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OPERATING EXPERIENCE

Measurement of a Conductor Cross Sectional Area

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Abstract—An electrical method of indirect measurement of the cross sectional area of a conductor covered by a dielectric covering at casting in a suspended state is considered. The description of the method, the structural and electric equivalent schemes, and a mathematical treatment describing the essence and correctness of the method, as well as an error analysis of the method and its permissible possibilities, are given. Conditions under which the error of the measurement of the cross sectional area is the least are given.

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The continuity of a conductor dielectric cover obtained according to professor A.V. Ulitovskii's method [1] makes it impossible to use the traditional methods for measuring the resistance per unit of the conductor length, since such a conductor fails to have a full galvanic connection with the measuring circuit. Regardless of the measuring method and the circuit, at least one of the poles of the connection between the conductor being measured and the measuring circuit should be nongalvanic [2]. As a rule, the transient resistance of such a pole exceeds by several orders the resistance that is being measured. Therefore, for measuring the resistance per unit of length of the above conductor with sufficient accuracy for practice, there should be developed methods that exclude the poles resistances from the circuit of the conductor section being measured or at least minimize their values to ones sufficient for measuring the above resistance with sufficient accuracy for practice.

The present paper describes a measuring method including the Z resistance minimizing of one of the poles (of a complex character, as a rule) in a two-pole measuring circuit where the other pole that connects the measured conductor of R_1 resistance with the measuring circuit is resistive and the value of its junction resistance R_{pf} is significantly lower than the one being measured.

The essence of the method is explained by the structural scheme (Fig. 1a), the electrical equivalent circuits, and the voltage oscillograms (Fig. 2), which show that the measured conductor section of equivalent resistance $R_1 = r_x l_x$ and a sample conductor section of $R_{10} = r_0 l_0$ equivalent resistance with R_{pf} resistance of the preform (a preform is a thin metal bar whose end turns into a metal drop after being fused) are connected in a series circuit and together produce a negative feedback circuit of the OA operational amplifier, which is connected as a reversing circuit.

The sum of the $R_1 + R_2$ resistances, which are indicated in the scheme, form the amplifier load simultaneously with one arm of a four-arm bridge. The second arm of the bridge is formed by a series circuit $r_x l_x + R_{pf} + r_0 l_0$. The produced bridge is fed by the OA output voltage. The conductor section of l_x length that is being measured and a measuring arm of the bridge make a series circuit from one end via the PF preform of R_{pf} equivalent resistance (Fig. 1a) and from the other end via the CM coil of Z_{pf} equivalent and R_{inOA} input resistance of the operational amplifier.

The analysis of the method can be done with the help of the equivalent circuits presented in Fig. 1.

As one can see in the figure, the resistance $Z_{03} = (Z_b + R_{inOA})\left\|\frac{r_x l_x + r_0 l_0}{A}\right.$ between points 0–3 comes into the measured arm of the above bridge. The bridge equilibrium considering this resistance will occur when

$$R_l r_0 l_0 = R_2 |r_x l_x + Z_{03} + R_{pf}| \quad \text{or}$$

$$R_l r_0 l_0 = R_2\left[r_x l_x + R_{pf} + (Z_b + R_{inOA})\left\|\frac{r_x l_x + r_0 l_0}{A}\right.\right], \quad (1)$$

where A is the OA amplifying and r_x, r_0, l_0, and l_x are the resistances per unit of length and the length of the conductor section being measured and the one being formed, correspondingly.

The final values of the resistances $Z_{03}(Z_{03} \neq 0)$ and R_{pf} ($R_{pf} \neq 0$) give the conductor manufacturing error of the resistance per unit of length. The error value, as is shown below, significantly depends on the A amplification.

In order to determine these errors, let us examine two cases.

(a) The operational amplifier is ideal, i.e., $A = \infty$, $R_{inOA} = \infty$.

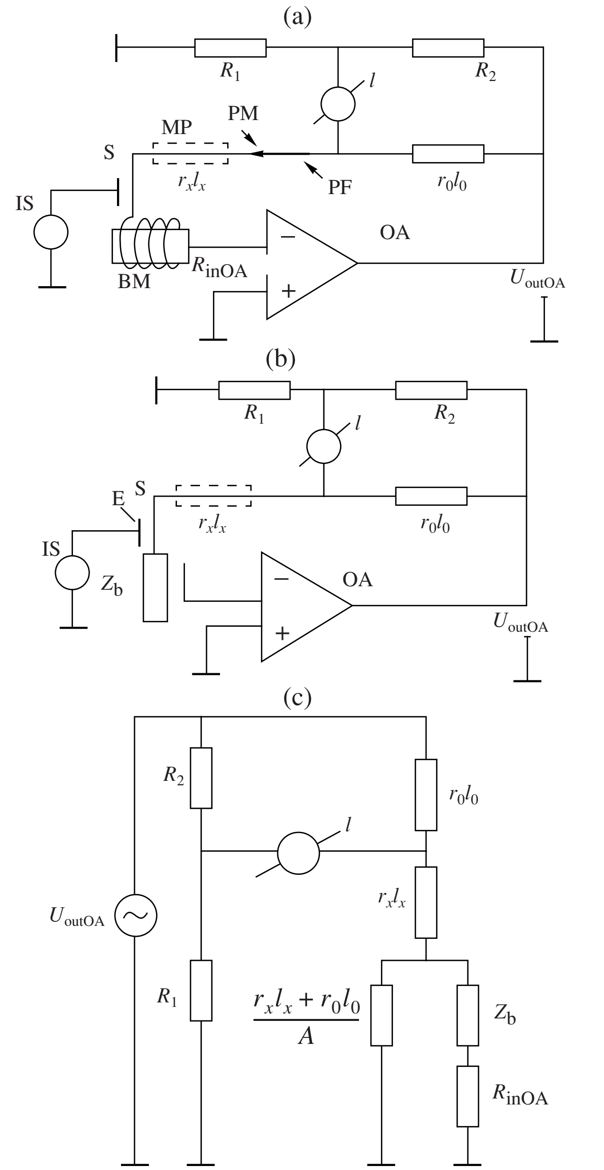

Fig. 1. Structural (a, b) and equivalent (c) circuits explaining the measuring principal of the cross section area with a dielectric cover at its casting.

In this case,

$$Z_{03} = (Z_b + R_{inOA}) \Bigg\| \frac{r_x l_x + r_0 l_0}{A} = 0, \qquad (2)$$

and, when $R_1 = R_2$, the resistance of the measured conductor section is equal to

$$r_x l_x = r_0 l_0 - R_{pf}, \qquad (3)$$

Table 1

A	10	100	1000	10000	10^5
∂d, %	–10.55	–1.01	–0.10	–0.01	0.00

(b) The operational amplifier is real, i.e., $A \neq \infty$, $R_{inOA} \neq \infty$.

In this case, at $R_1 = R_2$, the resistance $r_x l_x$ is equal to

$$r_x l_x = r_0 l_0 - R_{pf} - (Z_b + R_{inOA}) \Bigg\| \frac{r_x l_x + r_0 l_0}{A} < r_0 l_0. \qquad (4)$$

Actually, $(r_x l_x + r_0 l_0) \ll |Z_b + R_{inOA}|$, $A \gg 1$, and, as a consequence,

$$Z_{03} = \frac{r_x l_x + r_0 l_0}{A} \Bigg\| (|z_b| + R_{inOA}) \ll 1.$$

If we take into consideration the obtained inequality

$$Z_{03} = R_{03} = (r_x l_x + r_0 l_0)/A \qquad (5)$$

the condition of the bridge equilibrium becomes as follows:

$$R_l r_0 l_0 = R_2 \left(r_x l_x + R_{pf} + \frac{r_x l_x + r_0 l_0}{A} \right). \qquad (6)$$

When $R_1 = R_2$ and $l_x = l_0 = 1$, the resistance of the conductor section being measured (provided equation (6) is taken into account) has the following form

$$l r_x = r_0 l - \frac{l(r_x + r_0)}{A} - R_{pf}. \qquad (7)$$

From (7), it follows that the $l r_x$ resistance of the manufactured conductor differs from that of the sample conductor by the value

$$r_0 l - l r_x = \frac{l(r_x + r_0)}{A} + R_{pf}, \qquad (8)$$

and its relative divergence (in a percentage ratio) is

$$\partial r = \left(\frac{r_x + r_0}{A r_0} + \frac{R_{pf}}{r_0 l} \right) \times 100\%. \qquad (9)$$

The resistance of the conductor section being measured and the resistances of the sample conductor section and of the preform with their physical values (length l and diameter d) and with the specific resistances of the conductor thread and of the preform have the following relations:

$$r_0 l = 4\rho_0 \frac{l_0}{\pi d_0^2}, \qquad (10)$$

$$r_x l_x = 4\rho_x \frac{l_x}{\pi d_x^2}, \qquad (11)$$

$$R_{pf} = 4\rho_{pf} \frac{lL}{\pi D^2}, \qquad (12)$$

where L and D are the length and diameter of the perform, respectively.

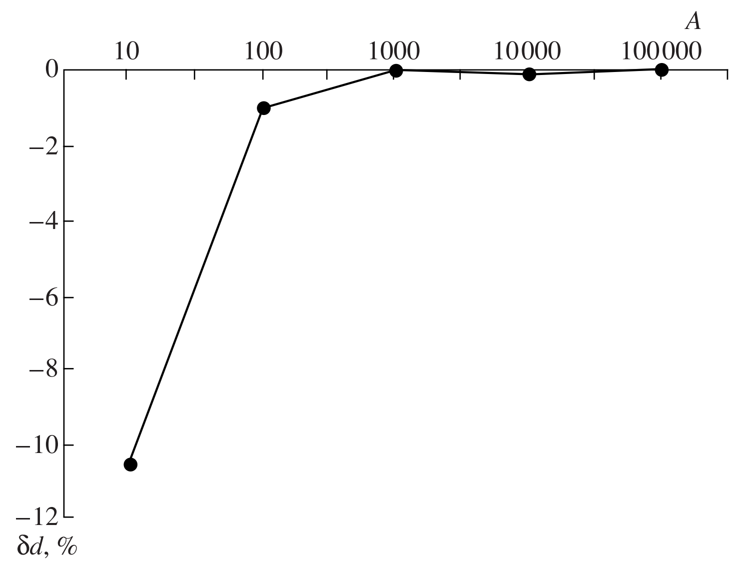

Fig. 2. The dependence of the measuring error of the conductor diameter as a function of the amplification of the operational amplifier.

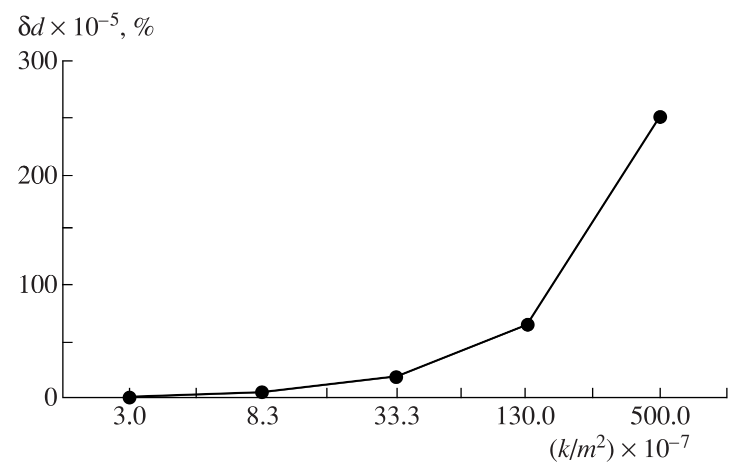

Fig. 3. Dependence of the measurement error of the conductor diameter as a k/m^2 function.

Substituting into (7) the relations (10), (11), (12) and considering that $\rho_0 = \rho_x = \rho_{pf}$, $l_0 = l_x = 1$, after the obvious reduction, we obtain

$$\frac{1}{d_x^2} = \frac{1}{d_0^2} - \frac{1}{A}\left(\frac{1}{d_x^2} + \frac{1}{d_0^2}\right) - \frac{kl}{m^2 d_0^2}, \quad (13)$$

where $k = L/1$, and $m = D/d_0$.

In order to simplify the estimation of the A and m values influence upon the divergence of the actual value of the d_x diameter from the sample d_0 diameter, equation (13) is disintegrated.

1. $k/m^2 = 0$, $A \neq \infty$, and then equation (13) obtains the following form:

$$\frac{1}{d_x^2} = \frac{1}{d_0^2} - \frac{1}{A}\left(\frac{1}{d_x^2} + \frac{1}{d_0^2}\right), \quad (13a)$$

hence,

$$d_x = d_0\sqrt{\frac{1 + \frac{1}{A}}{1 - \frac{1}{A}}}. \quad (13b)$$

Table 2

$K = L/l$	0.3	0.3	0.3	0.3	0.5
$m = D/d_0$	$3 \times 10^{-3}/3 \times 10^{-6}$	$3 \times 10^{-3}/5 \times 10^{-6}$	$3 \times 10^{-3}/10^{-5}$	$3 \times 10^{-3}/2 \times 10^{-5}$	$3 \times 10^{-3}/3 \times 10^{-5}$
k/m^2	$0.3/10^6 = 3 \times 10^{-7}$	8.3×10^{-7}	3.33×10^{-6}	1.3×10^{-5}	5×10^{-5}
$\partial d, \% = \left(1 - \frac{1}{\sqrt{1 - \frac{k}{m^2}}}\right) \times 100$	$1.5 \times 10^{-5}\%$	4.15×10^{-5}	16.6×10^{-5}	65×10^{-5}	250×10^{-5}

2. $k/m^2 \neq 0, A = \infty$, and then equation (13) appears as follows:

$$\frac{1}{d_x^2} = \frac{1}{d_0^2}\left(1 - \frac{k}{m^2}\right), \tag{14}$$

hence,

$$d_x = \frac{d_0}{\sqrt{1 - \frac{k}{m^2}}}. \tag{15}$$

From (13b) and (15), we can determine the divergence value (relative) of the diameter d_x from the preset d_0 for the first and second cases, correspondingly:

(1) $k/m^2 = 0, A \neq \infty$,

$$\partial d \longrightarrow \partial d_{\begin{cases} k/m^2 = 0 \\ A \neq \infty \end{cases}} = \left[1 - \sqrt{\frac{1 + \frac{1}{A}}{1 - \frac{1}{A}}}\right]; \tag{16}$$

(2) $k/m^2 \neq 0, A = \infty$,

$$\partial d_{\begin{cases} k/m^2 \neq 0 \\ A = \infty \end{cases}} = \left[1 - \frac{1}{\sqrt{1 - \frac{K}{m^2}}}\right]. \tag{17}$$

The total divergence is equal to

$$\partial d_{\begin{cases} k/m^2 \neq 0 \\ A \neq \infty \end{cases}} = \left[2 - \left(\sqrt{\frac{1 + \frac{1}{A}}{1 - \frac{1}{A}}} + \frac{1}{\sqrt{1 - \frac{K}{m^2}}}\right)\right]. \tag{18}$$

Tables 1 and 2 and Fig. 3 present the dependencies $\partial d_{\begin{cases} K/m^2 \neq 0 \\ A = \infty \end{cases}} = \varphi(A)$ and $\partial d_{\begin{cases} K/m^2 = 0 \\ A = \infty \end{cases}} = \varphi\left(\frac{k}{m^2}\right)$ for certain possible values of A and $\frac{k}{m^2}$, which show that, already at $A = 100$, the divergence of the real d_x diameter from the preset d_0 hardly exceeds one percent.

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