On the approximation of linear systems with delay and their stability

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In the present paper, we consider applications of the approximation schemes for differential-difference equations to approximate finding nonasymptotic roots of quasipolynomials and analysis of stability for solutions of system of linear differential equations with delay. Considered a linear system of differential equations with many delays

\[
\frac{dx(t)}{dt} = \sum_{i=0}^{k} A_i x(t - \tau_i),
\]

(1)
where \( x \in \mathbb{R}^n, A_i, i = 1, k - n \times n \) are constant matrices, \( 0 = \tau_0 < \tau_1 < \ldots < \tau_k = \tau \).

In accordance with the scheme [1-3], we associate with equation (1) the system of ordinary differential equations

\[
\frac{dz_0(t)}{dt} = \sum_{i=0}^{k} A_i z_i(t), \quad l_i = \left[ \frac{\tau_m}{\tau} \right],
\]

\[
\frac{dz_i(t)}{dt} = \mu [z_{i-1}(t) - z_i(t)], \quad i = 1, m, \quad \mu = \frac{m}{\tau}, \quad m \in \mathbb{N}.
\]

The following theorem is important for constructing algorithms for studying the stability of system (1).

**Theorem.** If null solution of equation (1) is exponentially stable (not stable) then there exists \( m_0 > 0 \) such that for all \( m > m_0 \), null solution of system (3) is exponentially stable (not stable). If for all \( m > m_0 \) null solution of approximation system (3) is exponentially stable (not stable) then null solution of equation (1) is exponentially stable (not stable).

Using the above theorem, we can obtain an effective algorithm for stability analysis of the system

\[
\frac{dx(t)}{dt} = Ax(t) + Bx(t - \tau),
\]

where \( x \in \mathbb{R}^n, A, B - n \times n \) are fixed matrices, \( \tau > 0 \).

When evaluating zeros of the characteristic equation of the approximating system of ordinary differential equations for (3) with different values of \( \tau \) remaining stability of zero solution of the approximating system, we find the delay domain \( \tau \), making system (3) to be exponentially stable.

\[
A = \begin{pmatrix} -0.9 & -6.5 \\ 4.8 & -0.9 \end{pmatrix}, \quad B = \begin{pmatrix} -1,39 & -0,65 \\ 0,48 & -1,39 \end{pmatrix}
\]

is asymptotically stable at \( m = 500 \) in system (2) if where \( \tau \in (0, \tau_1) \cup (\tau_2, \tau_3) \) where \( \tau_1 = 0,2862, \tau_2 = 0,8330, \tau_1 = 1,2290 \).

**Bibliography**

