_{\overline{d}} = \sqrt{s^2_{\overline{x}_1} + s^2_{\overline{x}_2}}$.
3. Determine the significant differences, relating the difference of means to the error of differences: $t = \dfrac{\overline{d}}{s_{\overline{d}}}$.

References:

1. N. Băbeanu. *Biometrie*, UASM, 2012.
2. J. McDonals. *Handbook of Biological Statistics.* University of Delaware, Sparky House Publishing, 2009.

SOME PARTICULARITIES IN ASSESSMENT OF ONLINE INFORMATION CREDIBILITY

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The online environment has become very large and very difficult to control. If before only a small number of people had access to place information on the Internet, now this has become very easy, including due to the advent of social media.

If we talk about approaches to assess credibility online, we must say that there are several approaches: automatic, manual and mixed [1]. All these 3 approaches can be achieved by linguistic analysis of the text and/or information from social networks. Even the manual assessment of online credibility involves the use of online tools such as: crowd-sourcing or online questionnaires.

Speaking about linguistic approach, most dubious news writers use their mother tongue strategically to avoid being caught. Despite trying to control what I say, mistakes occur in some verbal aspects that are difficult to monitor, such as the frequencies and patterns of use of pronouns, conjunctions, and words that express negative feelings [2].

Concerning the analysis of social networks, we mention their properties and behavior, which are ways to complete content-based approaches that are based on misleading language and clues to anticipate deception. As the real-time content of current events is increasingly proliferated through micro-blogging applications such as Twitter, deception analysis tools are all the more important [2].

Regardless of the approach chosen, any of them involves conducting experiments to evaluate the approach. Under such conditions, special data sets are required for such experiments. Research has shown that there are many data sets available for English and very few for other languages.

Regarding the existing data sets, they are textual information, but in some places have a different presentation structure. The differences in structure refer to the particularities of the experiments carried out, which depend on the spatial and social contexts, which are taken from social media networks.

Under these conditions, the basic goal should be to develop representative data sets in order to initiate experiments with reference to assessing the credibility of online information for the Romanian language.

References:

1. M. A. AlRubaian. Credibility in Online Social Networks. *IEEE Trans. Comput.*, **7** (2019), 2828–2855.

2. A. Iftene. *Exploiting Social Networks. Technological Trends.* Habilitation Thesis submitted at Alexandru Ioan Cuza University, December 2019.

C++ PROGRAMMING. LABORATORY NOTEBOOK FOR TECHNICAL VOCATIONAL EDUCATION

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The issue addressed: implementation in the technical vocational education institutions (TVE) of the Republic of Moldova of the Curriculum implement in 2020 for the school discipline of "Informatics", which requires the adaptation / updating of curricular supports.

The purpose of the intervention: providing qualitative support for TVE students, including a unique framework for carrying out practical introductory work in C++ programming, starting from the elaboration of logical schemes and program code in pseudocode language to the development and debugging of code in C++ with obtaining the expected results of the respective tasks. The paper is also useful for teachers who manage the educational process in accordance with the approved curriculum.

The results obtained: editing / publishing and distributing the paper with the above title, which corresponds to the National Curriculum, Curricular Area Mathematics and Sciences, Computer Science classes X-XII, high school, beginner level. The laboratory notebook is intended to guide and assist students in carrying out practical and individual C++ programming activities; to highlight and suggest what and how students should learn, developing their intellectual work style. The paper contains 120 solved problems and examples, generic (self) assessment tasks, projects and case studies, necessary to obtain the expected knowledge and skills, to be acquired by a student in the context of formal training established by the curriculum. The set of tasks offers students an extension and deepening of understanding of the studied subject. In compiling the tasks, the different needs and possibilities of students were taken into account, formulating tasks with three different levels of complexity: low, medium / moderate, high. This allows each student to start from their own level of knowledge and understanding and progress to the desired level.

An achievement of the work is the compilation, running and debugging of the developed code with GDB – a free debugger with portable code and GNU license, accessible to anyone at https://www.onlinegdb.com/. GDB is simple, transparent, easy to use and supports multiple interface languages; allows demonstration in the classroom; runs on the Web and UNIX systems; supports many programming languages, including C, C++, Free Pascal, Free BASIC, Ada, Fortran, Python3, Swift etc.

THE MAXIMUM EQUALITY-FREE STRING FACTORIZATION PROBLEM: GAPS VS. NO GAPS[2]

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A factorization of a string w is a partition of w into substrings $u_1, \ldots, u_k$ such that $w = u_1 u_2 \cdots u_k$. Such a partition is called equality-free if no two factors are equal: $u_i \neq u_j, \forall i, j$ with $i \neq j$. The *maximum equality-free factorization problem* is to decide, for a given string w and integer k, whether w admits an equality-free factorization with k factors.

[2]This work was supported by project PN19370401 "New solutions for complex problems in current ICT research fields based on modelling and optimization", funded by the Romanian Core Program of the Ministry of Research and Innovation (MCI), 2019-2022.

Equality-free factorizations have lately received attention because of their application in DNA self-assembly. Condon et al. (CPM 2012) study a version of the problem and show that it is $\mathcal{NP}$-complete to decide if there exists an equality-free factorization with an upper bound on the length of the factors. At STACS 2015, Fernau et al. show that the maximum equality-free factorization problem with a lower bound on the number of factors is $\mathcal{NP}$-complete. Shortly after, Schmid (CiE 2015) presents results concerning the Fixed Parameter Tractability of the problems.

In this paper we approach equality free factorizations from a practical point of view i.e. we wish to obtain good solutions on given instances. To this end, we provide approximation algorithms, heuristics, Integer Programming models, an improved FPT algorithm and we also conduct experiments to analyze the performance of our proposed algorithms.

Additionally, we study a relaxed version of the problem where gaps are allowed between factors and we design a constant factor approximation algorithm for this case. Surprisingly, after extensive experiments we conjecture that the relaxed problem has the same optimum as the original.

MEMBRANE COMPUTING MODEL BASED ON JSON LANGUAGE

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Membrane computing (P-system) is a branch of natural calculation, which is inspired by the structure and way of activities of living cells [1,2]. The main concepts were formulated by Gh. Paun in 1998 and then were developed by many researchers, particularly, from the field of theoretical calculations. Membrane computing is based on the notion of membrane that structures the calculation system (i.e. creates a topology of calculation units (cells)) in conformity with the mathematical model or data processing algorithm. A membrane defines a region that may comprise objects, rules or other regions functioning in parallel or concurrently. This thesis suggests a technique of formal description of topology of Membrane computing systems as based on JSON (JavaScript Object Notation) language, thus allowing to translate them automatically to reconfigurable Hardware structures (FPGA) for parallel or concurrent data processing. We provide here the topology of membranous calculation system as defined basing on the model:

$$\Big[_{M1} \cdots [_{M2} \cdots]_{M2} [_{M3} \cdots]_{M3} \Big]_{M1}$$

where M_i is the membrane i, $\cdots$ is the region of the membrane.

The JSON model of the topology of the Membrane computing system as defined basing on () is the following:

```
{       //Membrane Computing JSON Model
    "psystem" : [ {"name" : "M1", "regiondescription" : "..."},
    "membrane" : {"name" : "M2", "regiondescription" : "..."},
    "membrane" : {"name" : "M3", "regiondescription" : "..."} ] }
```

The JSON model gives a possibility to save the topology of the Membrane computing system to files, to organize the exchange of topologies between different applications and to realize dynamical processes in the topology thereof.

References:

1. Gh. Paun, G. Rozenberg, A. Salomaa, eds. *The Oxford Handbook on membrane computing.* Oxford University Press, Oxford, 2009.
2. R. Lefticaru, M. Gheorghe, F. Ipate. An empirical evaluation of P-system testing techniques. *Natural Computing*, **10** (2011), 151–165, Springer. DOI: 10.1007/s11047-010-9188-y.

STEPS IN CONTENT GENERATION FOR E-COURSES

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One of the new educational technologies that has shown its undoubted effectiveness is e-learning. In developed countries, e-learning covers all levels of education and is widely used not only in universities, but also in high school and in the organization of corporate (postgraduate) education [1].

Such platforms require the elaboration of high-quality and relevant teaching resources, the constant updating of existing ones. This, in turn, is a complex process consisting of processing a variety of materials, their analysis, synthesis, creative development and processing of all elements to build a single harmonious structure [2]. Up to now, far too little attention has been paid to dynamic content generation for e-learning courses.

The aim of this work is to describe development steps of the application that generates the content from the user request (key words). The program is based on our model for creating dynamic content of online training courses in Romanian, English and Russian languages.

According to our approach of the program model for the dynamic creation of training courses, we have 6 phases. At step 1 we managed to implement phase 1 and develop a web-crawler that creates networks of synonymous words. In

this article we describe the second step, where application uses original request and/or their selected synonyms for advanced search using Google search engine with the aim to extract only that paragraphs, which contain key words from the search-request and to store them in database. Such details as content type (html, pdf, audio, video) will be processed properly via "Content-Type" header of the web-page and web page markup. Thus, we obtain the behaviour model of a user performing manual scraping.

n the course of our research we found out that today's e-learning environment suffers from the cold start problem – lack of ready-made content for courses. At Phase 1 we used synonymous connections to search for dictionaries that are similar in meaning with the help of a crawler, and uses them at Phases 2–4 for advanced search using the Google engine. Our application succeeded in preprocessing searched data and exposed it in html format for user arrangement. The next 3 phases should be designed to present logically connected well merged content and export it in Moodle via Page and File standard plug-ins.

This article was written within the framework of the research project "20.80009.5007.22 Intelligent information systems for solving ill-structured problems, processing knowledge and big data"[3].

References:

1. D. Benta, G. Bologa, S. Dzitac. University Level Learning and Teaching via E-Learning Platforms. *Procedia Computer Science*, **55** (2015), 1366–1373.
2. J. Valverde-Berrocoso, M. Garrido-Arroyo, C. Burgos-Videla. Trends in Educational Research about e-Learning: A Systematic Literature Review (2009–2018). *Sustainability*, **12**(5153) (2020), 1–23.

APPLYING GENETIC ALGORITHMS TO SOLVE NON-LINEAR TRANSPORTATION PROBLEMS

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Non-linear transportation problems describe various real situations in the modern economy. These are complex models that can't be solved in reasonable time by the means of exact algorithms. Here heuristic algorithms come to our aid, a subset of which are genetic algorithms. Despite these algorithms not being able to guarantee a global solution, one can obtain a pseudo-optim in reasonable time even on large-scale problems. The use of these algorithms is recommended because it is not necessary to know the gradient or Hessian information and they are resistant to locks in a local minimum.

[3]http://www.math.md/en/projects/20.80009.5007.22/

We consider the transportation problem on a network described by a convex graph $G = (V, E), |V| = n, |E| = m$. On the finite set of vertices the real function of production and consumption $q : V \to R$ is defined. On the finite set of edges the concave non-decreasing functions of cost $\varphi_e(x(e))$ are defined. It is required to solve the non-linear optimization problem that consists in determining a flow x^* that minimizes the cost function:

$$F(x) = \sum_{e \in V} \varphi_e(x(e)) \ .$$

We must solve the non-linear problem:

$$F(x) \to min,$$

$$\sum_{e \in E^+} x(e) - \sum_{e \in E^-} x(e) = q(v),$$

$$x(e) \geq 0, e \in E,$$

where $E^-(v) = \{(v, u)|(v, u) \in E\}$, $E^+(v) = \{(u, v)|(u, v) \in E\}$ and X is the set of admissible solutions witch satisfies the conditions of the existence of the flow in the network.

The presented algorithms [1-2] can be used to solve several particular cases of the formulated problem. The algorithms were tested in Wolfram Mathematica.

References:

1. T. Paşa, V. Ungureanu. Solving the transportation problem with piecewise - linear concave cost function. *Review of the Air Force Academy, The Scientific Informative Review*, **XV**, no. 2(34) (2017), 49–56.

2. T. Paşa. The genetic algorithm for solving the non-linear transportation problem. *Review of the Air Force Academy, The Scientific Informative Review*, **XVI**. nr. 2(37) (2018), 37–44.

INVERSION OF CONSECUTIVE ALTERNATIONS

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The original notions of consecutive alternations and sums of consecutive alternations were introduced in the paper [1] in order to formulate a set of problems for the course "Compilation methods". The use of consecutive alternations for spiral traversal of square matrices, starting with the center element, was demonstrated in detail in [2]. The use of the inversions of consecutive alternations for the spiral traversal of the square matrices, starting with the element in the indicated corner, was demonstrated in detail in the paper [3].

This communication contains the classification and research of following types of traversal:

- LTCW (Left Top ClockWise) – the first movement from the top left corner to the right in a clockwise direction;
- LTCCW (Left Top Counter-ClockWise) – the first movement from the top left corner down in a counter-clockwise direction;
- RTCW (Right Top ClockWise) – the first movement from the top right corner downwards in a clockwise direction;
- RTCCW (Right Top Counter-ClockWise) – the first movement from the upper right corner to the left in the opposite direction of the clock;
- RBCW (Right Bottom ClockWise) – the first movement from the lower right corner to the left in a clockwise direction;
- RBCCW (Right Bottom Counter-ClockWise) – the first movement from the bottom right corner upwards in a counter-clockwise direction;
- LBCW (Left Bottom ClockWise) – the first movement from the bottom left corner upwards in a clockwise direction;
- LBCCW (Left Bottom Counter-ClockWise) – the first movement from the lower left corner to the right in a counterclockwise direction.

References:

1. A. Pereteatcu, S. Pereteatcu. Utilizarea problemei de calculare a sumelor alternanEJelor consecutive. In: *Conference MITRE-2019, Abstracts, June 24-26, 2019*, 93–94.
2. А. Перетятку, С. Перетятку. *Интерпретация чередований и их сумм (в дополнение к баазовым курсам по прогрпмированию).* РИО ИНФ, Кишинэу, 2019.
3. А. Перетятку, С. Перетятку. *Инверия последовательных чередований (в дополнение к баазовым курсам по прогрпмированию).* РИО ИНФ, Кишинэу, 2019.

MAXIMIZING A NEURAL NETWORK'S ACCURACY AND BOOSTING THE LEARNING PROCESS FOR DETECTING THE ABSOLUTE COLOR SIMILARITY BASED ON THE CENTROIDS OF NORMAL DISTRIBUTIONS

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Artificial neural networks (ANN) are a method of performing machine learning in which a machine learns to perform a task such as the detection of the absolute color similarity by analyzing training examples as a data set. Training is the process of identifying the ideal set of weights for maximizing the accuracy

of a neural network. However, the training process depends a lot on the training data, so that if the data contains a lot of noise then the learning process will take longer time to converge to a result with a level of accuracy that is not guaranteed to represent the highest accuracy and a ground thruth. The training data used by us, consists from tuples of triplets with parameters such as R,G,B channels of the RGB color space and target labels of 12 color classes which indicates the absolute color similarity for one of them. Our aim is to demonstrate that the ANN is prone to converge faster and with a higher level of accuracy using training data where tuples of triplets also represents the centroids of those 12 color classes, considering each centroid as a pixel for a color class, and as a mean, mode, and median for a normal gaussian distribution, instead of using the training data consisting from tuples of triplets randomly chosen. By the centroid of f (x) we mean the point with abscissa $\langle x \rangle$ such that the area of the function times $\langle x \rangle$ is equal to the first moment [1]. Thus,

$$\langle x \rangle = \frac{\int x f(x) dx}{\int f(x) dx} = x_0.$$

References:

1. R. Bracewell. *The Fourier Transform and Its Applications.* 3rd ed. New York: McGraw-Hill, 1999 (pp, 139-140, 156).

CONCEPT OF INFORMATIONAL TOOL FOR MAFLD AND COVID-19 SEVERITY INTERPLAY ASSESSMENT

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The earliest evidence suggested that obese patients, who often present Metabolic Associated Fatty Liver Disease (MAFLD), are even more vulnerable at severe presentation of COVID-19.

On the one hand, the effect of compromised liver function, as a result of MAFLD, on the severity of COVID-19 illness [1] has not been well studied yet. We aim to systematize the info and data of patients with fatty liver and evaluate the severity of COVID-19 in this cohort. On the other hand, most likely, these patients will receive drugs necessary for the treatment of acute respiratory

syndrome, antiviral agents and other drugs to treat systemic inflammation, combat bacterial and fungal superinfections. Among these pharmaceuticals can produce some drug-induced liver injury (DILI) [2]. The mechanisms of DILI are not always understood. There is a clear need for a deeper understanding of interplay between MAFLD and severity of COVID-19.

For more reliable and accurate results, we plan to create a partnership and combine the efforts of medical experts and the specialists in the field of medical informatics. The overall research objective is to elucidate relationship between fatty liver and COVID-19 by creating a data warehouse of MAFLD cases, based on a single ontology (protocol), and to provide intelligent information tool for quantifying and assessing the reciprocation of compromised liver function as a result of MAFLD and the severity of COVID-19 illness, and vice versa, COVID-19 treatment and MAFLD progression, using our existing experience [3].

Relevance of the assumed overall objective is enormous, as we have witnessed the wide spread of MAFLD, which predominantly affect people of working age, having a significant negative impact on social and economic development. COVID-19 pandemic only made things worse.

The research for this paper have been supported by the COST Action CA17112 "Prospective European Drug-Induced Liver Injury Network" and the project 20.80009.5007.22 "Intelligent information systems for solving ill-structured problems, processing knowledge and big data" in the framework of the State Program.

References:

1. P. Sharma, A. Kumar. Metabolic dysfunction associated fatty liver disease increases risk of severe Covid-19. *Diabetes Metab Syndr.*, **14** (2020), 825–827.
2. P.J. Ferron, T. Gicquel, B. Mégarbane, B. Clément, B. Fromenty. Treatments in Covid-19 patients with pre-existing metabolic dysfunction-associated fatty liver disease: a potential threat for drug-induced liver injury? *Biochimie*, **179** (2020), 266–274.
3. Iu. Secrieru, O. Popcova, E. Gutuleac. Quantification and Assessment of Diffuse Liver Diseases Using Deep Data Analysis. In: *Proceedings of the Conference on Mathematical Foundations of Informatics*, 2017, 166–167.

ADAPTIVE COMPUTING MODEL FOR DISTRIBUTED AND REAL-TIME APPLICATIONS

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The distributed computing systems as based on real-time applications provide advantageous solutions for complex process control. Such systems are a multitude of heterogeneous nodes that are integrated into data processing and

create a multicast communication network, where each node settles a part of a complex model, which is defined for the purpose of process control. The efficiency and performance of these systems are evident for distributed computing systems with a limited number of data processing nodes. However, in the context of the development of IoT technologies and Industry-4.0, which provides for the integration into a global network of all data processing nodes, delays in the communication process are possible, which will reduce the quality of the control process [1, 2]. In order to partially solve the above-mentioned problem, it is proposed to apply an adaptive calculation model. This model will reconfigure the interconnection topology of the data processing nodes which will reduce the delay time in the data transfer process. The process is defined as follows:

$P(t) = \{X(t), U(t), Q(t), M(t), \Delta t\}$, where

$$\begin{cases} X(t) \xrightarrow{F(t)} U(t+\Delta t), \\ M(t) \xrightarrow{Q(t)} M(t+\Delta t), \\ F(t) \xrightarrow{M(t)} F(t+\Delta t), \\ Q(t) \xrightarrow{F(t)} Q(t+\Delta t), \end{cases}$$

X(t) is the process state at a point of time t; *U(t)* is the action on the process; *Q(t)* is the system quality model; *F(t)* is the system control model; *M(t)* is the matrix of configuration of the topology of the network where data processing nodes interact; $t \to t + \Delta t$ is the dynamics of the process of adaptation of the system control model.

References:

1. O. Vermesan, P. Friess. *Digitising the Industry. IoT Connecting the Physical, Digital and Virtual Worlds.* River Publishers. 2016. 264p. ISBN: 978-87-93379-82-4.

2. M.P. Kumar, P. Sahithi, S.S. Revanth. Industry of Quality (IoQ) – An Industry 4.0 Perspective. *International Journal of Applied Research*, 6(4) (2020), 109-114. ISSN: 2394-5869.

ANALYSIS OF LEARNING MANAGEMENT SYSTEMS FEATURES

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Distance learning has not been widely used for several objective reasons in many countries until recently — mainly because of the underdevelopment of the technical means of new technologies. At present, technical premises have been created for the widespread use of distance learning in education and, with the restrictions imposed by Covid, the dynamics of their implementation was surprising.

E-learning software is represented by both simple static HTML pages and complex learning management systems (LMS), and learning content management systems (LCMS) used in computer networks. The further development is related, also, to the possibilities offered by the cloud environment.

Successful implementation [1] of e-learning is based on the correct choice of software that meets the specific requirements and objectives of the organization.

This article proposes an analysis of the usability and software functionality of the Open Source E-learning Platforms. Based on the survey of the state-of-art [2] science research we can highlight some of our results:

– a series of criteria for analysis of the LMS platforms were proposed: ∘ Functionality, ∘ Reliability, ∘ Stability, ∘ Availability of content development tools, ∘ SCORM support, ∘ Knowledge testing system, ∘ Easy to use, ∘ Modularity, ∘ Ensuring access, ∘ Scalability and extensibility, ∘ Multiplatform LMS.

– most popular distance learning platforms were analyzed,

– assessment of current LMS Integration capabilities was done.

LMS are commonly used for management of online learning activities inside single or many courses.

Modern trends in the development of Open Source LMS/LCMS are aimed at universalizing and increasing the functionality of systems. Most of open source systems have multiple platforms and are not related to specific operating systems or specific web browsers. In terms of their capabilities, the most advanced systems are not inferior to their commercial counterparts, and some are even superior. They are usually rich enough in different tools and options in order to allow modern interactive learning environment.

A wide-range comparative analysis of 8 electronic learning management systems was presented. All of them support the use of multimedia elements, creating and editing the lectures, exercises and course assignments.

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References:

1. N. I. Alhothli. *Investigating the impact of using moodle as an elearning tool for students in an English language institute*, 2015.
2. J. M. Fuentes, A. Ramírez-Gómez, A. I. García, F. Ayuga.. Web-based education in Spanish Universities. A Comparison of Open Source E-Learning Platforms. *Journal of Systemics, Cybernetics & Informatics*, (2012).

THE WOLFRAM LANGUAGE: MACHINE LEARNING, PROSE AND POETRY RESEARCH

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For almost 33 years, the Wolfram Language is a major computation and programming environment for millions of researchers, educators, students, and many other categories of creative people [1,2]. It offers to everyone the possibility to easily apply the most advanced computations and knowledge even in areas that seem to be extremely far from programming, numeric, symbolic, and technical computations.

In this work we present an illustration how the Wolfram Language, and Mathematica System, can be used to investigate poetry and prose, translations of literary works to other languages, and evaluation of translation quality.

First, we are starting from computation of some numerical characteristics of works in the original languages, and their translations into other languages, e.g., from Romanian to English, French, and Russian. Evidently, a good translation must preserve most characteristics of the original works. Is this so in reality? To make more objective conclusions, we apply some graphical, image, and sound tools. But we can use also some advanced mathematical tools such as the interpolation, and the curve fitting. Based on interpolation-functions (or fit-functions) that correspond to original works and their translations, we can evaluate good translation-works as ones for which interpolation functions (fit-functions) differ insignificantly from the interpolation-functions (fit-functions) of the original works and may be very close one to other if the parallel mathematical (geometric) translation is applied.

Second, we use Machine Learning to train a function that may recognize poetry and prose texts, that may find text's author, too. Should we train a function for every language, or it is enough to train one function for all languages?

May a trained function have a “polyglot” feature? If the trained author-function “understand” more than one language, may it be applied to evaluate good work-translation? Should the trained author-function understand who is the original language author of translated work?

Our work presents not only answers to the above questions, but it highlights a series of other interesting subjects, and the answers, and comments to them. The final discussion and conclusions may be seen as a good starting point to an interesting area of research: computational recognition of the original language author for a translated work.

References:

1. S. Wolfram. *An Elementary Introduction to the Wolfram Language*, Wolfram Media, 2018, second edition.

2. C. Hastings, K. Mischo, M. Morrison. *Hands on Start to Wolfram Mathematica: And Programming with the Wolfram Language*, Wolfram Media, 2020, third edition.

IV. DIDACTICS

PATTERN OF STUDENTS' KNOWLEDGE ASSESSMENT UNDER THE CONDITIONS OF DISTANCE TEACHING OF TECHNICAL SUBJECTS

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The educational process is a collective multi-criterial activity as based on different spheres of natural sciences, social sciences, economic sciences, humanities and engineering sciences [1]. The distance teaching and assessment process at higher educational establishments of professional training of engineering staff creates certain difficulties for the process of right assessment of knowledge gained. Such difficulties are conditioned by the lack of possibility for direct monitoring of the development and assessment process, but this may entail defrauding of the results. However, the challenge mentioned may be settled by using certain mathematical patterns for knowledge assessment, which are based on an ongoing process that comprises any and all tasks, curricular activities, knowledge shown and results achieved.

We define the teaching and assessment process

$$E = \{TP, P, L, C, F\},$$

where TP - contents of the teaching process; P - realization of practical works; L - realization of laboratory works; C - current assessments; F - final assessment.

The dynamics of the teaching and assessment process is defined by the following equalities:

$$\begin{cases} dp = (TP \cap P(t))dt;\ dl = (TP \cap L(t))dt; \\ dc = ((TP \cup P \cup L) \cap C(t))dt;\ df = ((TP \cup P \cup L) \cap F(t))dt. \end{cases}$$

The results of knowledge assessment are calculated as correlated with implication factors k_p, k_1, k_c, k_f from expression $Q(E) = (k_p p + k_l l + k_c c + k_f f)$ for time interval $[0, T]$, where

$$\begin{cases} p = \int\limits_0^T (TP \cap P(t))dt;\ l = \int\limits_0^T (TP \cap L(t))dt; \\ c = \int\limits_0^T ((TP \cup P \cup L) \cap C(t))dt;\ f = \int\limits_0^T ((TP \cup P \cup L) \cap F(t))dt. \end{cases}$$

References:

1. M.G. Lodico, D.T. Spaulding, K.H. Voegtle. *Methods in Educational Research. From Theory to Practice.* Wiley, 205p. ISBN: 978-0-7879-7962-1.

UTILIZAREA IMAGINILOR LA STUDIEREA AUTOMATELOR FINITE[4]

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Teoria şi aplicarea practică a automatelor, în particular a automatelor finite (AF), reprezintă unul dintre cele mai vechi şi activ studiate domenii ale informaticii. De rând cu aplicaţiile tradiţionale ale AF ce ţin de proiectarea compilatoarelor, inteligenţă artificială şi procesarea textelor, sunt elaborate aplicaţii moderne pentru procesarea limbajului natural, recunoaşterea vorbirii, modelarea şi testarea produselor software, probabilităţi (lanţuri Markov), jocuri video, procesarea imaginilor, teoria codificării etc.

La elaborarea acestor aplicaţii inevitabil apare necesitatea unor transformări echivalente asupra AF [1], cum ar fi:

- eliminarea stărilor inaccesibile şi neproductive,
- eliminarea ε-tranziţiilor,
- convertirea automatului nedeterminist în automat determinist echivalent,
- minimizarea AF şi altele.

Pentru majoritatea acestor transformări este foarte utilă reprezentarea AF sub formă de graf. Dacă reprezentările analitică şi tabelară nu implică dificultăţi la vizualizare, reprezentarea şi vizualizarea AF sub formă de graf este mult mai dificilă. În lucrare se propun algoritmi pentru generarea automată a reprezentărilor grafice ale automatelor finite (AF) şi vizualizarea lor. Metodele cunoscute sunt modificate pentru a explora proprietăţile specifice ale AF: existenţa unui singur nod sursă, flux informaţional bine determinat, absenţa nodurilor izolate.

Pornind de la definiţia AF (reprezentarea analitică) în mod automat se generează reprezentarea AF sub formă de graf $\Gamma = (Q, E, F)$, unde Q - mulţimea nodurilor (stările automatului), E - mulţimea de arce (q_i, a, q_j), iar F - mulţimea nodurilor-stări finale. Pentru a obţine reprezentarea grafică a fost elaborat un compilator, care generează pentru acest graf un program LaTeX/Ti*k*Z [2-4]. Acest compilator generează în mod aleator şi distribuie uniform coordonatele nodurilor (se are în vedere respectarea unei distanţe minime între oricare două

[4]Această lucrare a fost executată în cadrul proiectului de cercetare 20.80009.5007.22.

noduri, specificată în prealabil în dependenţă de numărul de noduri) şi şabloane pentru desenarea arcelor.

Aplicaţiile propuse sunt utilizate în procesul de instruire la studierea automatelor finite. Pentru a obţine o structură mai estetică a grafului este prevăzută posibilitatea intervenţiei manuale.

Bibliografie:

1. J.E. Hopcroft, R. Motwani, and J.D. Ullman. *Introduction to Automata Theory, Languages, and Computation.* Addison Wesley, 1979, 427 p.
2. Stefan Kottwitz. *LaTeX Cookbook.* Packt Publishing, 2015, 387 p.
3. *The TikZ and PGF Packages. Manual for version 3.1.6.* Institut für Theoretische Informatik Universität zu Lübeck, 2020, 1320 p.
4. Till Tantau. *Graph Drawing in TikZ.* Journal of Graph Algorithms and Applications, **17**(4), (2013), 495–513.

APLICAŢIILE NUMERELOR COMPLEXE ÎN REZOLVAREA DIFERITOR PROBLEME

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În această lucrare sunt considerate trei tipuri de probleme. Primul grup reprezintă probleme cu numere complexe, ce prezintă anumite greutăţi pentru elevi, legate atât de condiţii nonstandarde, cât şi de stabilirea metodelor de rezolvare [1].

În al doilea grup sunt incluse probleme din diferite compartimente ale matematicii, reformulând aceste probleme în mod corespunzător, se aplică pentru rezolvare metodele teoriei numerelor complexe, ce de regulă conduc la rezolvări mai simple şi mai efective comparativ cu metodele tradiţionale.

În ultimul grup au fost incluse diferite aplicaţii ale numerelor complexe în geometrie [2], probleme cu conţinut geometric, deducerea formulelor de bază pentru rezolvarea triunghiurilor, patrulaterelor convexe.

În raport vor fi prezentate diferite probleme rezolvate cu ajutorul metodelor enumerate mai sus.

Bibliografie:

1. T. Cohal, G. Popa. *Probleme de matematică clasa a X-a, ediţia 3.* Piteşti, Paralela 45, 2010, 296p.
2. N. Muşuroia, D. Heuberger. *Matematica de excelenţă – clasa X-a – pentru concursuri, olimpiade şi centre de excelenţă.* Piteşti, Paralela 45, 2016, 264p.

PRODUSE SOFTWARE PENTRU MODELAREA 3D

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Grafica tridimensională (3D) şi animaţia sunt componentele obligatorii ale unei serii vizuale moderne, utilizate în realitatea virtuală. Oricare ar fi proiectul - *realitate virtuală* sau *vizualizare arhitecturală*, *joc distractiv pe calculator* sau *producţie cinematografică*, va avea cu siguranţă grafică pe calculator şi, aproape sigur, animaţie. De fapt, animaţia şi grafica 3D sunt folosite în diverse domenii de activitate şi reprezintă baza spaţiului virtual de astăzi.

În domeniul graficii din realitatea virtuală, dezvoltările tehnologice sunt deosebit de dinamice: apar în permanenţă echipamente din ce în ce mai performante; apar numeroase biblioteci, limbaje şi programe utilitare (toolkits) care propun diferite modalităţi de abordare a aplicaţiilor grafice.

Generarea imaginii vizuale în realitatea virtuală implică două aspecte importante: *crearea modelului scenei virtuale* şi *vizualizarea scenei virtuale*. *Crearea modelului scenei virtuale* este un proces off-line şi, de cele mai multe ori, de durată considerabilă, prin care se creează colecţia de modele ale obiectelor 3D care constituie cea mai adecvată reprezentare a mediului virtual. *Vizualizarea scenei virtuale* este un proces on-line, care se desfăşoară în timp real, cu participarea uneia sau mai multor persoane, în care scena virtuală este explorată în mod interactiv şi, în fiecare moment, imaginea scenei redată pe display depinde de condiţiile de explorare (poziţie de observare, acţiuni interactive, etc.).

Prin urmare, ne-am propus analiza celor mai populare şi avansate aplicaţii pentru lucrul cu spaţiul volumetric, ce îndeplinesc cerinţele moderne ale profesionistilor din domeniul divertismentului, designului şi ingineriei, celor cărora le place să creeze/să modifice conţinut 3D personalizat. În analiza noastră, am abordat, de asemenea, problema complexităţii studierii programului şi a timpului necesar pentru adaptarea acestuia, deoarece lucrul cu modelarea 3D ar trebui să fie raţional, rapid şi convenabil, iar rezultatul să fie de înaltă calitate şi cel mai creativ. Rezultatul analizei este prezentat în tabelul următor:

CERINŢE	PRODUSE
1* Modelare interioară	Visicon, Sweet Home 3D, Sketch Up
2* Vizualizare	Autodesk 3Ds Max, Cinema 4D, Blender
3* Proiectare 3D subiect	Autodesk 3Ds Max, Maya, Blender, Cinema 4D, AutoCAD, NanoCAD
4* Sculpting	Sculptris, Blender, Cinema 4D, Autodesk 3Ds Max, Autodesk Maya
5* Creare de animaţie	Autodesk 3Ds Max, Autodesk Maya, Cinema 4D, iClone
6* Modelarea 3D a pieselor	Autodesk 3Ds Max, Autodesk Maya, Cinema 4D, iClone, Sketch Up, ZW3D
7* Modelare de divertisment	Lego Digital Designer, Sculptris, Paint3D

Deci, în concluzie, putem afirma că atunci când alegem un mediu pentru modelare 3D, în primul rând, este necesar să cunoaştem gama de sarcini pentru care este potrivit.

EVALUAREA E-LEARNING CU AJUTORUL LISTELOR DE VERIFICARE

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Metodele de învăţare electronică sunt utilizate din ce în ce mai mult în organizaţiile de astăzi în scopuri de predare, învatare si formare. Identificarea eficienţei metodelor de e-learning este esenţială pentru orice organizaţie. Modelele şi procesele de evaluare pot fi utilizate pentru a determina impactul e-learning-ului asupra organizaţiilor. Acest studiu trece în revistă principalele modele de evaluare e-learning existente. Se efectuează un studiu al listelor de verificare propuse pentru evaluarea învăţării electronice. Listele de verificare pentru evaluarea învăţării electronice s-au dovedit a fi un instrument util şi eficient, care poate ajuta la îndrumarea atât a evaluatorilor profesionişti, cât şi a celor care nu sunt evaluatori, atunci când efectuează o evaluare a învăţării electronice.

AN ALTERNATIVE APPROACH ON NEWTON AND LAGRANGE COEFFICIENTS OF THE INTERPOLATING POLYNOMIAL

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It is already well known that polynomials have a great theoretical importance, because they can approximate any continuous function over an interval, with a relatively small error. Nowadays, various interpolation algorithms exist in the literature (see [1,2]), and they propose approximating the value of a function at a point in an interval, knowing a finite number of values in points (nodes) of that interval. However, we can also program the computing of coefficients of the interpolating polynomial (Newton or Lagrange), if we take into account the formulas they proposed and the Viète's relations. The intention of this 'approach' is to have access, through the same program, not only to the approximate value of the function at a specified point, but also to its interpolating polynomial. We have thus tried to construct two interpolation algorithms, which provide the possibility of obtaining both the interpolating polynomial and the approximate value of the function at the specified point. Starting from the formulas developed by Newton and Lagrange, and from the relatively simple idea of using the Viète's relations (see [3]) for obtaining the coefficients of interpolating polynomials (Newton or Lagrange), two algorithms were obtained, and then implemented, and they allow an effective access to the coefficients of the interpolant polynomial of a function, and so to its approximate values at various points belonging to the studied interval. These algorithms also bring, in addition to obtaining the approximate value at a point, the "visualization" of the interpolating polynomial by which this approximation was achieved. Because of practical (numerical) reasons, Newton's polynomial interpolation is privileged to Lagrange interpolation, and since the approach (in terms of the algorithms we developed) is similar for both types of polynomials, we will present in detail the Newton interpolation, for Lagrange interpolation we will highlight only briefly the steps of the algorithm, so it can then be programmed using the desired programming environment. We chose to use the C language because it has been used in our courses corresponding to the first year students classes. The programs obtained are then run on a few examples (which we extracted from our own activities in the courses/laboratories classes), to test their functionality. The running tests performed confirm the functionality of these algorithms, as well as their usefulness.

References:

1. E. Meijering. A chronology of interpolation – from ancient astronomy to modern signal and image processing. *Proceedings of the IEEE*, **90**, no. 3, (2002), 319–342.
2. R. Militaru, A. Calin, G. Calugaru, A. Georgescu. *Pachet utilitar de Calcul Numeric – Numerical Engineering Software.* Ed. Sitech, Craiova, 2011.
3. M. Racila, *Metode numerice pentru studentii automatisti.* Ed. Universitaria, Craiova, 2020.

UTILIZAREA ALTERNANŢELOR CONSECUTIVE PENTRU PARCURGEREA ÎN SPIRALĂ A MATRICELOR PĂTRATICE, ÎNCEPÂND CU ELEMENTUL DIN CENTRU

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Autorii în lucrarea [1] au introdus noţiunile originale de alternanţe consecutive şi de sume ale alternanţelor consecutive. Aceste noţiuni au fost descrise cu scopul de a formula pentru cursul "Metode de compilare" o problemă, care poate fi rezolvată cu ajutorul calculatorului prin două metode diferite: (a) scriind programul cu utilizarea buclei **while**; (b) scriind programul cu utilizarea instruţiunilor **if** imbricate.

În lucrarea [2] s-a demonstrat detaliat utilizarea alternanţelor consecutive pentru parcurgerea în spirală a matricelor pătratice, începând cu elementul din centru.

Comunicarea de faţă conţine clasificarea şi cercetarea următoarelor tipuri de parcurgere: LCW (Left ClockWise) - prima mişcare la stânga şi apoi în direcţia acelor ceasornicului; LCCW (Left Counter-ClockWise) - prima mişcare la stânga şi apoi în direcţia opusă acelor ceasornicului; RCW (Right ClockWise) - prima mişcare la dreapta şi apoi în direcţia acelor ceasornicului; RCCW (Right Counter-ClockWise) - prima mişcare la dreapta şi apoi în direcţia opusă acelor ceasornicului; TCW (Top ClockWise) - prima mişcare în sus şi apoi în direcţia acelor ceasornicului; TCCW (Top Counter-ClockWise) - prima mişcare în sus şi apoi în direcţia opusă acelor ceasornicului; BCW (Bottom ClockWise) - prima mişcare în jos şi apoi în direcţia acelor ceasornicului; BCCW (Bottom Counter-ClockWise) - prima mişcare în jos şi apoi în direcţia opusă acelor ceasornicului.

Bibliografie:

1. A. Pereteatcu, S. Pereteatcu. Utilizarea problemei de calculare a sumelor alternanţelor consecutive. In: *Intern. Conference]]Mathematics & Information Technologies: Research and Education" (MITRE-2019), Abstracts, June 24-26, 2019.* Chişinău, CEP USM, 93–94.

2. А. Перетятку, С. Перетятку. *Интерпретация чередований и их сумм (в дополнение к баазовым курсам по прогрпмированию).* РИО ИНФ, Кишинэу, 2019.

TEHNOLOGII WEB, UTILIZATE ÎN DEZVOLTAREA CONŢINUTURILOR XR

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Realitatea extinsă (*XR*) cuprinde Realitatea Virtuală (*VR*) şi Realitatea Augmentată (*AR*). Realitatea virtuală este o realitate generată de computer şi poate fi accesată, folosind dispozitive imersive precum: căşti, mănuşi, ochelari etc. Realitatea augumentată, însă, este acea tehnologie, care face posibilă adăugarea de conţinut suprapus în lumea reală.

Pentru a fi posibilă transpunerea în lumea virtuală, sunt necesare echipamente costisitoare. În scopul diminuării cheltuielilor, au existat multiple tentative de a aduce XR-ul în mediul web. Extensia **WebXR Emulator**, pentru *GoogleChrome* şi *Firefox*, permite utilizatorilor şi dezvoltatorilor să ruleze şi să testeze conţinutul *WebXR* în browserele desktop, fără a utiliza un dispozitiv *XR* real.

Dezvoltarea *AR* în mediul web a căpătat amploare în ultimul timp, datorită posibilităţilor browserelor mobile, dar şi a dispozitivelor mobile. Din punctul de vedere al dezvoltării – utilizarea *WebAR* micşorează timpul de realizare a soluţiei. Ca bază este suficient un site, apoi se face conexiunea cu una dintre platformele *WebAR*, pentru a realiza contentul necesar [1].

WebXR cuprinde deja şi *WebVR*, termenul *WebVR* – nemaifiind utilizat. Pot fi recomandate două frameworkuri/biblioteci, care fac parte din top 3 cele mai utilizate tehnologii în domeniul WebXR: **Three.js** şi **Babylon.js** [3]. Aceste biblioteci sunt liber accesibile (au licenţă gratuită) – ambele sunt bazate pe limbajul de programare *JavaScript* – diferă doar sintaxa lor [2]. **Three.js** este o bibliotecă *JavaScript* care conţine un set de clase predefinite pentru crearea şi afişarea graficii 3D interactive în *WebGL*.

Bibliografie:

1. https://ar-js-org.github.io/AR.js-Docs/intro-image.gif
2. https://threejs.org/
3. https://habr.com/ru/post/494810/

DEZVOLTAREA COMPETENŢELOR DIGITALE LA STUDENŢII DE LA SPECIALITĂŢILE NON-IT ÎN CONDIŢIILE INSTRUIRII LA DISTANŢĂ LA UNITATEA DE CURS "TEHNOLOGII INFORMAŢIONALE ŞI COMUNICAŢIONALE"

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La etapa actuală, un loc important în formarea unui specialist care urmează să activeze în condiţiile societăţii informaţionale îl ocupă competenţa digitală. La o manifestare de înmânare a certificatelor de Aptitudini ECDL, Jose Manuel Barroso a declarat: "Aptitudinile IT reprezintă o competenţă de bază, precum cititul şi scrisul, acesta fiind mesajul pe care Comisia Europeană îl transmite către toţi cetăţenii europeni. În situaţia economică actuală dificilă, mulţi angajaţi vor avea nevoie să fie reinstruiţi şi competenţele IT pe care le pot dobândi îi vor ajuta să găsească noi oportunităţi pentru o viaţă mai bună".

În această lucrare ne-am pus ca obiectiv să descriem cum are loc dezvoltarea competenţelor digitale la studenţii de la specialităţile non-IT în condiţiile instruirii la distanţă, la unitatea de curs "Tehnologii informaţionale şi comunicaţionale".

Atât universitatea noastră, cât şi alte instituţii de învăţămuant superior din RM au ca obiectiv major pregătirea specialiştilor la un nivel compatibil cu standardele europene şi cu cerinţele pieţei muncii, ceea ce se realizează prin perfecţionarea planurilor de învăţământ, elaborarea şi modernizarea curriculumurilor la unităţile de curs orientate atât spre formarea, cât şi spre dezvoltarea competenţelor digitale şi asigurarea procesului de învăţământ cu cadre didactice de înaltă calificare.

Competenţa digitală este o abilitate esenţială a caracteristicii de calificare a unui specialist contemporan care activează în condiţiile unei societăţi informaţionale bazate pe cunoaştere. Exigenţele înaintate faţă de viitorii specialişti de la specialităţile non-IT vizează pe lângă nivelul înalt de pregătire teoretică şi practică, posedarea competenţelor digitale pentru activitatea profesională într-un mediu informaţional în permanentă schimbare. Activitatea unui specialist modern presupune utilizarea unei diversităţi enorme de date şi informaţii profesionale provenite din diferite surse şi având un grad diferit de prelucrare, ceea ce necesită posedarea diferitor tehnologii speciale pentru colectarea, analiza, prelucrarea, prezentarea şi aplicarea acestora.

De la un timp, tehnologiile informaţionale şi comunicaţionale au deschis uşile tuturor domeniilor profesionale, iar competenţa digitală a devenit un parametru indispensabil al specialiştilor din orice domeniu de activitate umană.

APLICAŢII ALE ELEMENTELOR DE COMBINATORICĂ ÎN CONTEXTUL ABORDĂRII STEAM ÎN EDUCAŢIE

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Matematica furnizează ştiinţelor naturii modele de raţionamente coerente şi clare. Tratarea riguros matematică a teoriilor din diverse domenii este indubitabil necesară. Problema dezvoltării capacităţii de a emite judecăţi de valoare pentru rezolvarea problemelor inventiv şi euristic, de a face conexiuni cognitive la nivelul disciplinei, a ariei curriculare şi inter-arii este una actuală în abordarea STEM/STEAM. Ea reflectă specificul activităţii intelectuale matematice la nivel de performanţe superioare, prin focalizarea atenţiei spre problemele de tip euristic-creative şi aplicative, în scopul formării unei imagini de ansamblu asupra ştiinţei ca sistem în permanentă evoluţie şi interacţiune cu mediul înconjurător.

Combinatorica este un compartiment al matematicii care vizează problemele de numuărare atât ca mijloc, cât şi ca scop al studierii proprietăţilor unor structuri. Ea are aplicaţii în contexte largi, traversează o multitudine de capitole şi se aplică în diverse domenii, precum logica, informatica, fizica, biologia, chimia etc.

Tipurile de probleme de combinatorică sunt variate, enunţurile referindu-se la stabilirea prin enumerare (numărare): a unor structuri specifice, ordonări, configuraţii; existenţa diferitor tipuri de structuri cu proprietăţi predefinite; construcţia unor structuri specifice şi identificarea versiunii optime a acestora.

Rezolvarea problemelor de combinatorică se bazează pe câteva strategii interconectate cu domeniul algoritmilor. Cea mai generală strategie împrumutată din domeniul algoritmilor este considerată "împarte şi cucereşte", care constă în divizarea unei probleme într-o serie de probleme mai mici, soluționarea acestor porțiuni şi combinarea ulterioară a rezultatelor obţinute într-o soluţie pentru "întreaga problemă" [1, p. 111]. Strategia reuneşte: regula sumei, regula produsului, regula sumei-produs, cernerea (ciuruirea), construirea grafului, recursivitatea, numărarea prin bijecţii, numărarea în două moduri ş.a.

Bibliografie:

1. A. Enghel. *Problem-Solving Strategies.* Springer Verlag, New York. 1998.

APLICAREA DERIVATEI FUNCŢIEI DE O VARIABILĂ ÎN DIVERSE DOMENII ALE MATEMATICII

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Noţiunea de derivată a unei funcţii într-un punct a fost introdusă de către Leibniz în scopul acordării unui sens dreptei tangente într-un punct $(x_0, f(x_0))$ la graficul acestei funcţii [1]. Studiind problema vitezei instantanee a unui punct material, Newton a ajuns la studiul limitei unui raport.

Făcând la lecţie analiza problemelor de acest tip, elevilor li se propune să găsească legătura dintre modul de studiere a problemelor de către Leibniz şi Newton. În acelaşi timp, se atrage atenţia la domeniul, în care au fost aplicate aceste cercetări. Se observă că noţiunea de derivată a funcţiei într-un punct este strâns legată de noţiunea de limită a funcţiei.

Sarcinile, care folosesc aplicarea derivatei funcţiei:

1. Una din problemele clasice, care a condus la noţiunea de derivată este *definirea ecuaţiei tangentei la o curbă plană.* Este important ca elevul să înţeleagă şi să facă legătura dintre valoarea derivatei funcţiei în punctul de tangenţă, coeficientul unghiular (panta), ecuaţia tangentei şi tangenta unghiului format de dreapta tangentă şi direcţia pozitivă a axei Ox [1].

2. În *studiul variaţiei unei funcţii* este important să cunoaştem în ce condiţii funcţia este constantă sau monotonă pe un interval dat. Monotonia se determină cu ajutorul semnului primei derivate.

3. Determinarea *numărului rădăcinilor reale ale ecuaţiei* - se aplică teorema Rolle.

4. Studiul *multiplicităţii rădăcinii unui polinom* folosind derivata.

5. *Problemele de maxim şi minim* dau posibilitatea să exemplificăm eficacitatea aplicării metodelor analizei matematice la rezolvarea unor probleme practice. Împărţirea clasei în mai multe centre ne dă posibilitatea să demonstrăm folosirea derivatei în mai multe domenii.

6. Aplicarea derivatei la *calculul limitei* - fiind puşi în situaţii concrete din viaţă, elevii se conving de necesitatea cunoaşterii derivatei pentru a-şi optimiza cheltuielile.

7. *Funcţii cu derivata nulă.*

8. Demonstrarea ıegalităţilor sau *inegalităţilor.*

Pentru fiecare caz sunt prezentate exemple rezolvate, selectate din mai multe resurse [2, 3]. De asemenea, este cercetată aplicarea pachetului MATLAB la calculul derivatei [4].

Bibliografie:

1. 1. I. Achiri, V. Garit, A. Poştaru, N. Prodan. *Matematică. Manuale pentru clasele a X-a, XI-a.* Chişinău, Prut Internaţional, 2010.
2. G. Geangalău. *1001 probleme de analiză matematică.* Chişinău, Universitas, 1994.
3. I. Achiri, M. Efros, P. Efros, V. Garit, A. Popa, V. Popa, N. Prodan, P. Sârbu. *Matematică: Culegere de exerciţii şi probleme pentru clasa X.* Chişinău, Prut Internaţional, 2001.
4. R. T. Trîmbiţaş. *Analiză numerică. O introducere bazată pe MATLAB.* Cluj-Napoca, Presa Universitară Clujeană, 2005.

ASPECTE DE APLICARE A VECTORILOR PENTRU SIMULAREA OPERAŢIILOR CU MULŢIMI ÎN C++

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Mulţimile au o aplicaţie practică foarte variată: sunt binecunoscute mulţimile de numere naturale, întregi, reale, complexe, etc. De asemenea, mulţimea angajaţilor unei companii sau instituţii, a elevilor, a studenţilor, a culorilor, etc. De aceea, apare necesitatea de a determina proprietăţile de aplicare a mulţimilor, a submulţimilor; a aranja elementele uneia sau a mai multe mulţimi într-o anumită ordine etc.

Un set este o structură de date echivalentă cu mulţimile din matematică. Un set constă din diverse elemente de un anumit tip şi acceptă operaţiunile de adăugare a unui element într-un set, eliminarea unui element dintr-un set şi verificarea apartenenţei elementului. Aceeaşi valoare este stocată în set doar o singură dată [1].

Deoarece în limbajul de programare C++ nu sunt definite mulţimile ca structuri separate, pentru implementarea acestora se folosesc vectorii. Vectorul este alcătuit dintr-un şir ordonat de elemente de acelaşi tip şi este definit ca tablou unidimensional. În versiunile noi este adăugat un container *set*, cu ajutorul căruia poate fi simulată gestiunea elementelor mulţimilor [2].

În lucrare sunt analizate modalităţile aplicării vectorilor-mulţimi, a containerului *set*, sunt cercetate operaţiile cu elementele mulţimilor şi este elaborat un set de programe la implementarea operaţiilor cu elementele mulţimilor: aplicarea

container-ului set la gestiunea mulţimilor de numere, determinarea submulţimilor de vocale şi consoane şi a numerelor prime, aplicând ciurul lui Eratosthene. De asemenea, sunt rezolvate 10 probleme pentru prezentarea operaţiilor cu elementele mulţimilor cu aplicarea vectorilor în C++ şi 10 probleme de determinare a cardinalului diverselor mulţimi, bazate pe utilizarea datelor studenţilor, elevilor, angajaţilor, a unor persoane referitor la ocupaţiile şi preferinţele lor.

Astfel, pentru realizarea operaţiilor cu elementele mulţimilor este eficient de folosit vectorii, elementele vectorilor, în prealabil, trebuie ordonate crescător. Utilizarea vectorilor statici în calitate de mulţimi duce la un surplus de memorie, deoarece numărul de elemente ale rezultatului final al operaţiilor cu elementele mulţimilor nu este cunoscut iniţial şi se alege maximal posibil.

Exemplele prezentate în setul de programe prezintă interes în practică şi permit de a înţelege aplicarea operaţiilor cu elementele mulţimilor-vectori.

Bibliografie:

1. I. Achiri, V. Garit, A. Poştaru, N. Prodan. *Matematică. Manual pentru clasa a X-a.* Chişinău, Prut Internaţional, 2002.
2. D. Lucanu. *Bazele proiectării programelor şi algoritmilor.* Vol. I, II, III. Iaşi, Ed. Universităţii "Al. I. Cuza", 1996. 265 p.

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