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Evaluation of spatial interpolation methods for groundwater: case study, the Republic of Moldova

Abstract

In this case study the geostatistical analysis of digital hydrogeological map accuracy was carried out. Maps were modeled in isolines by means of different interpolators. The occurrence of groundwater levels was mapped on the representative area in limits of the Republic of Moldova. Using the software Surfer 11 eleven models of groundwater levels were generated by the following interpolators: Kriging, Radial Basis Function, Inverse Distance to a Power, Modified Shepard's, Minimum Curvature, Polynomial Regression, Triangulation with Linear Interpolation, Nearest Neighbor, Natural Neighbor, Moving Average and Local Polynomial. The comparability between maps was estimated applying methodology of the statistical residuals as the subtraction between Natural model –Artificial model and the absolute average value of the statistical residuals. Modified Shepard's method is defined as the most accurate interpolator. Nevertheless the correlation analysis shows that this interpolation method is very close with Kriging, Local Polynomial, Minimum Curvature, Natural Neighbor and Triangulation with Linear Interpolation (r> 0.70).

Rezumat

În studiile prezente a fost analizată precizia hărţilor hidrogeologice digitale, care au fost produse folosind metodele diferitor interpolatori în izolinii. Pentru un teritoriu reprezentativ, în cadrul R.Moldova, s-a cartat nivelul apelor freatice în condiţii naturale. Utilizînd softul Surfer 11 au fost întocmite 11 modele a nivelului freaticului prin intermediul interpolatorilor Kriging, Radial Basis Function, Inverse Distance to a Power, Modified Shepard's, Minimum Curvature, Polynomial Regression, Triangulation with Linear Interpolation, Nearest Neighbor, Natural Neighbor, Moving Average şi Local Polynomial. Diferenţa între hărţile obţinute a fost estimată folosind metodologia reziduurilor statistice ca diferenţa dintre Model natural –Model artificial şi a valorii reziduurilor absolute medii. Cel mai exact interpolator a fost considerat Modified Shepard's Method. Tot odată în rezultatul analizei corelaţionale această metodă este foarte apropriată de interpolatorii Local polynomial, Kriging, Minimum curvature, Natural neighbor şi Triangulation with linear interpolation (r >0.70).

Резюме

В данной работе проведён геостатистический анализ точности компьютерных гидрогеологических карт, которые были построены в изолиниях посредством разных интерполяторов. Для репрезентативной территории в пределах Рес. Молдова был картирован уровень грунтовых вод. Используя программное обеспечение Surfer 11 были построены 11 моделей уровня грунтовых вод посредством методов Крайгинга, Радиальных базисных функций, Обратных расстояний, Шепарда, Минимальной кривизны, Полиномиальной регрессии, Триангуляции с линейной интерполяцией, Ближайшего соседа, Естественного соседа, Скользящего среднего и Локальных полиномов. Сравнимость между картами была оценена с использованием методологии статистических остатков от разницы Природная модель –Исскуственная модель и абсолютной величины статистических остатков. Метод Шепарда определен как наиболее точный интерполятор. Тем не менее корреляционный анализ показывает, что данный метод очень близок с методами Крайгинга, Локальных полиномов, Минимальной кривизны, Естественного соседа и Триангуляции с линейной интерполяцией (r > 0.70).

Introduction

Geologists and particularly hydrogeologists represent their data in different map formats. Mapping, as a rule, is associated with data interpolation and extrapolation. Interpolation is a procedure of predicting value of attributes at unsampled sites from measurements made at point location within the same area, and extrapolation is predicting value of attribute at sites outside the area covered by existing observation [5]. Interpolation is used more frequently by specialists. It is necessary to note that at present one unique classification of methods for interpolation does not exist. There are two types of methods: global estimation (over a large area within there are many samples) and local estimation (a smaller area, one in which there are a few samples; in such situations we use nearby samples locate outside the area being estimated) [9].We will predominantly use the second type. Additionally, authors [5] notes, that global methods use trend surfaces on geometric coordinates, regression models on surrogate attributes methods of spectral analysis; local methods are deterministic and use Thiessen polygons and pycnophylastic methods linear and inverse distance thin plates spines. Despite the major improvements in interpolation methods over the last 20-30 years, the problem of mapping accuracy remains. Evaluation and comparison of different special interpolators were well analyzed [18, 19]. New interpolation approaches are proposed periodically [1, 3, 7, 13]. A comparison of different interpolators has been exploited in studying rainfall [10, 16], urban air pollution [4], geomorphology [6], geophysics [2] and other fields.

Groundwater, compared with other natural mapped phenomena, is located below the land surface and measurements for aquifers are possible only using special wells. These wells are limited in number because of high cost for drilling and special permission for its location. In such cases all available data are unique and need to be processed accurately and with high degree of confidence. Groundwater level is the complex indicator of the aquifer existence. Water level is indirectly connected with hydraulic rocks properties, water movement, artificial and natural variations, and aquifer water resources. The work reported here has been motivated by the practical need for understanding how accurate our groundwater level map is.

Case study area

General data. The case study area is located in the south of Moldova (fig.1). From the administrative point of view the area of research belongs to Cimişlia's region, it is in 0.5 km to the North from the town of Cimişlia, between 46° 32' 28.9" North Latitude - 28° 45' 27.5" East Longitude and 46° 34' 03.5" North Latitude - 28° 47' 09 0.3" East Longitude. Authors [21] give the detailed description of natural conditions of the research area.

Climate. Climate in case study area, as well as over all country, is moderate continental. Winter is short and rather soft, summer - long and warm. Average annual temperature is positive and as a result of long-term supervision data notes 9.20°C. Average monthly temperature remains positive during nine months. Average air temperature of the hottest month July reaches + 26.0 °C, and in the coldest month January it is - 11.0°C. The average annual absolute maximum rises to + 39 °C, and the average annual absolute minimum goes down to -29 °C.

Due to amount of precipitation the territory of research makes part of areas with insufficient natural moistening. The average annual amount of precipitation is 360.0–420.0 mm. In warm periods of time there is 344 mm of precipitation, and in the cold ones - 95 mm (only 10% of precipitation is firm). Relative humidity increase is observed in the winter period (84-87%), and it falls to 62% in the warm periods of time.

In the area of research winds of the northern and northwest direction prevail. Winds are weak or moderate. Average wind speed is equal to 3.90 m.

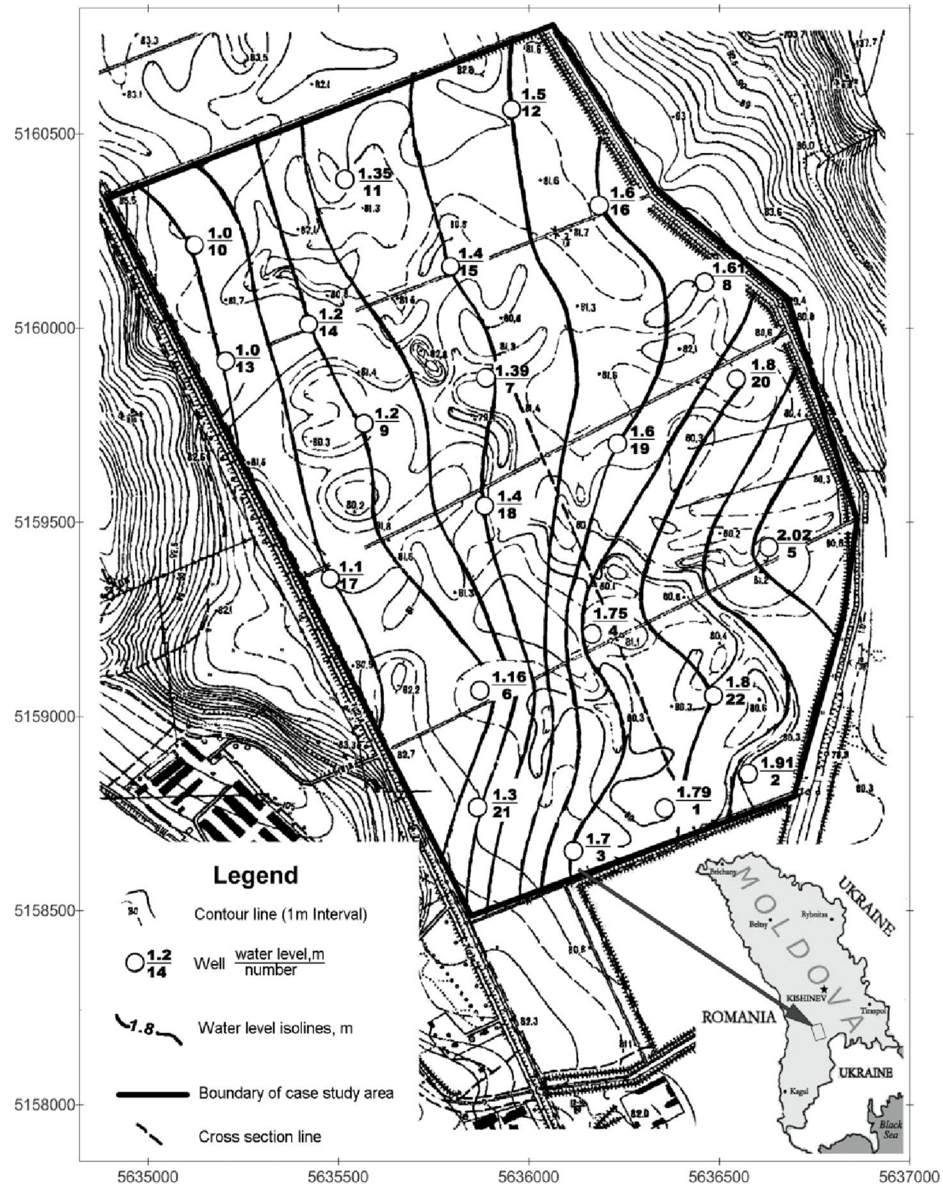

Fig. 1. Location map and groundwater level of the study area

Geomorphology. From the geomorphologic point of view the area of research is situated within the Southern Moldavian Wavy Plain. Watersheds are presented by Pliocene surface of alignment. Definitely the case study area is located on the right bank of the Cogîlnic River, and it is an almost flat site in comparison with the elevation number, which goes down from the northwest to the southeast. The elevation difference is 1-2 m.

Exogenous factors are involved in shaping surface of the research area. *Gully* erosion and *landslides occur on this territory*. Landslide density makes 3-5 landslides per 10 km^2 [22]; its values can sometimes reach 5-10 per 10 km^2[22]. Ravines have density of 10-20 ravines per 10 km^2[22]. Landslides form hilly land surface. Older landslides belong to frontal type; they are widespread on the right bank of the Cogîlnic River where the research area is. This area is characterized by various types of sediments: eluvial, colluvial, alluvial, inclusive alluvial deposits of the Cogîlnic River terraces (IX terraces).

Hydrology. Basic elements of the hydrographic network of the case study area are the rivers, floodplain lakes and ponds. The majority of the southern rivers of the country (where the town of Cimişlia is located) discharge into the Black Sea basin. The Cogîlnic River flows into Lake Sasîc. This lake is a certain liman of the Black Sea. River water comes from rain and snow; also underground water supplies the rivers. About 50 % of the annual drain fall on the period from March to May, and the minimum drain is observed in February.

The largest river of the research area is the Cogîlnic (Cunduc) River with the Northwestern direction of the water course. The Cogîlnic River has length of 243 km (125 km of them flow on the territory of Moldova), and also the water-collecting area is 3910 km (from which 1030 km^2 belong to RM). The annual average discharge of the river makes 0.30 m^3/s, absolute maximum value of 6.47 m^3/s (1962) and absolute minimum value of 0.0006 m^3/s (1964) were registered. In droughty years the river starts drying up because of low rainfall amounts.

The valley of the river is well defined within the research territory. Depth of the river-bed fragmentation makes 159.0 m in the North and 130.0 m in the South. The width of the bed is 10.0 m and it curves strongly on flat, marshland places. The right side of the river valley is rather steep (elevation is 30.0-350.0 m) and crossed by numerous ravines. The left bank is rather flat and gradually moves to the watershed.

The hydrographic mode of the river depends on the amount of precipitation. During summer season, in connection with uneven distribution and shortage of rains, the rivers and its tributaries are almost dried completely up and restore the stream with summer rainfall and snow thawing during spring season.

Geology and Hydrogeology. Deposits of Proterozoic, Paleozoic, Mesozoic and Cainozoic groups take part in the geological structure of the described area. Because of the studied aquifer is dated for Quarternary deposits it is expediently to provide only their characteristics. The geological and hydrogeological cross-section of the study territory is shown in fig.2.

Genetically Quaternary sediments are presented by all complexes of continental sediments, except glacial and lake ones. There are eluvial (elQ) and eluvial deluvial (el +dlQ) geological and genetic complexes on watersheds and inclinations (slopes). Lithologically they are presented by loess loams with thickness of 20-25 m. In flood bed of the rivers alluvial (al Q) sediments which compose terraces of valleys are widely spread. Lithologically they are presented by gravel with some pebbles and sandy formations. Sediment thickness is 5–10 m thick.

In the regional plan it is possible to allocate the aquifers and complexes in deposits of Quaternary System, the Sarmatian Stage and Cretaceous System. For the economic and drinking purposes and industrial water-supply has been generally identified Sarmatian aquifer.

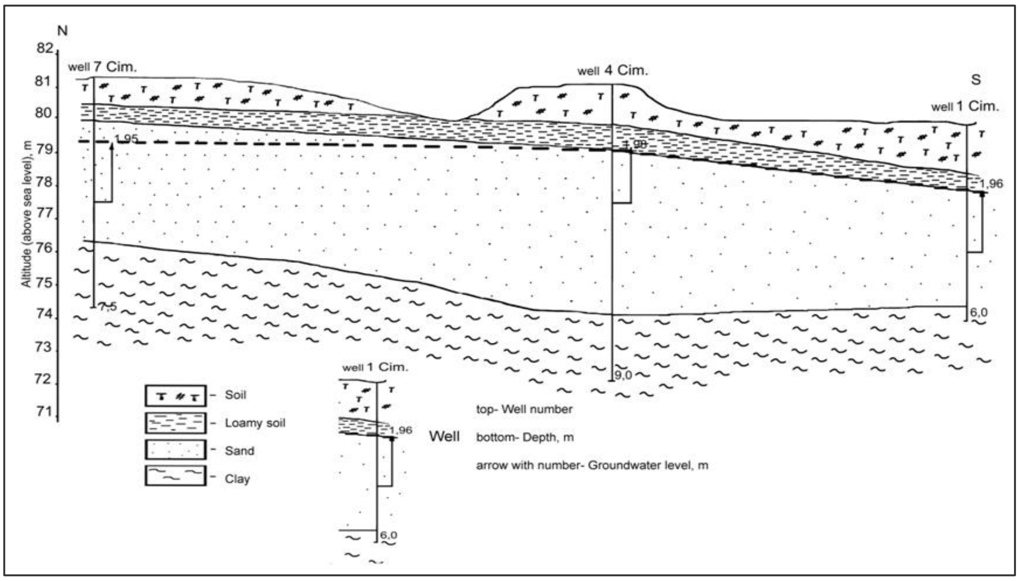

Fig. 2. Geological and hydrogeological cross – section (cross section line –see fig.1)

Quarternary deposits contain three aquifers which are related for various genetic types:

1) Aquifer of old alluvial deposits;

2) Aquifer of eolian-deluvial deposits;

3) Aquifer of early alluvial and alluvial-deluvial deposits.

Geological and hydrogeologic features of the case study area come as the consequence of the regional structure. For our purposes we provide only the detailed characteristics of the research area. These data are obtained as a result of water well drilling (see fig.1), hydrogeologic and geochemical workings. The representative geological section is offered in fig.2. From top to down the following layers occur: 1) soil (0.8–1.8 m), 2) loams (0.3–2.0 m), 3) sand (3.0–4.2 m) and 4) blue clay (thickness is more than 3 m). The aquifer of phreatic type is related to sand. Phreatic water level varies in a narrow interval – from 1.0 to 2.02 m from the earth surface. It is important to note that the water level doesn't depend on topographic landforms. The difference in water levels is caused by the direction of water current from the NW to the SE (see fig.1).

Ground water is salty; the value of mineralization varies from 2.19 to 9.3 g/l; water corresponds to the SO_4 – Na type.

The aeration zone (1.0–2.02 m thick) represents a layer of rocks unsaturated with water. Rocks contain a large amount of the salts which concentration changes from 0.07 to 3.0 mg/l (in a water extract). The most salted is the top part of the zone (an interval of 0.0-0.5 m). Below this depth the content of salts decreases drastically.

Model characteristics

The main objective of this research paper is to make a comparison of different interpolators and to mark down which of them work more accurately for mapping groundwater levels. But first of all, we generated a model (see fig.1) based on topographic map with scale of 1:10000 and hydrogeological data.

The flat relief of the case study area was taken as an important condition in order not to influence on the occurrence of the groundwater levels. The elevation difference is 1-2 m on the investigated territory (altitudes vary in 80–82 m above sea level). In addition this fact caused the strict boundaries of the research area. Outside of the territorial limits, the ground features differ significantly (elevation is 30-50 m).

The model map represents the spatial occurrence of the groundwater levels. During the field investigations, 22 sample wells were drilled. The representative wells are of a random selection. The minimum value of the groundwater levels was registered in the first well (1.0 m) and the maximum one – in the fifth well (2.02 m) (see fig.1). In that way, we designate that the values of the groundwater levels rises from the northwest to the southeast. Undoubtedly, that 22 sample wells are not enough for generating a more realistic manual or "eyeballing" [8] interpolation by integrating expert knowledge, modeling capabilities and experience into the construction of the geological surface. In this case the grid with the following parameters was elaborated.

Grid Name: Natural Model
Grid Size: 24 rows * 20 columns
Total Nodes: 480
Filled Nodes: 322
Grid Geometry:
X Spacing: ≈ 100 m
Y Spacing: ≈ 100 m

The quantities of Filled Nodes and Total Nodes differ from each other because of territorial boundaries and some interpolator features to draw lines strictly in limits of data points (The Triangulation with Linear Interpolation Method and the Natural Neighbor Method). The cell is in the shape of regular tetragon (highly approximately to foursquare). According to Descriptive Statistics for the Natural Model, the values of phreatic water levels vary in 0.62–2.05 m. Statistic parameters of the model are presented in the tab.1

Table 1

Statistic parameters of the model

Parameters	**Value**
Mean	1.46
Standard Error	0.02
Median	1.45
Mode	1.39
Standard Deviation	0.30
Sample Variance	0.09
Kurtosis	-0.86
Skewness	0.11
Range	1.43
Minimum	0.62
Maximum	2.05

Interpolation Methods

Interpolation methods which are used in GIS applications and other software are described in a large spectrum of publications [5, 8, 9]. We used as a platform for our purposes commercial software Surfer 11 which is representative regarding the number of interpolators. Totally we test 12 methods and namely Moving Average, Kriging, Nearest Neighbor, Local Polynomial, Minimum Curvature, Data Metrics, Inverse Distance to a Power, Natural Neighbor, Polynomial Regression, Modified Shepard's Method, Radial Basis Function and Triangulation with Linear Interpolation.Earth science users of these interpolators, as a rule and predominantly, do not understand in a full way how interpolators work and that is the difference among them. Fundamentals of these methods are well described in many sources [5, 9, 12, 15, 18]. In this context a brief description of each method which is based on data from indicated sources is analyzed.

1) *The Moving Average Method* is based on the averaging values of grid node. It is like a moving “window” with defined by author a radius for searching ellipse. For each ellipse a new value of (Z) value is calculated using general approach as:

$$Z = \sum (Z_i / n) ,$$

where Z is the new calculated value; Z_i is value of parameter (i) inside the ellipse and (n) is number of such parameters. Accuracy of the method depends of the used radius of the moving ellipse and density of initial data.

2) *The Kriging Method* is considered a popular one in interpolating surfaces. Numerous different kinds of Kriging are in actual use: for instance, punctual (simple) Kriging is used where the ensured variable is stationary; universal (general) Kriging where it is not; Kriging with Splines – any curve fitted by a spline function can also be indentified using Kriging and vice – versa [5, 12]. We used simple or ordinary Kriging which is operated according to assumption: N points are used to estimate grade at unknown point A; the weights w can be estimated by different number of ways, but it is clear that w will vary intensively in distance between points d; the w is obtained by solving N simultaneous equations of the form below, together with equation $\sum w_i = 1$ [12]:

$$w_1 \mu(h_{i1}) + w_2 \mu(h_{i2}) + \ldots + w_N \mu(h_{iN}) = \mu(h_{iA}) ,$$

where $\mu(h_{ij})$ is the semivariance corresponding to the separation h_{ij} between points i and j. The variance of the estimate E_A can also be obtained using semivariance $\mu(h_{ij})$. When the sample variance has been calculated a function is fit to it.

3) *The Nearest Neighbor Method* (otherwise known as Thiessen polygons or Dirichlet or Voronoi) is characterized by: a) using distances between closest pair of points and b) requiring no choice of quadrant size. The simplest algorithm for this method compares the observed mean distances d between points with means expected from particular distributions ∂ determined by the Poisson distribution:

$$\partial_{ran} = 0.5\ (A/N)^{1/2} \text{ for a random distribution}$$

$$\text{and } \partial_{hex} = 1.0743\ (A/N)^{1/2} \text{ for a hexagonal distribution [12].}$$

4) *The Polynomial Regression Method* allows powers of the independent X value in an equation like this type [15]:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \ldots + \beta_n X^n$$

According to [16] Polynomial Regression is not really an interpolator because it does not attempt to predict unknown X values. Nevertheless it can be used to define type of trend surfaces. Both in Surfer 10 and Surfer 11 surface definition is determined by user selected options namely as simple planar surface, bi-linear saddle, quadratic surface, cubic surface and user defined polynomial with different order of equation.

5) *The Minimum Curvature Method* in the Surfer code implements the concepts of tension similar to a simple planar model using least squared regression [14]:

$$AX + BY + C = Z(X,Y),$$

where Z is variably interpolated.

Four steps are to generate the final grid using this method [16]: a) the least squares regression model is fit to data, b) substation of data location from data values – as a result, a set of residual data values is obtained, c) interpolation of the residuals at the grid nodes and d) the values of the regression model at the grid nodes are added to the interpolated residuals.

6) *The Data Metrics Method* uses five groups of statistics and namely Z Order Statistics, Z Moment Statistics, Other Z Statistics, Data Location Statistics, and Terrain Statistics [15]. This method just creates grids of information about the data on a node-by-node basis. Generally speaking, Data Metrics statistics is not interpolator and this one is not used in our study.

7) *The Inverse Distance to a Power Method* combines the ideas of proximity exposed by the Thiessen polygons with the gradual change of the trend surface [5]. Authors of this reference analyzed the weighted Moving Average equation used for computing inverse distances:

$$Z\,(x_0) = \sum_{i=1}^{n} \lambda z(Xi) \sum_{i=1}^{n} \lambda i = \mathbf{1},$$

where the weights λi are given by $\Phi(d(x, x_i)$.

A requirement is that $\Phi(d)$ tends the measured value as d tends to zero. Given equation after some transformations becomes a linear interpolator in which the weights are computed from a linear function of distance. Generally this method is exact and is used in many earth science mapping applications.

8) For *the Natural Neighbor Method* interpolation algorithm uses a weighted average of the neighboring observations, where the weights are proportional to the "borrowed area". Method does not extrapolate contours beyond the convex is full of the data locations (i.e. the outline of the Thiessen polygons) [15].

9) *The Local Polynomial Method* is very similar to the method of Polynomial Regression. In Surfer code gridding method assigns values to grid nodes by using a weighted least squares fit with data within the grid node's search ellipse [15]. User is free to set the power to a number between 0 and 20, and then to select a polynomial order, 1, 2, or 3.

10) *The Modified Shepard's Method* is similar to the Inverse Distance to a Power method, but the use of local least squares eliminates or reduces the "bull's-eye" appearance of the generated contours. The Modified Shepard's method can be either an exact or a smoothing interpolator [15].

11) *The Radial Basis Function Method* is based on a group of functions. The multiquadric function is the most used. For this case interpolated surface is calculated on the base of equation [11]:

$$F(d_{ij}) = (\,d^2 ij + r^2)^{1/2},$$

where $F(d_{ij})$ is the radial base function and *d* is the distance between points.

This algorithm uses an R softening factor. The default value for R^2 in the Radial Basis Function gridding algorithm is calculated as follows: (length of diagonal of the data extent)^2 / (25 * number of data points) [15].

12) *The Triangulation with Linear Interpolation Method* holds much favor among Earth scientists. Method is simple and consists from the following steps: a) all points are covered by triangles point by point without crossing of triangles, b) changes of point values are considered to be described by a linear function. The equation of a plane can be expressed as:

$$Z = ax + by + c ,$$

where Z is the predicted value.

Given the coordinated and V values of three nearby points we can calculate the coefficients a, b and c [9]. Isolines are countered from triangle to triangle.

Results

Groundwater maps, constructed using different interpolators, are presented in fig. 3–5. Visual analysis of maps shows that all methods do not exactly fit with Natural surface model. Essential difference of accuracy is observed for Moving Average, Inverse Distance to a Power, Local Polynomial and Polynomial Regression methods. The rest methods have different degree of coincides with natural groundwater level position.

In order to estimate geostatistical relationship between model and modeled surfaces linear correlation was applied (tab.2). In this case it is necessary for interpretation of the correlation to describe shortly which parameters are compared. Because of random location of hydrogeological wells (see fig.1) each interpolator made a regular grid of the data with centered values in nodes. For correlation processing data from each grid have been prepared and finally the matrix of groundwater level values has been arranged. Structure of the matrix is simple: horizontal line is associated with the interpolation methods and vertical grid contains computed data for groundwater level. Analysis of the tab. 3 indicated that the rank of correlation is following (from maximum to minimum): Natural Neighbor, Kriging, Triangulation with Linear Interpolation, Radial Basis Function, Minimum Curvature, Local Polynomial, Modified Shepard's Method, Inverse Distance to a Power, Nearest Neighbor, Polynomial Regression and Moving Average. Logical comparing of the maps (see fig. 3-5) and correlation coefficients (r) do not express the real situation. For instance r = 0.937 for Polynomial Regression and model, and visual analysis clearly indicate that linear correlation in our case is formal (fig.4). One important assumption need to be noted –data matrices for each method and procedure of creating surfaces (isolines of groundwater level) are different by algorithms (see description of interpolation methods). That is why correlation coefficients do not estimate real geostatistical relationships between selected methods.

Other procedures are applied, and namely slice techniques. In fact the slice in Surfer code is associated with the cross – section across the contour maps. Line for slicing was selected as unique for all maps including Natural model. Results of slices are summarized in fig. 6. Visual analysis of presented data shows that Kriging and Radial Basis Function methods are much closed to Natural model line. Correlation coefficients for all slices are presented in the tab. 3.

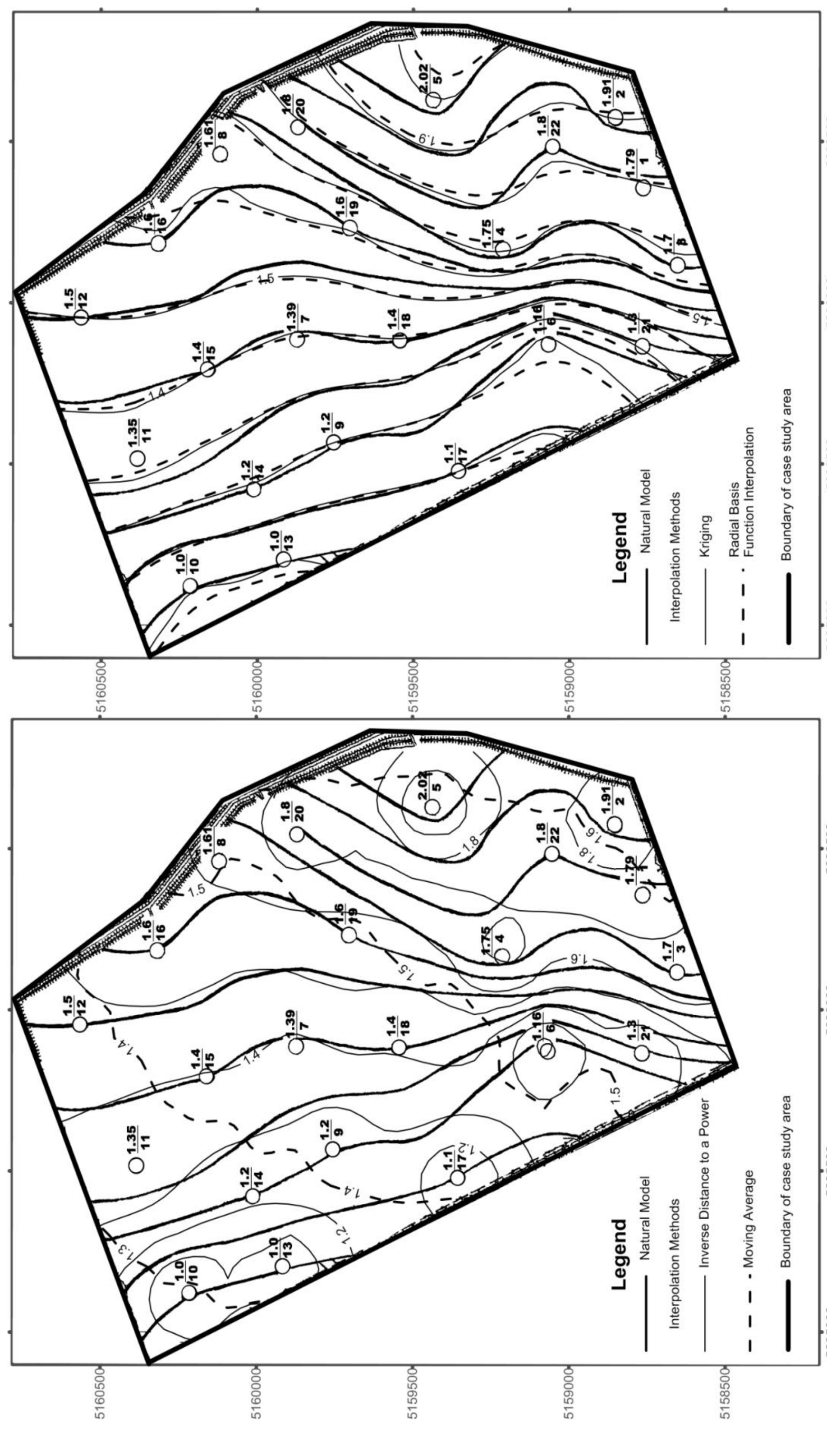

Fig. 3. Groundwater level maps for Inverse Distance to a Power, Moving Average, Kriging and Radial Basis Function methods (full legend is indicated in the fig.1)

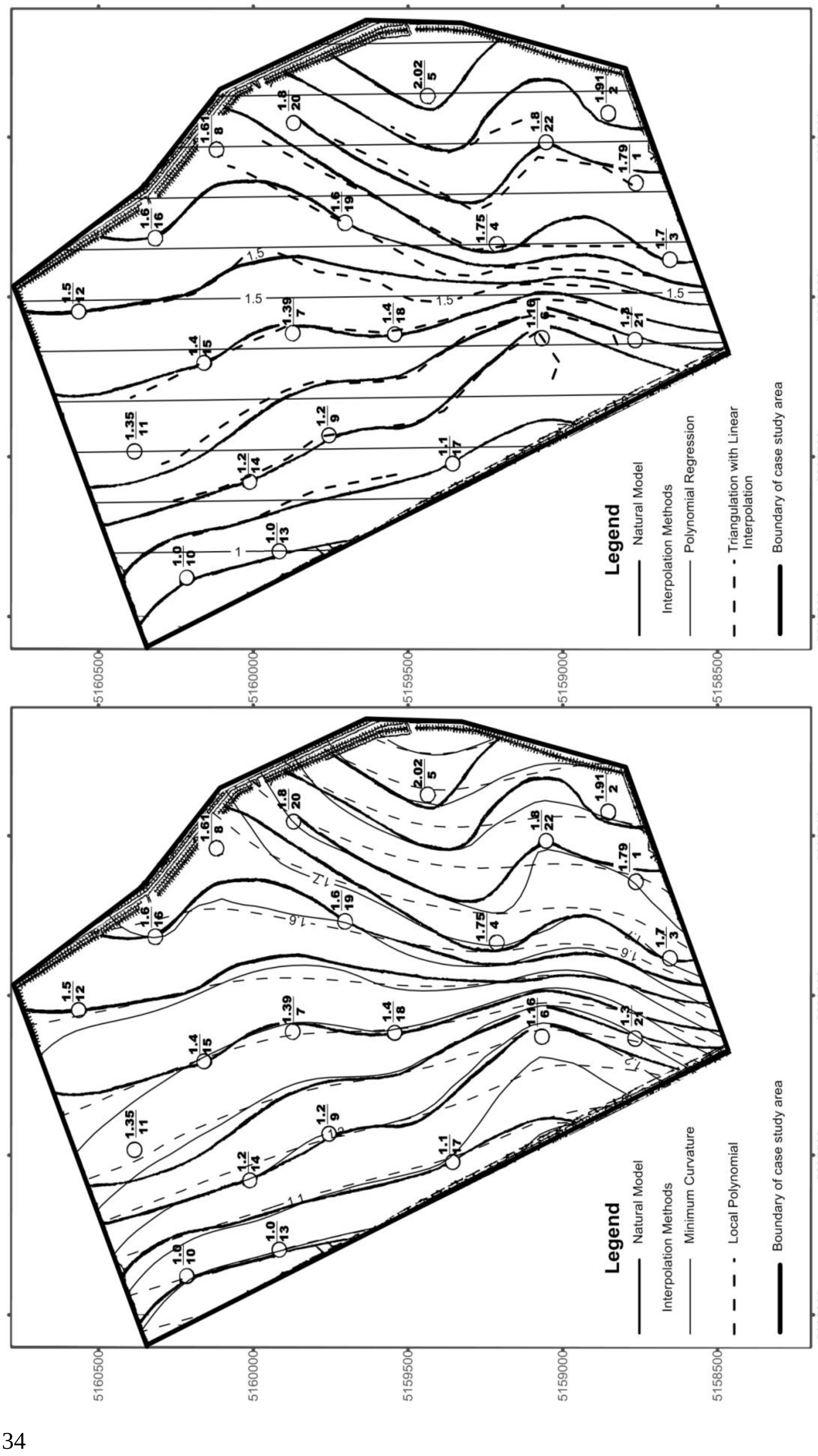

Fig. 4. Groundwater level maps for Minimum Curvature, Local Polynomial, Polynomial Regression and Triangulation with Linear Interpolation methods (full legend is indicated in the fig.1)

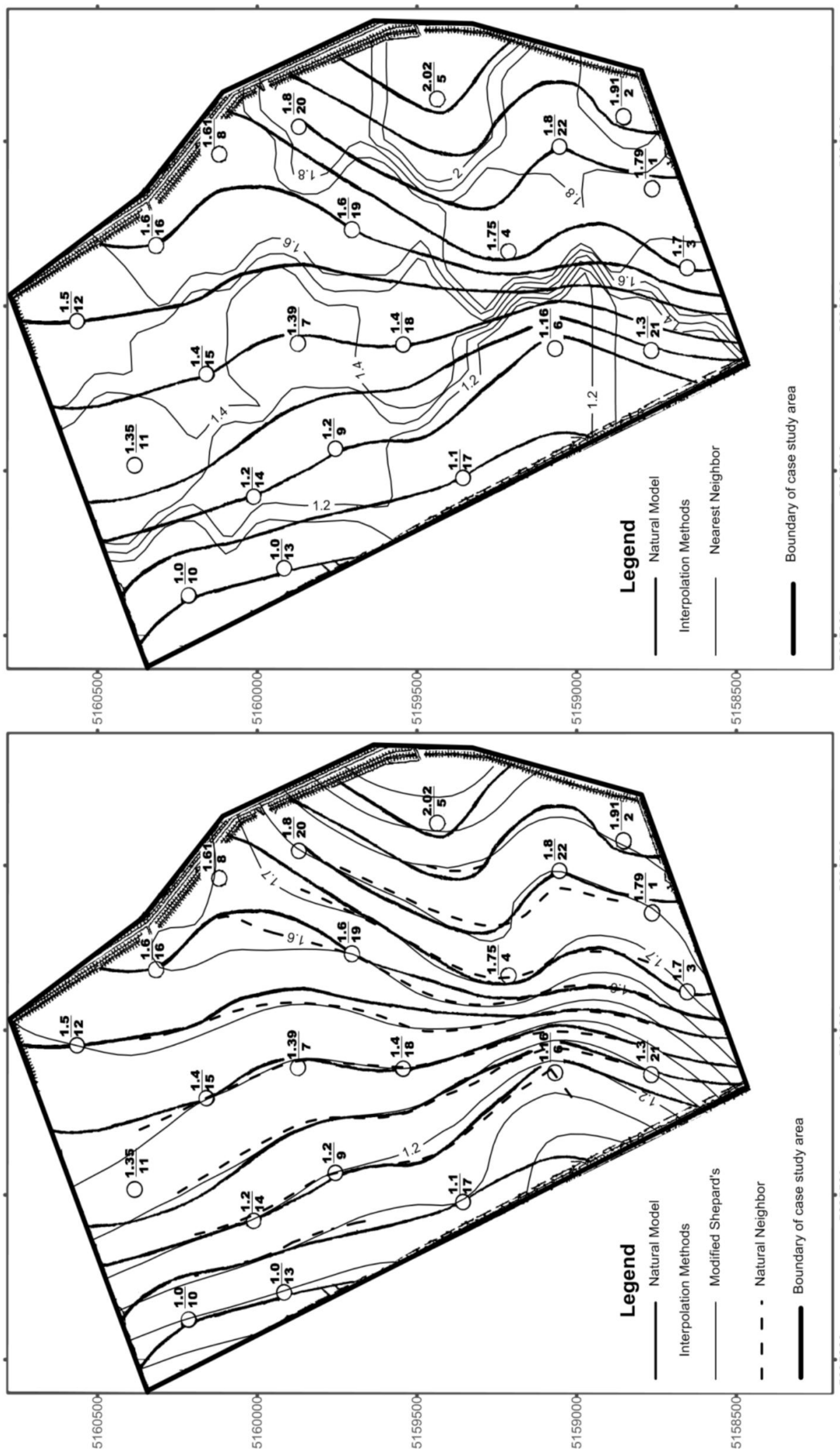

Fig. 5. Groundwater level maps for Modified Shepard's Method, Natural Neighbor and Nearest Neighbor methods (full legend is indicated in the fig.1)

Table 2

Correlation coefficients for modeled area and interpolated methods

Methods	Model	Kriging	Inverse Distance to a Power	Minimum curvature	Modified Shepard's	Nearest Neighbor	Radial Basis Function	Moving Average	Local Polynomial	Polynomial Regression	Natural Neighbor	Triangulation with Linear Interpolation
Model	1											
Kriging	0.980	1										
Inverse Distance to a Power	0.950	0.973	1									
Minimum curvature	0.976	0.993	0.955	1								
Modified Shepard's	0.970	0.987	0.946	0.995	1							
Nearest Neighbor	0.941	0.960	0.957	0.954	0.949	1						
Radial Basis Function	0.979	0.999	0.972	0.991	0.985	0.958	1					
Moving Average	0.770	0.789	0.814	0.762	0.768	0.753	0.795	1				
Local Polynomial	0.975	0.993	0.958	0.992	0.989	0.948	0.995	0.799	1			
Polynomial Regression	0.937	0.960	0.942	0.938	0.935	0.910	0.965	0.851	0.968	1		
Natural Neighbor	0.998	0.999	0.981	0.997	0.995	0.931	0.998	0.701	0.994	0.950	1	
Triangulation with Linear Interpolation	0.995	0.995	0.978	0.994	0.989	0.932	0.994	0.712	0.987	0.937	0.997	1

Table 3

Correlation coefficients for slice lines produced by different methods and natural model

Methods	Model	Kriging	Radial Basis Function	Triangulation with Linear Interpolation	Modified Shepard's	Inverse Distance to a Power	Local Polynomial	Minimum Curvature	Moving Average	Natural Neighbor	Nearest Neighbor	Polynomial Regression
Model	1											
Kriging	0.999	1										
Radial Basis Function	0.999	0.999	1									
Triangulation with Linear Interpolation	0.384	0.559	0.550	1								
Modified Shepard's	0.997	0.996	0.994	0.575	1							
Inverse Distance to a Power	0.994	0.996	0.995	0.555	0.988	1						
Local Polynomial	0.996	0.995	0.996	0.546	0.995	0.988	1					
Minimum curvature	0.999	0.999	0.998	0.554	0.997	0.994	0.995	1				
Moving Average	0.970	0.959	0.958	0.554	0.966	0.955	0.972	0.959	1			
Natural Neighbor	0.451	0.620	0.612	0.966	0.625	0.615	0.602	0.612	0.615	1		
Nearest Neighbor	0.991	0.992	0.990	0.571	0.990	0.993	0.985	0.991	0.952	0.629	1	
Polynomial Regression	0.989	0.986	0.986	0.587	0.990	0.980	0.994	0.985	0.987	0.642	0.978	1

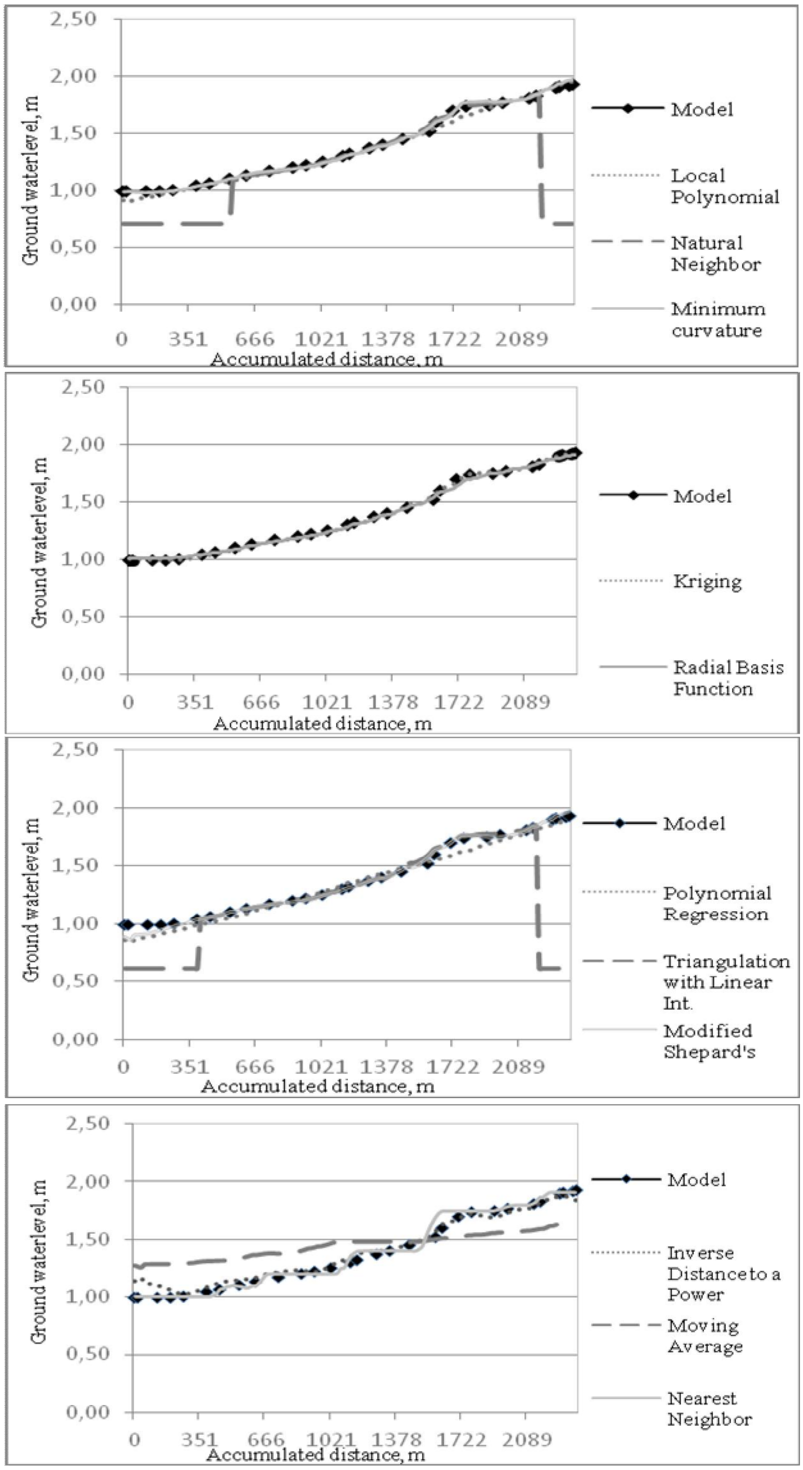

Fig. 6. Slices for compared methods

Data from the fig.6 can be ranked in the following sequence (from maximum to minimum): (Kriging, + Radial Basis Function + Minimum Curvature), Modified Shepard's Method, Inverse Distance to a Power, Moving Average, Local Polynomial, Nearest Neighbor, Polynomial Regression and Triangulation with Linear Interpolation.

The first three methods tend to be more accurate compared with the rest of interpolators. In some way one slice line is not representative for all used methods and in other one - does it make sense to use more lines, and statistically how many lines will be enough?

The best solution in this case is connected with residual analysis. Generally, the Residuals mean the vertical computing subtraction between the Z value of the initial data and the interpolated Z_1 value of the gridded (generated) surface. It gives a quantitative measure of how well the grid data agrees upon the original data [14]. In our study we compute the difference between data model grid and interpolated values of groundwater level by selected interpolators. Graphical representation of residuals is shown in the fig. 7. Distribution of residuals represents intervals of values compared with the model. Suffice it to say that these graphs express comparative situation which is shown in fig. 3-5. For residuals correlation coefficients cannot be computed because data for model are in real values (e.g. groundwater level). Averaged absolute values of residuals for each method are useful and indicative (tab.4).

The lowest value of averaged absolute residuals indicates the best method fits to the initial model.

Table 4

Averaged absolute values of residuals

Method	AAVR
Inverse Distance to a Power	0.435
Kriging	0.013
Local Polynomial	0.029
Minimum curvature	0.009
Modified Shepard's	0.003
Natural Neighbor	0.138
Nearest Neighbor	0.005
Polynomial Regression	0.077
Radial Basis Function	0.027
Moving Average	0.200
Triangulation with Linear Interpolation	0.013

Remark: AAVR is averaged absolute values of residuals.

Data from tab.4 indicate that 3 methods are more suitable (AAVR > 0.2) and namely Inverse Distance to a Power, Natural Neighbor and Moving Average. There is interest how these methods are interrelated between them. Quantitatively the measure of statistical relationship can be expressed by linear correlation (tab.5). According to these data the Modified Shepards's method is highly correlated with Local polynomial (r = 0.852), Kriging (r = 0.811), Minimum curvature (r = 0.728), Natural neighbor (r = 0.775) and Triangulation with Linear Interpolation (r = 0.738).

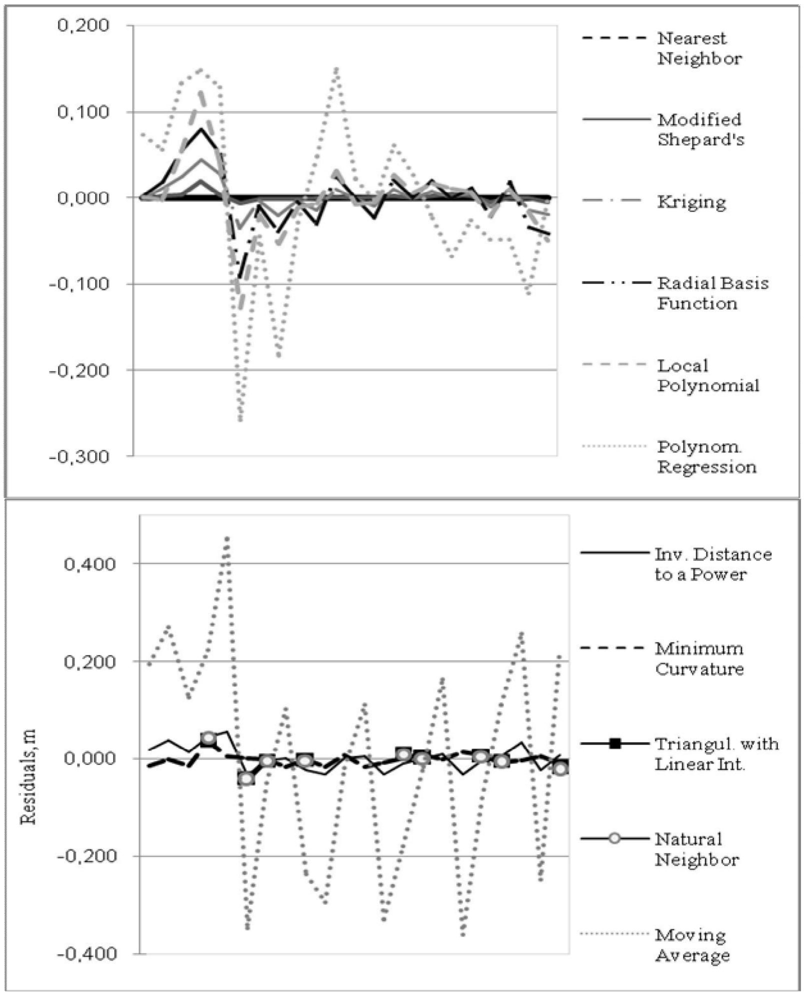

Fig. 7. Residuals for different interpolation methods

Table 5

Correlation coefficients for residuals of different interpolators

Methods	Inverse Distance to a Power	Kriging	Local Polynomial	Minimum Curvature	Modified Shepard's	Natural Neighbor	Nearest Neighbor	Polynomial Regression	Radial Basis Function	Moving Average	Triangulation with Linear Interpolation
Inverse Distance to a Power	1										
Kriging	0.704	1									
Local Polynomial	0.530	0.932	1								
Minimum Curvature	0.175	0.500	0.493	1							
Modified Shepard's	0.407	0.811	0.852	0.728	1						
Natural Neighbor	0.763	0.993	0.995	0.775	0.911	1					
Nearest Neighbor	-0.383	-0.668	-0.821	-0.331	-0.712	-0.981	1				
Polynomial Regression	0.536	0.783	0.821	0.183	0.535	0.892	-0.569	1			
Radial Basis Function	0.691	0.989	0.944	0.426	0.768	0.995	-0.703	0.81	1		
Moving Average	0.955	0.528	0.365	-0.049	0.193	0.460	-0.255	0.44	0.541	1	
Triangulation with Linear Interpolation	0.750	0.983	0.994	0.738	0.882	0.997	-0.990	0.90	0.996	0.463	1

Summary and Conclusions

At present computer-based mapping is associated with a large spectrum of interpolators which are used to generate different surfaces. In hydrogeological research and practice contour maps are widely applied. Different parameters are mapped and mainly groundwater level, hydraulic properties and hydrogeochemical element compositions. In fact, all interpolators on the basis of real data distribution in plane use transformed regular grids. Also hydrogeological medium is considered homogeneous in all directions from one point to other (for instance lithological properties or hydraulic conductivity). These main assumptions lead to errors in surface modeling and estimation.

Ideal interpolator algorithms do not exist and final results of any computer mapping is an approximation of natural conditions. The main question is how closed are modeled hydrogeological conditions to the natural state. In most software, including family of the GIS, number of proposed method is limited or one from the methods is considering more popular and correct. In such context our study was oriented to compare and estimate results of contour mapping using 11 well recognized methods and namely Kriging, Radial Basis Function, Inverse Distance to a Power, Modified Shepard's Method, Minimum Curvature, Polynomial Regression, Triangulation with Linear Interpolation, Nearest Neighbor, Natural Neighbor, Moving Average and Local Polynomial. As a software platform Surfer 11 was exploited. Modeled territory is representatively selected and hydrogeologically simple. Groundwater level was simulated and compared with its natural state of occurrence. On the basis of these simulations several general statements and conclusions are made:

1) Geostatistical comparison of simulated mapping results was caring out using visual analysis, correlation of grid matrices, slice techniques and correlation, residual data processing. Residual analysis is an useful tool to evaluate statistical differences between interpolation methods. Graphics of residuals permits visualization of results. Matrix of residuals has been used to calculate average value and absolute average value for each method. Minimum average absolute value indicates which method is more effective.

2) According to average absolute value the Modified Shepard's Method is considered the most accurate. Nevertheless this method is statistically very similar with Local Polynomial, Kriging, Minimum Curvature, Natural Neighbor and Triangulation with Linear Interpolation ($r > 0.70$). It is logically to conclude that all these methods reflect the better coincidence of simulated surfaces with natural groundwater model.

3) It is safe to assume that the differences between interpolators and elaborated maps are due to the choice of comparison methodology. Additional data is needed to improve the conceptual principle for statistical estimation of differences between hydrogeological maps which are generated with different methods.

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