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Thermal stimulation of cooperative two-photon decay in a microcavity

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Abstract

The cooperative two-photon spontaneous decay of an excited atomic system in a microcavity is investigated. We demonstrate that the presence of a small number of thermalized photons in the microcavity mode stimulate the cooperative generation rate of the coherent entangled photon pairs.

It is well known that the spontaneous decay of a single atom in a microcavity differs substantially from decay in free space [1]. Such behavior of an atom inside a microcavity can be explained by the fact that near the transition frequency the rate of spontaneous decay is proportional to the EMF mode density.

The study of two-photon light generation has attracted much attention [2-4]. The first report of an experimental observation of two-photon coherent light generation in which excited Li atoms were used was made by Nikolaus *et al* [3]. Experiments involving Rydberg atoms have demonstrated the real possibility of building a two-photon micromaser [4]. Since in a microcavity the rate of two-photon decay in a three-level cascade system increases substantially, it is of interest to analyze in such a setting the cooperative emission of radiation by Rydberg atoms in microcavities.

In the present paper we study the cooperative decay of an ensemble of such atoms with a cascade pattern of the levels, where at a finite temperature the intermediate level is arbitrarily offset from resonance with a microcavity mode. In this case the intermediate level is essentially vacant, and the fact that it lies between the excited and ground states leads to a sizable increase in the two-photon cooperative transition amplitude. Using the statistical method of eliminating operators of the EMF and the virtual intermediate state of electrons in the atoms [2] one can obtain the following equation for an operator of the atomic subsystem, $O(t)$, in the process of two-photon spontaneous decay inside the microcavity.

$$\frac{d}{dt}\langle O(t)\rangle = i\omega_{21}\sum_{j=1}^{N}\langle[R_{zj},O(t)]\rangle$$

$$+\frac{1}{\hbar^4}\sum_{k_1k_2}\sum_{j,l=1}^{N}\Big[\frac{(\vec{g}_{k_1}\cdot\vec{d}_{31})(\vec{g}_{k_2}\cdot\vec{d}_{32})}{\omega_{23}-\omega_{k_2}}-\frac{(\vec{g}_{k_1}\cdot\vec{d}_{32})(\vec{g}_{k_2}\cdot\vec{d}_{31})}{\omega_{31}-\omega_{k_2}}\Big]^2$$

$$\times\Big[[1+n(k_1)+n(k_2)]\Big(e^{-i[(\vec{k}_1+\vec{k}_2)\cdot(\vec{r}_j-\vec{r}_l)]}\frac{\langle R_l^\dagger[R_j^-,O(t)]\rangle}{i(\omega_{k_1}+\omega_{k_2}-\omega_{21}+2i\Gamma)}+e^{i[(\vec{k}_1+\vec{k}_2)\cdot(\vec{r}_j-\vec{r}_l)]}$$

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$$\times \frac{\langle [R_j^\dagger, O(t)] R_l^- \rangle}{i(\omega_{k_1} + \omega_{k_2} - \omega_{21} - 2i\Gamma)} \Big) + n(k_1) n(k_2) \Big(e^{-i[(\vec{k}_1+\vec{k}_2)\cdot(\vec{r}_j - \vec{r}_l)]} \frac{\langle [R_l^\dagger, [R_j^-, O(t)]] \rangle}{i(\omega_{k_1} + \omega_{k_2} - \omega_{21} + 2i\Gamma)}$$

$$+ e^{i[(\vec{k}_1+\vec{k}_2)\cdot(\vec{r}_j - \vec{r}_l)]} \frac{\langle [[R_j^\dagger, O(t)], R_l^-] \rangle}{i(\omega_{k_1} + \omega_{k_2} - \omega_{21} - 2i\Gamma)} \Big) \Big], \quad (1)$$

where the operators $R_j^\pm$ and R_{zj} satisfy the usual commutation relations for spin operators. Here ω_{21} is the atomic transition frequency between the dipole-forbidden $|2\rangle$ and $|1\rangle$ states; $d_{3\beta}$ is the dipole moment of the transition between levels $|3\rangle$ and $|\beta\rangle$ (β=1,2); $\vec{g}_k = \sqrt{2\pi\hbar\omega_k/V}\vec{e}_\lambda$, with $\vec{e}_\lambda$ the polarization vector of a photon ($\lambda = 1, 2$) of frequency ω_k and V the cavity volume. In deriving equation (1), we have considered the offset $\Delta = |\omega_k - \omega_{23}|$ ($\Delta = |\omega_k - \omega_{31}|$) much greater than the damping factor $\Gamma(k)$ of the microcavity in order to two-photon process can occur. Next we limit ourselves to one microcavity mode. When the number of atoms is large, the process of cooperative two-photon decay becomes appreciable stronger. If we ignore fluctuations in the number of particles when the number of atoms is large ($N \gg 1$), one can easily obtain for the atomic inversion operator

$$\frac{d}{dt}\langle R_z(t)\rangle = -\frac{1}{\tau^{(b)}}\langle R_z\rangle - \frac{1}{\tau_0^{(b)}}(j(j+1) - \langle R_z\rangle^2 + \langle R_z\rangle), \quad (2)$$

where $1/\tau_0^{(b)}$=$4g^4 d_0^4(1+2n)/(\Gamma\Delta^2\hbar^4)$, $1/\tau^{(b)}$=$8g^4 d_0^4 n^2/(\Gamma\Delta^2\hbar^4)$ and $j = N/2$. The solution of this equation is

$$\langle R_z(t)\rangle = \frac{1+q}{2} - \frac{c}{2}\tanh\left[\frac{1}{2\tau_r}(t - t_0)\right],$$

where $\tau_r = \tau_0^{(b)}/c$ is the time of cooperative spontaneous decay of the ensemble of atoms, $t_0 = \tau_r \ln\frac{N-[1+q-c]}{[1+q+c]-N}$ is the time lag of the pulse of collective emission of a pair of photons in the microcavity, $q = \tau_0^{(b)}/\tau^{(b)}$, and $c = \sqrt{(1+q)^2 + 4j(j+1)}$. The Eq. (2) describing two-photon cooperative spontaneous decay implies that a thermalized field affects not only the Einstein coefficient $1/\tau^{(b)}$ corresponding to stimulated decay but also the rate of two-photon spontaneous decay, $1/\tau_0^{(b)}$. Clearly, a thermalized field facilitates the process of cooperative two-photon decay. This constitutes one of main difference between two-photon dipole-forbidden emission and one-photon cooperative spontaneous emission. Evidently, two-photon cooperative spontaneous emission prevails over stimulated thermalized transition only if $N(1+2n) > n^2$. These estimates suggest that when $n < 1$, the term $1/\tau^{(b)}$, which corresponds to induced decay, is negligible in comparison to the term $1/\tau_0^{(b)}$, which corresponds to spontaneous decay.

In the absence of radiators inside the microcavity, one can calculate the fluctuations of the electromagnetic field operators:

$$\delta_0^2 = \langle a^{\dagger 2} a^2\rangle - \langle a^\dagger a\rangle^2 = n^2.$$

It would be very interesting to find the fluctuations in the number of photons of the electromagnetic field that are generated by the excited radiators in the process of two-photon emission in the microcavity. In doing this one introduce a function that accounts for fluctuations of the electromagnetic field in relation to thermalized fluctuations:

$$\delta_r^2 = \delta^2 - \delta_0^2, \tag{3}$$

where $\delta^2 = \langle a^{\dagger 2}(t)a^2(t)\rangle$-$\langle a^\dagger(t)a(t)\rangle^2$. Since experiments often monitor the dynamics of the population difference of the population difference of the atomic subsystem in the microcavity, we express the electromagnetic field fluctuations δ_r^2 in terms of the kinetics of atomic population inversion.

Let $G^{(1)}(t) = \langle a^\dagger(t)a(t)\rangle$ then

$$\frac{d}{dt}G^{(1)}(t) = \frac{d}{dt}\langle a^\dagger(t)a(t)\rangle = \langle \frac{da^\dagger(t)}{dt}a(t) + a^\dagger(t)\frac{da(t)}{dt}\rangle. \tag{4}$$

Eliminating the heat-bath operators, in the Born-Marcov approximation $\frac{d}{dt}G^{(1)}(t) \ll \Gamma G^{(1)}(t)$, one can represent the function $G^{(1)}(t)$ via the atomic inversion operator:

$$G^{(1)}(t) = n - \frac{1}{\Gamma}\frac{d}{dt}\langle R_z(t)\rangle. \tag{5}$$

For a large number of excited atoms in the Born-Marcov approximation $\frac{d}{dt}G^{(2)}(t) \ll \Gamma G^{(2)}(t)$ we find for the second-order correlation function $G^{(2)}(t) = \langle a^{\dagger 2}(t)a^2(t)\rangle$:

$$G^{(2)}(t) = 2n^2 - [3n + 1/2]\frac{1}{\Gamma}\frac{d}{dt}\langle R_z(t)\rangle + [\frac{1}{\Gamma}\frac{d}{dt}\langle R_z(t)\rangle]^2. \tag{6}$$

One can observe that as the two-photon absorption probability $w \sim \langle a^{\dagger 2}a^2\rangle$=$G^{(2)}(t)$, at low-temperatures it is proportional to the two-photon flux Φ:

$$w \sim d/dt\langle R_z(t)\rangle \sim \Phi.$$

We note that the probability w also depends on the square of the two-photon flux, but this dependence is ignored in our approximation. Note that for one-photon superradiance, the function $G^{(2)}(t)$ is proportional to the square of the one-photon flux, or Φ^2. This occurs because $a^\dagger \sim R^+$ for one-photon emission, while for two-photon emission we have $a^{\dagger 2} \sim R^+$. Hence

$$w \sim \langle R^{+2}R^{-2}\rangle \sim \langle [d/dtR_z]^2\rangle$$

for one-photon superradiance, and

$$w \sim \langle R^+R^-\rangle \sim d/dt\langle R_z\rangle$$

for two-photon superradiance.

Now we can easily derive a formula for the relative fluctuations of the electromagnetic field inside the microcavity:

$$\delta_r^2 = -(n + 1/2)\frac{1}{\Gamma}\frac{d}{dt}\langle R_z(t)\rangle. \tag{7}$$

This implies that in each decay event, photons are generated in pairs and the emission intensity becomes proportional to N^2, while the second-order correlation function for the photons remains much greater than the square of the first-order correlation function. In this case, at low temperatures and for large numbers of atoms, we can speak of photon superbunching, i.e. $\delta_r^2/\delta_0^2 \gg 1$.

References

1. E. M. Purcell, Phys. Rev. **69**, 681 (1946); D. Kleppner, Phys. Rev. Lett. **47**, 233 (1981).

2. N. A. Enaki, Sov. Phys. JETP **67**, 2033 (1988); N. A. Enaki, M. A. Macovei, Phys. Rev. A **56**, 3274 (1997); N. A. Enaki, M. A. Macovei, J. Phys. B: At. Mol. Opt. Phys. **33**, 2163 (2000).

3. B. Nikolaus, D. Z. Zhang, and P. E. Toshek, Phys. Rev. Lett. **47**, 171 (1981).

4. M. Brune, J. M. Raimond, P. Goy, L. Davidovich, and S. Haroche, Phys. Rev. Lett. **59**, 1899 (1987).