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Uncontrollable Distortions in Inverse Problems for Dynamical Systems

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Abstract

The influence of errors in the initial conditions on the solution of the inverse problem for a dynamical system is studied. It is shown that these errors lead to uncontrolled distortions of the solution of the inverse problem. A method is proposed for special filtering of initial data which allows to exclude the uncontrolled distortions.

Keywords: dynamical systems, inverse problem, uncontrolled distortions, filtering.

1 Introduction

The inaccuracy is inevitable in experimental measuring of physical values. It consist of inaccuracy of measuring instruments, noise value and inaccuracy of visual means. The value of this inaccuracy can be evaluated by technical indicators of measuring instruments. They do not exceed 5-10 percent as a rule.

The experimental measuring are chosen as initial data for the following calculations with the use of mathematical models in many practical important problems. For example, the inverse problems for evolution processes [1], the control problems with the use of experimental data [2] belong to this class.

2 Statement of the Problem

Let us consider the certain dynamic system the motion of which is describing by the equation

$$\dot{X} = CX(t) + BZ(t), \tag{1}$$

where initial conditions

$$X(0) = X^0, \tag{2}$$

where $Z(t)$ is the vector function of external impacts, $X(t)$ is the vector-function of state variables, C is matrix of system, B is matrix of control.

The vector function $Z(t)$ is given in direct problems. The matrices C and B are also given. The vector function $X(t)$ is an unknown function. The initial conditions (2) have been given. The solution of system (1) can be presented in the form

$$X(t) = F(X^0, Z(t)). \tag{3}$$

If we consider the inverse problems, for example, when the vector function $Z(t)$ is searched, then we use the vector function $X(t)$, values X^0 and matrixes C, B, as initial data. If we have all components of $X(t)$ then we have the values X^0. But as a rule in practice we can't measure all components of $X(t)$. One or two components of vector function $X(t)$ are measured usually, for example, only the first component $x_1(t)$. Then it is necessary to have the values $\dot{x}_1(t), \ddot{x}_1(t)$... for the search of vector function $Z(t)$. But the inaccuracy of $\dot{x}_1(t), \ddot{x}_1(t)$... cannot be evaluated in principle as the function $x_1(t)$ was obtained by experimental way with error. This inaccuracy equals infinity in general case. It leads to approximate solution which will be equal zero if the regularization method was used [3]. The indicated inaccuracy was called the uncontrollable inaccuracy [4].

3 The Filtration of Initial Data

In work [4] it was shown that uncontrollable inaccuracy lead to an additional uncontrollable distortions of the desired solution of the form

$$c_0 + c_1 t + c_2 t^2 + ... + k_1 \delta_+(t) + k_2 \dot{\delta}_+(t) + ... \tag{4}$$

where c_0, c_1, k_1, k_2 are constants, $\delta_+(t)$ is symmetrical delta function.

Let us consider the inverse problem for the dynamical system (1) as the solution of equation

$$Az = u, \tag{5}$$

where z is searched function, u is given function, A is given compact operator.

The following method of influence removal of uncontrollable inaccuracy on result of inverse problem solution is suggested: the items which determine the uncontrollable values of initial conditions are excluded from function u in equation (5) by means of special filtration. For example, for the inverse problem of astrodynamics [5] in the equation (5) the items c_0, c_1 are excluded by us from function u as very these items determine the uncontrolled distortions. Further we used the properties of Legander's polynomials. Let us define the values of c_0, c_1 from expressions $c_0 = \tilde{c}_0 - 0.5\tilde{c}_2, c_1 = \tilde{c}_1 - 1.5\tilde{c}_3$, where $\tilde{c}_0, \tilde{c}_1, \tilde{c}_2, \tilde{c}_3$ are the coefficients of Fourier of function u on Legander's polynomials. Then in equation (5) we use the function $\hat{u} = u - c_0 - c_1 t$ instead of function u.

4 Conclusion

An algorithm for eliminating an uncontrolled error in solving an inverse problem for a dynamical system based on a special filtering of the initial data is proposed.

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