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ABSTRACT

The field of researches is connected with problems of restoration of images on the incomplete information of objects, which are represented in the digital image form. Questions of application of artificial intelligence systems for image restoration are considered.

Keywords: associative memory, image, neural network, restoration

I. INTRODUCTION

The problem of restoration of true images of objects exists, first of all, at the decision of classical problems of recognition of images and clusterization of objects under the incomplete and deformed information on objects or under the damaged images of objects (figure 1).

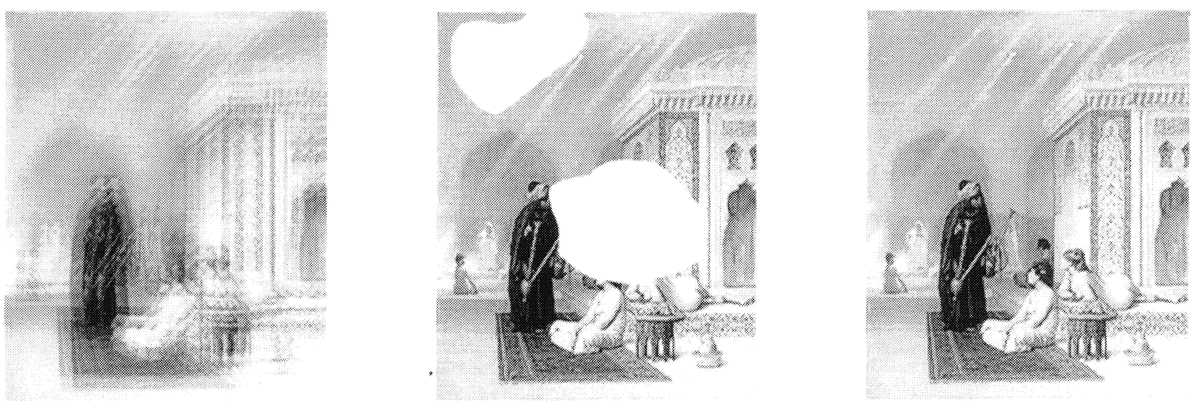

Figure 1. Damaged and true images of object.

Besides this, problem of restoration of images arises in archeology when are identified partly effaced coins and decayed manuscripts, as well as damaged figures on rocks and patterns on earthenware (figure 2). The similar problem takes place in restoration works and in reconstruction of images of historic persons on basis of mummificated remains.

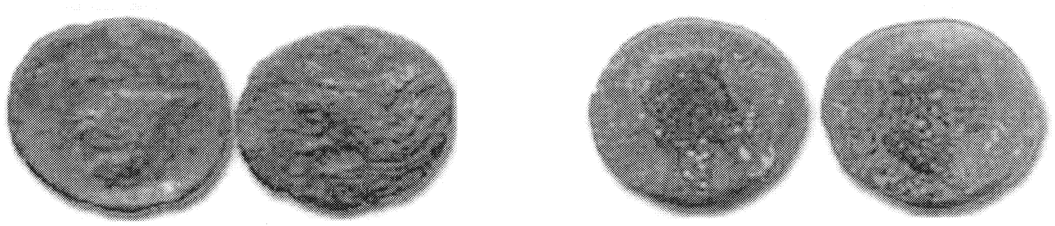

Figure 2. The images of coins demanding recognition-restoration.

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In criminalistics the specified problem demands to be solved at presence of indistinct fragments of prints of fingers and approximate identikits of criminals. In physics there are situations when image is deformed by devices and environment.

Especially frequently it is necessary to deal with such images in astronomy. At supervision and photographing of the removed galaxies, planets, heavenly bodies the true image is necessary to restore under the received images. The problem of restoration of images of a spreading surface of the Earth observable from satellites in conditions of deforming influence of atmosphere and influence of translucent cloudiness is still very important. In meteorology the problem of forecasting of a place and time of abnormal phenomena and accidents of various nature is extremely actual. In haildefence, during formation of clouds it is required to complete (to restore) the image of object in order to define probable hail danger clouds. In telecommunication, images of objects transmitted in noises and not error-controlled channels demand filtration, correction and restoration. In economy the analysis and forecasting of exchange situations, forecasting of exchange rates and securities, studying of tendencies of share market is necessary.

The problem of restoration of image of object is important both from applied and. From the applied point of view the decision of this problem opens an opportunity of automation of many processes, which were connected only with activity of an alive brain. From principle points of view decision of this problem concerns questions of development of ideas of cybernetics: what can, and what the machine essentially cannot do? In what rate opportunity of the machine can be approached to opportunities of an alive brain? For the present it is clear that if a person can realize itself his skill, and then describes it, for example, restoration of image of object with missing elements up to the true one with its further recognition, then such skill without basic difficulties can be transferred to the machine. If a person possesses skill, but cannot explain it, for example, recognition of the familiar person in crowd, then there is only one way of transfer of skill to the machine – training by examples.

2. THE ANALYSIS OF IMAGES RESTORATION METHODS

When solving unfamiliar problems, appears natural desire to represent them as some easily understood model, which would allow to comprehend problem in terms easily reproduced by human imagination. The most clear for a person is existential interpretation of problems. Classical approaches of solving the problems are based on determinate and probability technologies.

For using of determinate technologies for problems of recognition - restoration of images it is necessary that the problem was completely described by certain determinate model – some set of known functions and parameters. The image of object can be restored by topological or frequency methods. Frequency methods do not distinguish different images with an identical spectrum, that's why in some cases topological methods are more preferable. For example, recognition of images consists in comparison of certain characteristics of digital image of object with the standard. Depending on a degree of conformity of characteristics the object is distinguished whether or not. The simplest way of recognition assumes overlapping the center of coordinate grid with the geometrical center of object image and comparison of coordinates of points of image with coordinates of points of standard. Coordinates of points of television image of object ***O*** are defined as follows:

$$\boldsymbol{x_0} = \frac{1}{a}\sum_{y=1}^{m}\sum_{x=1}^{n} x \cdot \boldsymbol{B}(x,y), \qquad (1)$$

$$\boldsymbol{y_0} = \frac{1}{a}\sum_{y=1}^{m}\sum_{x=1}^{n} y \cdot \boldsymbol{B}(x,y), \qquad (2)$$

where m – number of lines of linear scanning; n – number of points of line raster; á – number of points of image raster;
$\boldsymbol{B}(x, y)$ – indicate function, such, that:

$$\boldsymbol{B}(x,y)=\begin{cases} 1, \text{ for points of image of object;} \\ 0, \text{ for points outside of image of object.} \end{cases}$$

The algorithm is represented as functional dependence as two sums of product of indicate function $\boldsymbol{B}(x, y)$ on the corresponding coordinate x or y. Parameters are the coordinates of points of image. Comparison of coordinates is made, first of all, for points laying on borders of the image, and then for the internal points forming various apertures, cracks, etc.

For problems of recognition - restoration of images connected with random variables are applied probability technologies. Parameters of probability models are the distributions of random variables, their average values, dispersions, etc. As a rule, these parameters are initially unknown, and for their estimation the statistical methods are used, which are applied for samples of observable values (images to be recognized).

Such approach also means, that some probability model of a problem is known. For example, in a problem of identification of object it is possible to assume, that on recognition the object $\boldsymbol{x}_{\mathrm{n}}$ will come under condition that two previous objects also were objects $\boldsymbol{x}_{\mathrm{n}}$. If it is true, the subsequent objects, which have come on recognition, allow to estimate precisely enough the coefficients of this dependence and to predict identification of object $\boldsymbol{x}_{\mathrm{n}}$ in the future.

However, determinate and probability methods appear ineffective in many practical problems. It is connected with that it is impossible to describe full enough real objects with the help of a small number of parameters of model, or calculation of model demands too much time and computing resources. In particular, at the decision of problems of recognition and restoration of images the determinate algorithms are applicable only in case of linear functions. However in such problems this condition is not carried out. Though some functions can be approximated as linear ones, the result will be far from optimum. If function is not linear one of following traditional ways of decision of a problem is applied:

- gradient method for search of maximum of function. If form of function is difficult and has several local maxims, method can get not optimum decision;
- full review of variants. Sometimes for full review some days of work of a modern computer are necessary.

There is also a question on ability of the determinate system to reflect adequately all completeness of a reality not only as a whole, but even in any of its aspects. One of consequences of Gedely's theorem will be, that the reality by its nature can not be presented as formal - logic circuit and basically cannot be full and adequately reflected by determinate means, i.e. basically it is impossible to construct logically full sign model absolutely isomorphic with realities.

Probability approach has been illustrated on simple linear model, however practical dependences frequently are not linear. Besides the form of even simple dependences is beforehand unknown. But statistical methods are well advanced basically for one-dimensional random variables. If we use several interconnected factors, then construction of multivariate statistical model is required. However, such models assume the Gauss distribution that in practice is not carried out, or are not proved theoretically.

Other technology based on completely other principles is necessary for indemnification of basic incompleteness of determinate and probability methods. It is, first of all, the figurative, analytical thinking constructed on great set of associations and connections of amorphous images. Experience and intuition are capable to solve problems in which the logic is powerless. In multivariate statistics quite often are applied heurism, which are inherently closed to technology of neural networks. They are based on the principles ***of artificial intelligence***, which simulate activity of neurons of a brain.

With the help of neural networks various "fuzzy" problems are solved such as recognition of images, speeches, hand-written text, classification, forecasting. In such problems traditional technologies are powerless, and neural networks frequently are the unique technique of the decision. Neural networks allow to create flexible, fast and effective systems of data analysis and processing.

For solving intellectual problems (restoration - recognition of images) are needed systems capable not only to carry out the programmed sequence of actions on beforehand determined data, but also to analyze new coming information, to find some laws in it, to make forecasting, etc. In this area of application the best are the ***analytical technologies*** – techniques that by means of models, algorithms or theorems allow on known data to estimate values of unknown values. Neural networks represent the analytical technologies created and verified by the nature. They enable to solve problems of forecasting, classification, search of optimum variants in cases when the decision of a problem is based on intuition or experience, instead of strict mathematical description. Examples of analytical technologies are ways of processing of information by human brain, such as recognition of familiar persons in a crowd. When solving problem of recognition - restoration of objects, neural networks generate a great number of regressive models and are the universal mean of approximation of functions. Generally, the entrance vector $\boldsymbol{x}_{\mathrm{n}}$ will be transformed by latent layer to some new space, which can have other dimension, and then to hyper planes corresponding to neurons of output layer, divide it into classes $\boldsymbol{C}_{\mathrm{q}}$. Thus, network recognizes not only characteristics of initial data, but also "characteristics of characteristics", generated by latent layer.

For decision of problem of recognition - restoration, first of all, it is necessary to define level of complexity of a problem. There are three basic levels of complexity: images are linearly devisable, not linearly devisable and probability devisable at crossing images. At limited number of objects definition of a level of complexity becomes rather inconvenient. In an ideal variant after preliminary processing linearly devisable problem must be obtained, after what construction of system of recognition - restoration considerably becomes simpler. Unfortunately, at the decision of real problems there is a limited number of objects on which basis construction of system of recognition - restoration is made. And carrying out of such preliminary data processing at which linear divisibility of objects will be achieved is impossible.

The initial data on object can be submitted as a vector $\boldsymbol{x}_n$ which components are parameters of object $\boldsymbol{x}_n=(x_1, x_2, ..., x_K)$. The system of recognition carries object $\boldsymbol{x}_n$ to one of classes $\boldsymbol{C}_q$. Definition of parameters of object influencing on decision-making on belonging of object to some image, is connected with two problems:

- for small number of parameters the same set of initial data can corresponds to the examples belonging to different classes and then training of network is impossible;
- for a great number of parameters the number of examples can be insufficient for training of network, and it instead of generalization will simply remember examples from training sample and cannot function correctly.

Thus, at definition of number of parameters of object the compromise in a choice of their number is necessary.

The algorithm of recognition – restoration process on basis of neural networks is represented as follows:

- *data processing*
  - drawing up of a database from examples, characteristic for given problem;
  - splitting of a data set into two sets: training and test;
  - choice of system of attributes and transformation of data. Desirable are the linearly divided sets of objects;
  - choice of system of representation of output values;
- *designing, training and estimation of network quality:*
  - choice of topology of a network: number of layers and number of neurons in layers;
  - choice of function of neurons activation;
  - choice of algorithm of training of a network;
  - estimation of quality of functioning of a network and optimization of architecture;
  - choice of variant of a network with the best generalizing ability and estimation of quality of work by test set;
- *application and diagnostics*
  - definition of influence of various factors on accepted decision;
  - definition of accuracy of classification of a network;
  - definition of necessity of return on the first stage, with change of way of representation of objects or databases;
  - use of a network for solving a problem.

Solving a problem of recognition - restoration provides presence of qualitative data and enough full and representative set of examples.

3. THE ANALYSIS OF IMAGES

The important property of human brain is *the figurative perception of the world.* This property enables on the basis of learning of final number of objects to find out with certain reliability infinite number of their variations. For example, under the incomplete, deformed or defective images of objects to learn (to restore) their true image. Thus, the human brain is capable to complete missing elements of objects.

Another interesting property of human brain is *classification of coming information.* This property means ability of a brain to answer on infinite set of states of external world by final number of reactions. The person breaks data into groups of similar, but not the identical phenomena. And, different people training on various materials of supervision, independently from each other classify the same objects equally. Objective character of images consists in it.

The image or class represents the classification grouping uniting certain group of objects to some attribute. Objects of the same image can differ strongly enough from each other. For example, image of a coin may be: free of defects; with effaced coin heads; with effaced coin tails; with damaged face of emperor; with scratches and etc. Other examples of images of triangular prism $\boldsymbol{A}=\{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_N\}$ and rectangular triangle $\boldsymbol{B}=\{\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_G\}$ are submitted on figure 3.

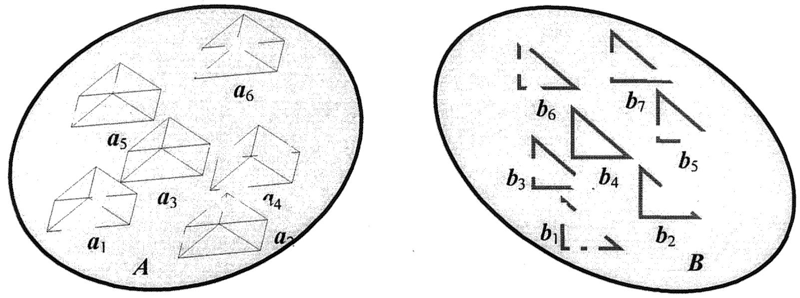

Figure 3: Image of triangular prism and rectangular triangle.

Objects of one class are equivalent from the point of view of criterion of splitting. Classes frequently are unknown beforehand, and are formed dynamically (for example, in networks Kohonen). In this case classes depend on showed objects and consequently addition of new object demands a correcting of system of classes.

Let objects be characterized by a vector of parameters $\boldsymbol{x}_n \in \boldsymbol{X}$, having K components: $\boldsymbol{x}_n=(x_1, x_2, ..., x_K)$, where $\boldsymbol{X}$ – space of objects. Set of classes $\boldsymbol{C}_q=\{\boldsymbol{C}_1, \boldsymbol{C}_2, ..., \boldsymbol{C}_Q\}$ in space of classes $\boldsymbol{C}$: $\{\boldsymbol{C}_1 \cup \boldsymbol{C}_2 \cup ... \cup \boldsymbol{C}_Q\} \subset \boldsymbol{C}_q$. Let's determine nucleus of classes $\boldsymbol{c}_q=\{\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_Q\}$ in space of classes $\boldsymbol{C}$ as true objects for their class. The quantitative estimation of affinity of object to a nucleus is defined by a measure of affinity $d(\boldsymbol{x}_n, \boldsymbol{c}_q)$ of object and a class nucleus, which the less is, than more object is similar to a nucleus of class. A measure of affinity of two objects is $d(\boldsymbol{x}_1, \boldsymbol{x}_2)$; of two nucleus of classes - $d(\boldsymbol{c}_1, \boldsymbol{c}_2)$. The Evklid measure represents geometrical distance between objects in multivariate space of attributes:

$$d(\boldsymbol{x}, \boldsymbol{y}) = \sqrt{\sum_{i=1}^{N} (x_i - y_i)^2}\,. \tag{3}$$

Each imaging of any object on perceiving elements of classification system is *the image* of object, and sets of such images incorporated by any general properties, also represent images. The image of object is represented in a digital form by spatial discretization and quantitization with respect to analog image brightness (figure 4). Let the digital image be discretizated on equal number of lines and columns $N=2^n$ (number of digitization levels) and has dimension $2^n x 2^n$, where n=0,1... . And, intensity of everyone pixel is quantitizated on one of M=2m levels, m=0, 1... . If N and M takes the greatest values possible for given system, then digital image will be as much as possible approached to the initial analog image.

The true image of object, for example, a rectangular triangle, represents the initial digital image of object (figure 4.1). Defective images of object represent the digital image consisting of: set of pixels with missed pixels (figure 4.2), noisy image (with superfluous pixel) (figure 4.3), or noisy image with missed pixel (figure 4.4).

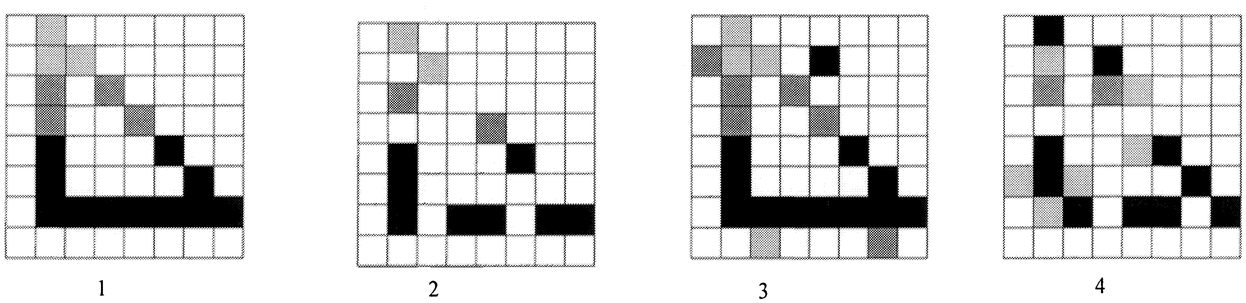

Figure 4: 1 – the true image of a rectangular triangle; 2 – the image with missed pixel; 3 – noisy image; 4 – noisy image with missed pixel.

Set of images shown on figure 4, defines an image of rectangular triangle $\boldsymbol{B}$. The class is formed by group of vectors, the distance between which inside group is less, than distance up to adjacent groups. Inside classes the objects should be closely connected with each other, and objects of different classes should be far from each other (the requirement of compactness of classes). Distributions of objects by classes should be uniform (the requirement of absence of association of separate groups of objects). For example, class $\boldsymbol{C}_1$ forms a nucleus of class $\boldsymbol{c}_1$ and group of defective objects $\boldsymbol{x}_n=\{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_N\}$: $\boldsymbol{C}_1=\{\boldsymbol{c}_1, \boldsymbol{x}_{n1}\}$, class $\boldsymbol{C}_2=\{\boldsymbol{c}_2, \boldsymbol{x}_{n2}\}$, …, class $\boldsymbol{C}_Q=\{\boldsymbol{c}_Q, \boldsymbol{x}_{nQ}\}$ (figure 5).

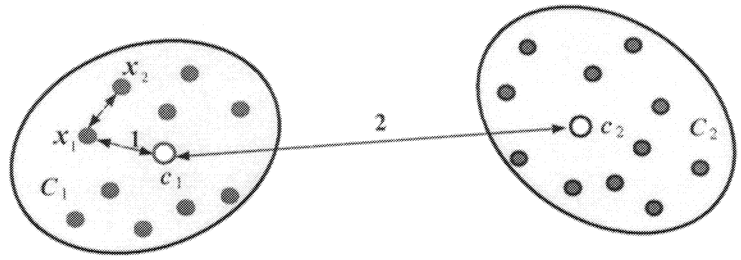

Figure 5: Distances between: 1 – object and nucleus; 2 – two nucleuses.

Vector representation of image of object allows to designate it as a point of some space of attributes. If it is affirms, that image unequivocally belongs to one of images $\boldsymbol{C}_1$, $\boldsymbol{C}_2$, …, $\boldsymbol{C}_Q$ thus affirms, that in some space there are areas which have no common points, and that images are the points from these areas. The areas corresponding to some images $\boldsymbol{C}_1$, $\boldsymbol{C}_2$, …, $\boldsymbol{C}_Q$ in space should be divided. For this purpose it is necessary to have some surfaces which would divide not only the chosen points (images), but also all other points (images) belonging to these areas, or presence of surfaces limiting these areas so that in each of them were only points (image) of one image (figure 6).

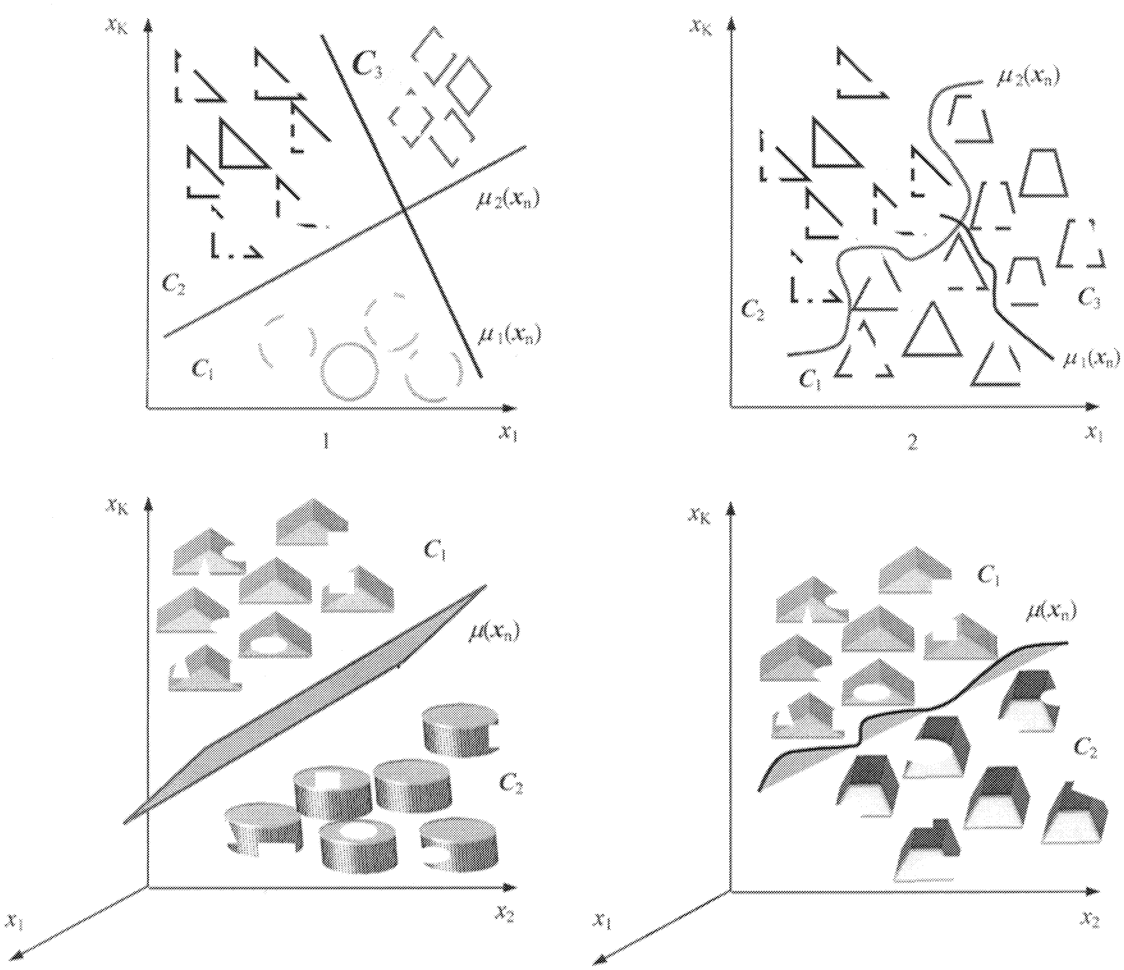

Figure 6: Dividing functions: 1 – simple; 2 – complex.

In other words, presence of such functions from vectors - images $\mu(\boldsymbol{x}_n)$, which would be, for example, are positive on all points of one and are negative on all points of other image – for two images; or the function accepting above points of each areas identical value, and above points from different areas various values – for several images. As these areas have no common points, then there is always a set of such dividing functions $\mu(\boldsymbol{x}_n)$, but there should be presence one of them.

The arrangement of image borders $\boldsymbol{C}_1$, $\boldsymbol{C}_2$, ..., $\boldsymbol{C}_Q$ in space is defined by allowable degree of similarity of defective image $\boldsymbol{x}_n$ and true image of object $\boldsymbol{c}_q$, that is determined by maximal Evklid distance $d_{max}(\boldsymbol{x}_n, \boldsymbol{c}_q)$, and is estimated by value of dividing function. For objects $\boldsymbol{x}_n$ of image $\boldsymbol{C}_Q$ value of dividing function $\mu(\boldsymbol{x}_{n\,Q}$ is more then values of other dividing functions $\mu(\boldsymbol{x}_{n\,1})$, $\mu(\boldsymbol{x}_{n\,2})$, ..., of objects of images $\boldsymbol{C}_1$, $\boldsymbol{C}_2$, ... For adjoining areas $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ the dividing them border is defined by equality:

$$\mu(\boldsymbol{x}_{n\,1}) = \mu(\boldsymbol{x}_{n\,2}) \,. \tag{4}$$

The parameters describing dividing functions have random values. For the image, every pixel of which can accept value equal to "zero" or "one" (x_i=0, x_i=1), dividing function is:

$$\boldsymbol{\mu}(\boldsymbol{x}_{nq}) = p(\boldsymbol{x}_{nq})\prod_{i}\left\{x_i\; p(i/\boldsymbol{x}_{nq}) + \bar{x}_i\,[1 - p(i/\boldsymbol{x}_{nq})]\right\} \,, \tag{5}$$

where $p(\boldsymbol{x}_{n\,q})$ – aprioristic probability of belonging of object $\boldsymbol{x}_n$ to class $\boldsymbol{C}_q$;

$p(i/\boldsymbol{x}_{n\,q})$ – conditional probability of equality of i-th pixel to "one" (x_i=1) at recognition of image of object $\boldsymbol{x}_n$ from class $\boldsymbol{C}_q$;

x_i – the read out value of i-th images pixel.

By classification of new object on basis of preliminary obtained information $p(\boldsymbol{x}_{n\,q})$ and $p(i/\boldsymbol{x}_{n\,q})$, and current values of image pixel x_i,are carried out calculations of values of dividing functions for each object, then comparison among themselves and revealing of class of objects with the greatest value of this function. Probabilities $p(\boldsymbol{x}_{n\,q})$ and $p(i/\boldsymbol{x}_{n\,q})$ are calculated on stage of training of system. The whole possible number of objects of each class n_{C_1}, n_{C_2}, ...,n_{C_Q} is defined by allowable degree of similarity of the defective image $\boldsymbol{x}_n$ and true image of object $\boldsymbol{c}_q$ i.e. by settled maximal Evklid distance $d_{max}(\boldsymbol{x}_n, \boldsymbol{c}_q)$. During training are showed all known objects of various classes, among which s_{C_1} objects of class $\boldsymbol{C}_1$,$s_{C_1} \le n_{C_1}$; s_{C_2} objects of class $\boldsymbol{C}_2$, $s_{C_2} \le n_{C_2}$;...;s_{C_Q} objects of class $\boldsymbol{C}_Q$. $s_{C_Q} \le n_{C_Q}$. Total number of presented objects is $S = s_{C_1} + s_{C_2} + ... + s_{C_Q}$. Then the probability of getting of object $\boldsymbol{x}_n$ from class $\boldsymbol{C}_q$ is:

$$p(\boldsymbol{x}_{nq}) = \frac{s_{C_q}}{S} \,, \tag{6}$$

and conditional probability:

$$p(i/\boldsymbol{x}_{nq}) = \frac{p(i) \cap p(\boldsymbol{x}_{nq})}{p(\boldsymbol{x}_{nq})} \,. \tag{7}$$

Thus, are founded functions, which divide not only shown during training objects, but also all other objects potentially belonging to images $\boldsymbol{C}_q$, or, the function limiting areas so that in each of them there were only objects of one image. The knowledge of only some number of objects $\boldsymbol{x}_n$ from areas $\boldsymbol{C}_q$ allows to separate all area as it is known, that a problem about approximation of function on information about it in the limited set of points, essentially narrower, than all set on which function is set, is a usual mathematical problem about approximation of functions. Decision of such problems demands introduction of certain restrictions on a class of considered functions, and the choice of these restrictions depends on character of additional information. Such additional information can be *a hypothesis about compactness of images*: to images there correspond compact sets in space of attributes. The compact set are understood as "clots" of points in space of images, assuming, that between these "clots" exist dividing them rareness. Intuitively it is clearly, that problem of approximation of dividing function will be as easy, as more compact and more far replaced in space are areas to be divided.

For example, in a case shown on figure 6.1, division it is more simple (objects considerably differ from each other), than in a case shown on figure 6.2 (objects are quite similar) since areas can be divided by a plane (linear dividing function), and even at big errors in definition of dividing function it nevertheless will continue to divide areas. In case on figure 6.2, division is carried out by a complex surface (nonlinear dividing function) and insignificant deviations in its form result in mistakes of division. The complex dividing surface takes place at minimally allowable Evklid distance between nucleuses of classes $d_{min}(\boldsymbol{c}_1, \boldsymbol{c}_2)$, that is in a case when images not strongly differ from each other.

For example of figure 6.1 with a vector of parameters x_n, having 2 components x_1 and x_2, dividing function is a direct line dissecting a plane. In case of three-componential vector of parameters $x_n=(x_1, x_2, x_3)$, dividing function is a plane dissecting three-dimensional space. For four and more componential vectors the K-dimensional space is dissected by a hyper plane. It is obvious, that linear divisibility defines an opportunity of correct classification.

If the hypothesis of compactness is well carried out (figure 6.1), then problems of classification find the simple decision. And on the contrary, if the hypothesis of compactness does not prove to be true (figure 6.2) problems of classification either are not solved at all, or are solved with difficulties.

Property of compactness of images does not depend on image nature, and defines ability to division of set of objects into images. The relative positioning of points in chosen space already contains the information on how it is necessary to divide set of points. This information defines property of divisibility of images. Therefore the hypothesis of compactness is represented as an attribute of opportunity of satisfactory decision of problems of classification.

In some random chosen space (abstract space) objects are distributed in random. And, in some parts of space they are settle down more densely, than in others. In this abstract space there are compact sets of points – abstract images. Usually images beforehand are not determined, and the problem will consist in definition of subsets of images in given space, representing images. In such understanding of a problem it is supposed existence of images in given space about which there was no representation earlier. Suitability of chosen space is defined by concurrence of abstract images with real. The more abstract images differ from real, the more inconveniently is chosen space.

4. RESTORATION OF THE IMAGES ON THE BASES OF THE OBJECTS CLASSIFICATION

On basis of figurative perception of world and classification of information, it is supposed to reduce a problem of restoration of defective objects to a problem of classification of objects. Thus, the problem of classification of objects is represented as a method of decision of problem of restoration of defective images of objects.

The problem of classification consists in splitting objects into classes on basis of vector of parameters of object x_n, and for given number of classes Q is formulated as follows: to find Q nucleus of classes $\{c_q\}$ and to break objects $\{x_n\}$ into classes $\{C_q\}$, i.e. to construct function q (n) so that to minimize a total measure of affinity for all set of input objects:

$$\min\left\{D=\sum_{n=1}^{N}\sum_{i=1}^{K}(x_{n\,i}-c_{q(n)\,i})^2\right\}. \tag{8}$$

Function q (n), determining number of a class on index n of sets of objects $\{x_n\}$, sets splitting into classes and is the decision of problem of classification. In elementary case $X=C$, the space of objects X is broken into areas $\{C_q\}$, and if $x_n \in C_q$, then q(n) = q and object is related to class with index q.

At splitting into classes the total measure of affinity for all set of objects $\{x_n\}$ should be minimized:

$$D=\sum_{n=1}^{N}\sum_{i=1}^{K}(x_{n\,i}-c_{q(n)\,i})^2. \tag{9}$$

Expression in a bracket can be submitted as scalar product:

$$D=\sum_{n=1}^{N}[(x_n,x_n)-2(x_n,c_{q(n)})+(c_{q(n)},c_{q(n)})]. \tag{10}$$

In this sum two components do not depend on way of splitting and are constant:

$$\sum_{n=1}^{N}(x_n,x_n)=\text{const},\quad \sum_{n=1}^{N}(c_{q(n)},c_{q(n)})=\text{const}.$$

Therefore the problem of searching of minimum of D is equivalent to search of a maximum of expression:

$$\min D \to \max\sum_{n=1}^{N}\sum_{i=1}^{K}x_{n\,i}c_{q(n)\,i}. \tag{11}$$

Algorithm of classification for search of maximum of this function is as follows:

- to establish a cycle: for each vector $\boldsymbol{x}_n$;
- to establish a cycle: for everyone q;
- to calculate the value $D_{qn} = \sum_{i=1}^{K} x_{n\,i} c_{q(n)\,i}$;
- to find q, for which $q : \max_q \{D_{qn}\}$.

Such algorithm is easily realized *by a neural network.* It needs Q summator determining all sums D_q, and interpreter finding the summator with maximal output (figure 7).

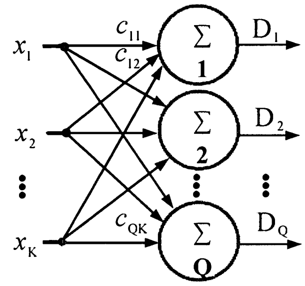

Figure 7: Network Kohonen.

The sum $\sum_{i=1}^{K} x_{n\,i} c_{q(n)\,i}$ reminds the weighed sum $\sum_{i=1}^{K} w_{ij} x_{ij}$ calculated by neuron, and value D_q – output value of neuron y_j. As input data is used vector of parameters of object $\boldsymbol{x}_n$ – the defective image, and components of nucleus c_{qi} – are used as weight factors w_{ij}. The total number of classes coincides with quantity of neurons. Everyone neuron remembers one nucleus of a class, being responsible for definition of objects in its class, and gives on output the sum y_j. The vector of defective image sent on an input of network Kohonen, makes active one of neuron. The size of an output of neuron is as large, as the defective image more closely to given nucleus of a class is. Neuron with maximal output defines the class $\boldsymbol{C}_q$ to which the object concerns.

Classification of objects can be realized also *by associative memory* – the neural network in which are available connections from outputs of neurons to their inputs, for example network Hopfield (figure 8).

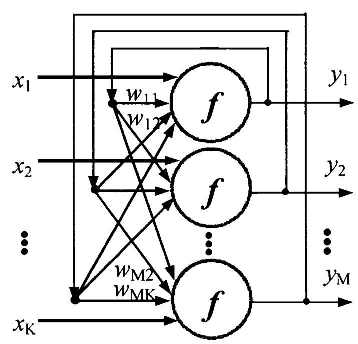

Figure 8: A two-layer neural network with feedback (network Hopfield).

The class is set by true image c_q and possible defective images $\boldsymbol{x}_n=\{x_1, x_2, ..., x_N\}$. Lets consider set of standard vectors $c_q=\{c_1, c_2, ..., c_Q\}$, stored in associative memory as weight vectors:

$$w_{ij} = \begin{cases} \sum_{q=1}^{Q} c_{q\,i} c_{q\,j}, & i \neq j \\ 0, & i = j, \end{cases} \tag{12}$$

where i, j for weight factors w_{ij} are the numbers of neurons.

Let as a measure of affinity of two vectors their scalar product is used: d($\boldsymbol{x}_n$, $\boldsymbol{c}_q$) = ($\boldsymbol{x}_n$, $\boldsymbol{c}_q$). The more similar vectors are, the more is measure of affinity. Then it is possible to expect, that change of vector $\boldsymbol{x}_n$ in time under the law

$$d\boldsymbol{x}_n = \sum_{q=1}^{Q} \boldsymbol{c}_q (\boldsymbol{c}_q, \boldsymbol{x}_n)\, dt, \tag{13}$$

will lead to concurrence of vector $\boldsymbol{x}_n$ with the most similar standard $\boldsymbol{c}_q$ and on outputs of network will appear vector $\boldsymbol{y}_n$ i.e. the required association $\boldsymbol{A}_{nq}$ will be found,

$$\boldsymbol{y}_n = \boldsymbol{c}_q = \boldsymbol{A}_{nq}(\boldsymbol{x}_n). \tag{14}$$

or in other words class $\boldsymbol{C}_q$ of belonging of vector $\boldsymbol{x}_n$ is determined.
As associative memory allows by a fragment of information $\boldsymbol{x}_n$ extract from memory all required information, then if set of standard objects $\boldsymbol{c}_q=\{\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_Q\}$ is stored in associative memory as binary vectors $\boldsymbol{w}_q=\{\boldsymbol{w}_1, \boldsymbol{w}_2, ..., \boldsymbol{w}_Q\}$, then defective images $\boldsymbol{x}_n=\{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_N\}$ according to (14), are capable to cause corresponding to them true image $\boldsymbol{c}_q$. Thus, "restoration" of image is carried out. For guaranteed classification without mistakes it is necessary that areas of associations were not crossed (figure 9).

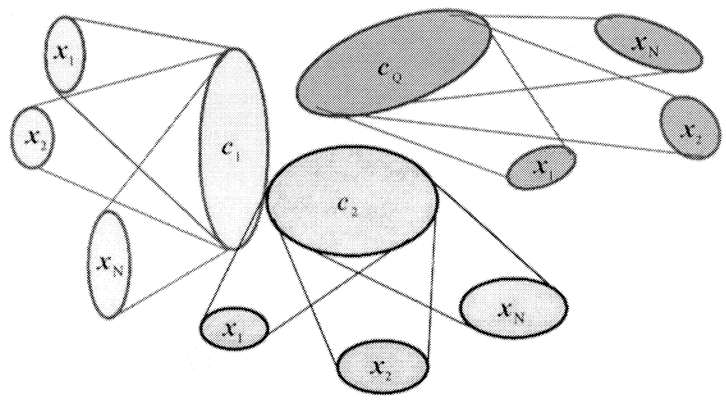

Figure 9: Vectors of defective images and vectors of reference objects corresponding to them.

The nucleus of a class $\boldsymbol{c}_q$ can be obtained by averaging of defective images $\boldsymbol{x}_n$. Let sl present the defective image $\boldsymbol{x}_n$ as sum of a vector of true image $\boldsymbol{c}_q$ and a vector of defect $\boldsymbol{x}_д$:

$$\boldsymbol{x}_n = \boldsymbol{c}_q + \boldsymbol{x}_д. \tag{15}$$

The purpose is to receipt true image by making some images with defects $\boldsymbol{x}_{n_1}, \boldsymbol{x}_{n_2}, ..., \boldsymbol{x}_{n_L}$. At averaging L of various defective images the new image is formed:

$$\bar{\boldsymbol{x}}_n = \frac{1}{L}\sum_{L} \boldsymbol{x}_{n_L}. \tag{16}$$

Expected value of image $\bar{\boldsymbol{x}}_n$ will be equal to the true image $\boldsymbol{c}_q$:

$$\mathbf{E}\{\bar{\boldsymbol{x}}_n\} = \boldsymbol{c}_q, \tag{17}$$

and deviation of vector $\bar{\boldsymbol{x}}_n$ from the true image will be:

$$\sigma^2\{\bar{\boldsymbol{x}}_n\} = \frac{1}{L}\sigma^2\{\boldsymbol{x}_д\}, \tag{18}$$

where $\sigma^2\{\boldsymbol{x}_д\}$ a deviation of vector $\boldsymbol{x}_д$.

Standard deviation of vector $\bar{\boldsymbol{x}}_n$ from true image is:

$$\sigma\{\bar{\boldsymbol{x}}_n\} = \frac{1}{\sqrt{L}}\sigma\{\boldsymbol{x}_д\}. \tag{19}$$

From the equation it is clear, that when increase L, the deviation of a vector $\overline{x}_n$ from the true image decreases. Thus, according to (19), the average image $\overline{x}_n$ comes nearer to the true one c_q by increasing of number of defective images used during averaging.

5. CONCLUSIONS

- It is enough to have one image of object by means of which it is possible to generate an image, using the metrics and property of compactness of image.
- According to a hypothesis of compactness, it is possible to refer exactly defective object to certain image.
- The problem of restoration of defective image of object consists only in definition of belonging of object to some class. The restored or true image will be a nucleus of a class.
- The problem of classification - restoration of images of objects can be effectively solved by neural networks.

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