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THERMOELASTIC EQUILIBRIUM OF SOME SEMI-INFINITE DOMAINS SUBJECTED TO THE ACTION OF A HEAT SOURCE

Victor Şeremet[1,2] and Hui Wang[2]
[1]*Laboratory of Green's Functions, Agrarian State University of Moldova, Chisinau, Moldova*
[2]*Department of Engineering Mechanics, Henan University of Technology, Zhengzhou, China*

By using the integral representations for main thermoelastic Green's functions (MTGFs) we prove a theorem about new structural formulas for MTGFs for a whole class of boundary value problems (BVPs) of thermoelasticity for some semi-infinite Cartesian domains. According to these new structural formulas many MTGFs for a plane, a half-plane, a quadrant, a space, a quarter-space and an octant may be obtained by changing the respective well-known GFPE and their regular parts. The crucial moment of our investigation consists of elaboration of a new technique for calculating some generalized integrals containing products of two different GFPEs. Also, the types of boundary conditions for volume dilatation considered and GFPE for temperature differ on a single boundary only. As example of application of the obtained new structural formulas, the new MTGFs for a concrete BVP of thermoelaesticity for an octant are derived in elementary functions. The MTGFs obtained are validated on a known example for a BVP for half-space. Graphical computer evaluation of the derived in elementary functions new MTGFs is included.

***Keywords*:** Elasticity; Green's functions; Heat conduction; Thermoelastic Green's functions; Thermoelasticity; Volume dilatation

INTRODUCTION

The main integral formulas of the traditional Green's function method (GFM) [1–9] have been extended onto uncoupled thermoelasticity [11–18] in the works [19–23]. However, the most difficult point for their successful application remains (as in traditional GFM) the problem of constructing the Green's functions (GFs), called the main thermoelastic Green's functions (MTGFs) $U_i(x, \xi)$. To derive these functions in the works [19–21] new general integral representations MTGFs and thermoelastic volume dilatation TVD $\Theta(x, \xi)$ were proposed.

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Address correspondence to Victor Şeremet, Laboratory of Green's Functions, Agrarian State University of Moldova, Mircesti Street 44, Chisinau 2049, Moldova. E-mail: v.seremet@uasm.md
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Some applications of the general integral representations to constructing MTGFs for three-dimensional (3D) BVPs for semi-infinite domains, which have no straight lines or parts (plane or its parts) parallel to the Cartesian coordinate axis (Cartesian coordinate planes) were presented in [19–21]. So, in the work [19], for one class of BVPs of thermoelasticity were constructed MTGFs, expressed in terms of Green's functions for Poisson's equation (GFPE), only. Mechanical boundary conditions are locally mixed (normal stresses and tangential displacements or normal displacements and tangential stresses on the boundary are given), but thermal boundary conditions for G_T are given in a such way that for TVD Θ and Green's function for heat conduction G_T are given the similar types of boundary conditions (Dirichlet's or Neumann's types). In these cases all integrals in integral representations vanish and final structural formulas for MTGFs are expressed in terms of GFPE, only.

In reference [20] are found structural formulas for one class of BVPs of thermoelasticity, when on one site are given displacements, only, but the others' sites are subjected to the previously mentioned locally mixed boundary conditions. For the TVD Θ and Green's function G_T are given the above-mentioned similar types of boundary conditions. In these cases the integrals in representations do not vanish, and the final structural formulas for MTGFs are not expressed in the terms of GFPE only. Finally, in the work [21] MTGFs for one BVP of thermoelasticity are obtained for an octant only. Boundary conditions for displacements and stresses are locally mixed, but types of boundary conditions for Θ and G_T are different (Θ has Neumann's, but G_T has Dirichlet's conditions).

In this case some integrals in integral representations do not vanish. Despite the fact that the general integral representations have been proposed their use requires yet special study for each specific BVP in particular for groups of BVPs when we want to derive a structural formula for MTGFs. Studies conducted in previous works [19–21] show that even a change of the type of boundary conditions for the temperature on the heat flux, or vice versa, at the same mechanical boundary conditions, lead to significant mathematical complications. This suggests that the construction of MTGFs for any new BVP or new class of BVPs needs to be studied separately, because these studies require elaboration of new techniques and mathematical procedures.

The main objective of this work is to overcome the new mathematical difficulties and build a technique to derive new structural formulas for MTGFs for new classes of two- and three-dimensional boundary value problems of thermoelasticity. Also, in one example was shown how to apply the derived structural formulas for the construction of new explicit expressions for a particular BVP for a thermoelastic octant.

So, in the present work we had found a new class of BVPs of thermoelasticity for some semi-infinite domains, which do not have straight lines or parts (plane or its parts) parallel to the Cartesian coordinate axis (Cartesian coordinate planes). On one site of these domains are given normal displacements and tangential stresses, but the types of the boundary conditions for Θ and G_T are different (Θ has Neumann's, but G_T has Dirichlet's types of boundary conditions). On the remaining sites are given any kind of mechanical locally mixed boundary conditions, but the types of the boundary conditions for Θ and G_T are the same. For the above-mentioned new class of BVPs of thermoelasticity we obtained new structural formulas for MTGFs, which are expressed in terms of GFPE and its regular parts. To achieve this result we elaborate a new technique to calculate some new integrals, which contain the products of two different GFPEs. So, as a final result, using the obtained new structural formulas we can easily write explicit expressions for MTGFs of 12 new BVPs of thermoelasticity, by changing well-known GFPEs and its regular parts. We think that these new results represent a considerable contribution to constructing MTGFs for BVPs of thermoelasticity.

Thus, we will derive new structural formulas for MTGFs in terms of GFPE and its regular parts. An application of the derived structural formulas to obtaining new explicit MTGFS $U_i(x,\xi)$ and solution in the form of integrals for a new BVP for thermoelastic octant is presented, followed by validation of the derived MTGFs for octant. Finally, the computer evaluation and graphical presentation of the MTGFs for octant are included in the Appendix.

New Structural Formulas for MTGFs in Terms of GFPE and its Regular Parts

Let us consider some canonical semi-infinite domains, whose surfaces represent planes (straight lines) of Cartesian system of coordinates. Also, these domains do not have parallel planes (parallel straight lines). For considered domains, if on the boundary planes (straight lines) are given homogeneous locally mixed boundary conditions for: (a) normal displacements, tangential stresses and Green's function G_T- on one boundary plane (straight line); (b) normal stresses, tangential displacements and G_T; or (c) normal displacements, tangential stresses and normal to the surface (line) derivatives of Green's function $\partial G_T/\partial n$- on the remain boundary planes (straight lines), then the following theorem is true.

Theorem. *Let the field of MTGFs for displacements $U_i(x,\xi)$ and temperature $G_T(x,\xi)$ at inner points $\xi \equiv (\xi_1,\xi_2,\xi_3)$ of the thermoelastic generalized octant $V(0 \le x_1, x_2, x_3 < \infty)$ be determined by non-homogeneous Lamé equations*

$$\mu\nabla_{\xi}^2 U_i(x,\xi) + (\lambda+\mu)\Theta_{,\xi_i}(x,\xi) = \gamma G_{T,\xi_i}(x,\xi) = 0; \quad i = 1,2,3, \tag{1}$$

where λ, μ- are Lamé's elastic constants; $\gamma = \alpha_t(3\lambda+2\mu)$-is the thermoelastic constant; α_t- is coefficient of linear thermal expansion; and $G_T(x,\xi)$ is Green's function in heat conduction, described by the Poisson-type equation

$$\nabla_x^2 G_T(x,\xi) = -\delta(x-\xi) \tag{2}$$

Also, for the points $y \equiv (0, y_2, y_3)$, $y \equiv (y_1, 0, y_3)$ and $y \equiv (y_1 y_2, 0)$ of boundary quadrants $\Gamma_{10}(y_1 = 0,\ 0 \le y_2, y_3 < \infty)$, $\Gamma_{20}(0 \le y_1 < \infty,\ y_2 = 0,\ 0 \le y_3 < \infty)$ and $\Gamma_{30}(0 \le y_1, y_2 < \infty,\ y_3 = 0)$ the following homogeneous locally mixed mechanical and thermal conditions are given:

$$U_1(x;\ 0,\xi_2,\xi_3) = \sigma_{12}(x;\ 0,\xi_2,\xi_3) = \sigma_{13}(x;\ 0,\xi_2,\xi_3) = 0; \quad G_T(x;\ 0,\xi_2,\xi_3) = 0 \tag{3}$$

– *on the boundary quadrant Γ_{10};*

$$\sigma_{21}(x;\ \xi_1,0,\xi_3) = U_2(x;\ \xi_1,0,\xi_3) = \sigma_{23}(x;\ \xi_1,0,\xi_3) = 0;$$
$$\partial G_T(x;\ \xi_1,0,\xi_3)/\partial n_{\xi_2} = 0 \tag{4a}$$

or

$$U_1(x;\ \xi_1, 0, \xi_3) = \sigma_{22}(x;\ \xi_1, 0, \xi_3) = U_3(x;\ \xi_1, 0, \xi_3) = 0; \quad G_T(x;\ \xi_1, 0, \xi_3) = 0 \tag{4b}$$

– *on the boundary quadrant* Γ_{20}, *and*

$$\sigma_{31}(x;\ \xi_1, \xi_2, 0) = \sigma_{32}(x;\ \xi_1, \xi_2, 0) = U_3(x;\ \xi_1, \xi_2, 0) = 0;$$
$$\partial G_T(x;\ \xi_1, \xi_2, 0)/\partial n_{\xi_3} = 0 \tag{5a}$$

or

$$U_1(x;\ \xi_1, \xi_2, 0) = U_2(x;\ \xi_1, \xi_2, 0) = \sigma_{33}(x;\ \xi_1, \xi_2, 0) = 0; \quad G_T(x;\ \xi_1, \xi_2, 0) = 0 \tag{5b}$$

– *on the boundary quadrant* Γ_{30}, *where* σ_{33} *and* σ_{21} σ_{31} σ_{23} *are the normal and the tangential stresses which are determined by the well-known Duhamel–Neumann law*

$$\sigma_{ij} = \mu\left(U_{i,j} + U_{j,i}\right) + \delta_{ij}\left(\lambda U_{k,k} - \gamma G_T\right); \quad i, j = 1, 2, 3. \tag{6}$$

Then the structural formulae for MTGFs $U_i(x, \xi)$ *and TVD* $\Theta(x, \xi)$ *for this class of BVPs of thermoelasticity in Eqs.* (4) *and* (3)–(14) *are the following*:

$$U_i(x, \xi) = \gamma\,[2(\lambda + 2\mu)]^{-1}\left[\xi_i G_T(x, \xi) - x_i G_i(x, \xi) + 2\left(x_i W_i(x, \xi) - x_1 \frac{\partial}{\partial x_i}\int W_i(x, \xi)dx_1\right)\right] \tag{7}$$
$$\Theta(x, \xi) = \frac{\gamma}{\lambda + 2\mu} G_T(x, \xi); \quad i = 1, 2, 3; \ k = 2, 3$$

where $G_T(x, \xi)$, $G_i(x, \xi)$ *are GFPE and* $W_i(x, \xi)$ *are those regular parts of the Green's functions* $G_i(x, \xi)$ *that contain inferior index* 1 (*those parts of the* $G_i(x, \xi)$ *that are reflected via boundary* Γ_{10}). *For functions* $G_T(x, \xi)$, $G_i(x, \xi)$ *on the boundary planes* (*straight lines*) *are given homogeneous conditions that are similar to boundary conditions for temperature and thermoelastic displacements* $U_i(x, \xi)$ *respectively. So, as example, under boundary conditions for* $G_i(x, \xi)$ *it means that, if* $U_i = 0$, *then* $G_i = 0$ *and if* $U_{i,n} = 0$, *then* $G_{i,n} = 0$.

Proof. First, we use the general integral representations [19–21] that in the case of the octant $V \equiv (0 \le x_1, x_2, x_3 \le \infty)$ can be rewritten in the following form:

$$\Theta(x, \xi) = \frac{\gamma}{\lambda + 2\mu} G_\Theta(x, \xi) + \sum_{j=1}^{3} \int_{\Gamma_{j0}} \left(\frac{\partial \Theta(y, x)}{\partial n_{y_j}} - \Theta(y, x)\frac{\partial}{\partial n_{yj}}\right) G_\Theta(y, \xi) d\Gamma_{j0}(y) \tag{8}$$

– for thermoelastic TVD $\Theta(x, \xi)$, and

$$U_i(x, \xi) = -\frac{\lambda + \mu}{2\mu}\xi_i \Theta(x, \xi) - \frac{\gamma}{2(\lambda + 2\mu)} x_i G_i(x, \xi) + \frac{\gamma \xi_i}{2\mu} G_T(x, \xi) - \sum_{j=1}^{3} \int_{\Gamma_{j0}} \left(V_i(x, y)\frac{\partial}{\partial n_{y_1}} - \frac{\partial V_i(x, y)}{\partial n_{y_1}}\right) G_i(y, \xi) d\Gamma_{j0}(y) \tag{9}$$

– for MTGFs $U_i(x, \xi)$, where $y \equiv (0, y_2, y_3) \in \Gamma_{10}$, $d\Gamma_{10}(y) \equiv dy_2 dy_3$, $\frac{\partial}{\partial n_{y_1}} = -\frac{\partial}{\partial y_1}$; $y \equiv (y_1, 0, y_3) \in \Gamma_{20}$,

$$d\Gamma_{20}(y) \equiv dy_1 dy_3, \quad \frac{\partial}{\partial n_{y_2}} = -\frac{\partial}{\partial y_2}; \quad y \equiv (y_1, y_2, 0) \in \Gamma_{30},$$

$$d\Gamma_{30}(y) \equiv dy_1 dy_2, \quad \frac{\partial}{\partial n_{y_3}} = -\frac{\partial}{\partial y_3}.$$

Second, we use the following hypotheses presented in works [19–21]:

1) Let the surfaces of some domains represent planes or their parts (straight lines or their parts) of Cartesian system of coordinates. If on the boundary planes or their parts (straight lines or their parts) are given zero normal displacements, zero tangential stresses and zero Green's function for temperature (see Eq. (3)), then the normal derivative of TVD is $\Theta,_n = [\gamma/(\lambda + 2\mu)][\partial G_T(y, \xi)/\partial n]$.
2) Respectively, if on the boundary planes or their parts (straight lines or their parts) are given zero normal displacements, zero tangential stresses and zero normal derivative of Green's function for temperature (see Eqs. (4a) and (5a)), then the normal derivative of TVD is equal to zero, $\Theta,_n = 0$.
3) If on the boundary planes or their parts (straight lines or their parts) are given zero normal stresses, zero tangential displacements and zero Green's function for temperature (see Eqs. (4b) and (5b)), then TVD $\Theta = 0$.

Next, let in the integral representations (8)–(9) the functions G_i, G_Θ and G_T are the GFPE those homogeneous boundary conditions are the similar to the boundary conditions for U_i, Θ and G_T, respectively. So, it means that, if on a boundary quadrant are known U_i and Θ, T, then $G_i = 0$ and $G_\Theta = G_T = 0$; and if on a boundary quadrant are known $U_{i,n}$ and $\Theta_{,n}$, $T_{,n}$, then $G_{i,n} = 0$ and $G_{\Theta,n} = G_{T,n} = 0$. In these cases, using hypothesis 1) in the work [20] is proved that the boundary conditions (3) lead to following equivalent locally mixed boundary conditions:

$$\begin{gathered} U_1 = \sigma_{12} = \sigma_{13} = 0; \quad G_T = 0; \quad \Rightarrow U_1 = 0; \quad U_{1,2} = U_{1,3} = 0; \quad U_{2,1} = U_{3,1} = 0 \\ \Rightarrow \Theta_{,1} = \gamma(\lambda + 2\mu)^{-1} G_{T,1}; \quad G_1 = G_{2,1} = G_{3,1} = G_{\Theta,1} = G_T = 0 \end{gathered} \tag{10}$$

– on the boundary quadrant Γ_{10}.

Also, in the cases presented earlier, using the hypotheses 2 and 3 in the work [19] is proved that the boundary conditions (4a), (4b) and (5a), (5b) lead to following equivalent locally mixed boundary conditions:

$$\begin{gathered} \sigma_{21} = U_2 = \sigma_{23} = 0; \quad G_{T,2} = 0 \quad \Rightarrow U_{1,2} = 0; \quad U_2 = 0; \quad U_{2,1} = U_{2,3} = U_{3,2} = 0 \\ \Rightarrow \Theta_{,2} = 0; \quad G_{1,2} = 0; \quad G_2 = 0; \quad G_{3,2} = 0; \quad G_{\Theta,2} = 0; \quad G_{T,2} = 0 \end{gathered} \tag{11a}$$

or

$$\begin{gathered} U_1 = \sigma_{22} = U_3 = 0; \quad G_T = 0; \quad \Rightarrow U_1 = U_{1,1} = U_{1,3} = U_3 = U_{3,1} = U_{3,3} = U_{2,2} = 0 \\ \Rightarrow \Theta = 0; \quad G_1 = G_{2,2} = G_3 = G_\Theta = G_T = 0 \end{gathered} \tag{11b}$$

– on the boundary quadrant Γ_{20}, and

$$\begin{aligned}&\sigma_{31} = \sigma_{32} = U_3 = 0; \quad \partial G_T/\partial n_{\xi_3} = 0; \quad \Rightarrow U_{1,3} = U_{2,3} = U_3 = U_{3,1} = U_{3,2} \\ &\quad \Rightarrow \Theta_{,3} = 0; \quad G_{1,3} = 0; \quad G_3 = 0; \quad G_{2,3} = 0; \quad G_{\Theta,3} = 0; \quad G_{T,3} = 0;\end{aligned} \tag{12a}$$

or

$$\begin{aligned}&U_1 = U_2 = \sigma_{33} = 0; \quad G_T = 0; \quad \Rightarrow U_1 = U_{1,1} = U_{1,2} = U_2 = U_{2,2} = U_{2,1} = U_{3,3} \\ &\quad \Rightarrow \Theta = G_1 = G_2 = G_{3,3} = G_\Theta = G_T = 0\end{aligned} \tag{12b}$$

– on the boundary quadrant Γ_{30}.

Substituting the boundary values of the TVD Θ and respective GFPE G_Θ from Eqs. (10)–(12b) into representation (8) we can see that integrals on boundary quadrants Γ_{20} and Γ_{30} are zero, so that we obtain:

$$\Theta(x, \xi) = \gamma(\lambda + 2\mu)^{-1} \left[G_\Theta(x, \xi) + \int_{\Gamma_{10}} \left(\partial G_T(y, x)/\partial n_{y_1}\right) \; G_\Theta(y, \xi) d\Gamma_{10}(y) \right] \tag{13}$$

As boundary conditions for $G_\Theta(x, \xi)$ and $G_T(x, \xi)$ on the boundary quadrants Γ_{20} and Γ_{30} are the same, but on boundary quadrant Γ_{10}, $G_\Theta,_1 = 0$, $G_T = 0$, from (14) follows:

$$\Theta(x, \xi) = \gamma(\lambda + 2\mu)^{-1} G_T(x, \xi) \tag{14}$$

Next, if we use boundary conditions (11a)–(12b) and expression (22) in representations (9) we can see:

1. In integral representation (9) rewritten for thermoelastic displacements $U_1(x, \xi)$ all surfaces integrals are zero, and

$$U_1(x, \xi) = \gamma \, [2(\lambda + 2\mu)]^{-1} \, [\xi_1 G_T(x, \xi) - x_1 G_1(x, \xi)] \tag{15}$$

   that coincides with the structural formula for MTGFs (7) at $i = 1$.
2. In integral representation (9) rewritten for thermoelastic displacements $U_k(x, \xi)$, $k = 2, 3$ the integrals on boundary quadrants Γ_{20} and Γ_{30} are zero, and

$$\begin{aligned}U_k(x, \xi) = &-\frac{\lambda + \mu}{2\mu} \frac{\gamma}{(\lambda + 2\mu)} \xi_k G_T(x, \xi) - \frac{\gamma}{2(\lambda + 2\mu)} x_k G_k(x, \xi) + \frac{\gamma \xi_k}{2\mu} G_T(x, \xi) \\ &+ \int_{\Gamma_{10}} \frac{\partial}{\partial n_{y_1}} \left[U_k(x, y) + (2\mu)^{-1} \, y_k \left((\lambda + \mu)\Theta(x, y) - \gamma G_T(x, y) \right) \right] \\ &\times G_k(y, \xi) d\Gamma_{10}(y)\end{aligned} \tag{16a}$$

Taking into account (10) and

$$2W_k(y, \xi) = G_k(y, \xi) \tag{16b}$$

from (16a) we obtain

$$U_k(x,\xi) = \gamma\,[2(\lambda+2\mu)]^{-1}\Bigg[(\xi_k G_T(x,\xi) - x_k G_k(x,\xi)) + 2\int_{\Gamma_{10}} y_k G_{T,1}(x,y) W_k(y,\xi)\,d\Gamma_{10}(y)\Bigg]; \quad k=2,3 \tag{16c}$$

The last integral can be taken as follows:

$$\begin{aligned} I_k(x,\xi) &= \int_{\Gamma_{10}} y_k G_{T,1}(y,x)\; W_k(y,\xi)\,d\Gamma_{10}(y) \\ &= \int_{\Gamma_{10}} y_k W_k(y,\xi) G_{T,1}(y,x)\,d\Gamma_{10}(y) = x_k W_k(x,\xi) - x_1 \frac{\partial}{\partial x_k}\int W_k(x,\xi)\,dx_1 \end{aligned} \tag{16d}$$

where $W_k(x,\xi)$ are those regular parts of the Green's functions $G_k(x,\xi)$ that contain inferior index 1 (those parts of $G_k(x,\xi)$ that are reflected via boundary Γ_{10}).

So, substituting (16d) into (16c) we obtain the final structural formulas for $U_k(x,\xi)$:

$$U_k(x,\xi) = \gamma\,[2(\lambda+2\mu)]^{-1}\Bigg[(\xi_k G_T(x,\xi) - x_k G_k(x,\xi)) + 2\left(x_k W_k(x,\xi) - x_1\frac{\partial}{\partial x_k}\int W_k(x,\xi)\,dx_1\right)\Bigg]; \quad k=2,3 \tag{16e}$$

that coincide with the structural formulas (7) at $i=k=2,3$.

Note that one of the most difficult moments of our investigations was the evaluation of the integral (16d). But this moment was avoided successfully, when we established the following properties of integral I_k:

1. The integral I_k is a harmonic function with respect to coordinates of both points: $\xi \equiv (\xi_1, \xi_2, \xi_3)$ and $x \equiv (x_1, x_2, x_3)$;
2. The values of integral I_k on boundary quadrants are determined by the boundary conditions of his integrands: the integrand $G_k(y,\xi)$ (with respect to coordinates of the point $\xi \equiv (\xi_1, \xi_2, \xi_3)$) and of the integrand $G_{T,1}(y,x)$ (with respect to coordinates of the point $x \equiv (x_1, x_2, x_3)$.

These boundary conditions are given in Eqs. (11a)–(12b).

So, these two properties of the left part of integral I_k help us to write its right part as is shown in Eq. (16d). Note that structural formulas for MTGFs (7) at $i=1,2$ are applicable also for 2D BVPs of thermoelasticity. Thus the proposed technique permits to derive many MTGFs in thermoelasticity, including 2D. Indeed, on the base of structural formula (7) we can easy (by changing the respective well-known analytical expressions for GFPEs $G_T(x,\xi)$, $G_i(x,\xi)$ and calculating some simplest integrals) to write MTGFs $U_i(x, \xi)$ and TVD $\Theta(x, \xi)$ in elementary functions for many BVPs of thermoelasticity: eight for 3D BVPs (one for space, one for half-space, two for quarter-space and four for octant) and four for 2D BVPs (one for plane, one for half-plane and two for quadrant). However, in this study we give only one example for constructing new explicit MTGFs $U_i(x, \xi)$ and TVD $\Theta(x, \xi)$in elementary functions for a BVP of thermoelasticity for an octant.

New Explicit MTGFS $U_i(x, \xi)$ and Solution in the Form of Integrals for a New BVP for Thermoelastic Octant

Let the field of displacements $U_i(x, \xi)$ and temperature $G_T(x, \xi)$at inner points $\xi \equiv (\xi_1, \xi_2, \xi_3)$ of the thermoelastic octant $V(0 \leq x_1, x_2, x_3 < \infty)$ be determined by non-homogeneous Lamé equations (4) and Poisson's equation $\nabla_x^2 G_T(x, \xi) = -\delta(x - \xi)$, but in the points $y \equiv (0, y_2, y_3)$, $y \equiv (y_1, 0, y_3)$ and $y \equiv (y_1 y_2, 0)$ of boundary quadrants $\Gamma_{10}(y_1 = 0,\ 0 \leq y_2, y_3 < \infty)$, $\Gamma_{20}(0 \leq y_1 < \infty,\ y_2 = 0,\ 0 \leq y_3 < \infty)$ and $\Gamma_{30}(0 \leq y_1, y_2 < \infty,\ y_3 = 0)$ the following homogeneous mechanical and thermal conditions are given:

$$U_1 = \sigma_{12} = \sigma_{13} = 0; \quad G_T = 0 \tag{17}$$

– on the boundary quadrant Γ_{10}

$$\sigma_{21} = U_2 = \sigma_{23} = 0; \quad \partial G_T/\partial n_{\xi_2} = 0 \tag{18}$$

– on the boundary quadrant Γ_{20} and

$$U_1 = U_2 = \sigma_{33} = 0; \quad G_T = 0 \tag{19}$$

– on the boundary quadrant Γ_{30}.

Then, according to the boundary conditions (3), the boundary conditions (4a) and the boundary conditions (5b) the respective boundary conditions for GFPE $G_i(x, \xi)$ are:

$$G_1 = G_{2,1} = G_{3,1} = G_T = 0, \tag{20}$$

on the boundary quadrant Γ_{10},

$$G_{1,2} = G_2 = G_{3,2} = G_{T,2} = 0, \tag{21}$$

on the boundary quadrant Γ_{20}, and

$$G_1 = G_2 = G_{3,3} = G_T = 0 \tag{22}$$

on the boundary quadrant Γ_{30}.

So, the expressions of GFPEs with boundary conditions (20)–(22) for octant V are [10]:

$$G_1(x, \xi) = G_T(x, \xi) = (4\pi)^{-1} \left(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} - R_3^{-1} + R_{13}^{-1} - R_{23}^{-1} + R_{123}^{-1}\right) \tag{23a}$$

$$G_2(x,\xi) = (4\pi)^{-1}\left(R^{-1} + R_1^{-1} - R_2^{-1} - R_{12}^{-1} - R_3^{-1} - R_{13}^{-1} + R_{23}^{-1} + R_{123}^{-1}\right) \tag{23b}$$

$$G_3(x,\xi) = (4\pi)^{-1}\left(R^{-1} + R_1^{-1} + R_2^{-1} + R_{12}^{-1} + R_3^{-1} + R_{13}^{-1} + R_{23}^{-1} + R_{123}^{-1}\right) \tag{23c}$$

where

$$\begin{aligned}
R &= \sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2 + (x_3-\xi_3)^2};\\
R_1 &= \sqrt{(x_1+\xi_1)^2 + (x_2-\xi_2)^2 + (x_3-\xi_3)^2}\\
R_2 &= \sqrt{(x_1-\xi_1)^2 + (x_2+\xi_2)^2 + (x_3-\xi_3)^2};\\
R_{12} &= \sqrt{(x_1+\xi_1)^2 + (x_2+\xi_2)^2 + (x_3-\xi_3)^2}\\
R_3 &= \sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2 + (x_3+\xi_3)^2};\\
R_{13} &= \sqrt{(x_1+\xi_1)^2 + (x_2-\xi_2)^2 + (x_3+\xi_3)^2}\\
R_{23} &= \sqrt{(x_1-\xi_1)^2 + (x_2+\xi_2)^2 + (x_3+\xi_3)^2};\\
R_{123} &= \sqrt{(x_1+\xi_1)^2 + (x_2+\xi_2)^2 + (x_3+\xi_3)^2}
\end{aligned} \tag{23d}$$

On the basis of GFPE $G_2(x,\xi)$, $G_3(x,\xi)$ from Eqs. (23b), (23c) and the proved theorem, we can rewrite their regular parts $W_2(x,\xi)$ and $W_3(x,\xi)$ that contains inferior index 1 (those parts of the $G_2(x,\xi)$ and $G_3(x,\xi)$ that are reflected via boundary Γ_{10}) and their integrals:

$$\begin{aligned}
W_2(x,\xi) &= (2\pi)^{-1}\left(R_1^{-1} - R_{12}^{-1} - R_{13}^{-1} + R_{123}^{-1}\right);\\
\int W_2(x,\xi)dx_1 &= (2\pi)^{-1}\ln\left(\frac{|x_1+\xi_1+R_1|\cdot|x_1+\xi_1+R_{123}|}{|x_1+\xi_1+R_{12}|\cdot|x_1+\xi_1+R_{13}|}\right)
\end{aligned} \tag{24}$$

– for $W_2(x,\xi)$, $\int W_2(x,\xi)dx_1$ and

$$\begin{aligned}
W_3(x,\xi) &= (2\pi)^{-1}\left(R_1^{-1} + R_{12}^{-1} + R_{13}^{-1} + R_{123}^{-1}\right)\\
\int W_3(x,\xi)dx_1 &= (2\pi)^{-1}\ln\Big(|x_1+\xi_1+R_1|\cdot|x_1+\xi_1+R_{13}|\cdot|x_1+\xi_1+R_{12}|\\
&\qquad\cdot|x_1+\xi_1+R_{123}|\Big)
\end{aligned} \tag{25}$$

– for $W_3(x,\xi)$, $\int W_3(x,\xi)dx_1$.

Substituting expressions (23a)-(25) in the structural formula (7) we obtain the final explicit expressions for MTGFs $U_i(x,\xi)$:

$$\begin{aligned}
&U_1(x,\xi)\\
&\quad = \gamma\,[8\pi(\lambda+2\mu)]^{-1}(\xi_1-x_1)\left[R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} - R_3^{-1} + R_{13}^{-1} - R_{23}^{-1} + R_{123}^{-1}\right]
\end{aligned} \tag{26a}$$

$$U_2(x,\xi) = \gamma\,[8\pi(\lambda+2\mu)]^{-1}\Big\{\xi_2\left(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} - R_3^{-1} + R_{13}^{-1} - R_{23}^{-1} + R_{123}^{-1}\right)$$
$$- x_2\left(R^{-1} + R_1^{-1} - R_2^{-1} - R_{12}^{-1} - R_3^{-1} - R_{13}^{-1} + R_{23}^{-1} + R_{123}^{-1}\right)$$
$$+ 2\Big[x_2\left(R_1^{-1} - R_{12}^{-1} - R_{13}^{-1} + R_{123}^{-1}\right)$$
$$- x_1\frac{\partial}{\partial x_2}\ln\left(\frac{|x_1+\xi_1+R_1(x,\xi)|\cdot|x_1+\xi_1+R_{123}(x,\xi)|}{|x_1+\xi_1+R_{12}(x,\xi)|\cdot|x_1+\xi_1+R_{13}(x,\xi)|}\right)\Big]\Big\} \tag{26b}$$

$$U_3(x,\xi) = \gamma\,[8\pi(\lambda+2\mu)]^{-1}\Big\{-\xi_3\left(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} - R_3^{-1} + R_{13}^{-1} - R_{23}^{-1} + R_{123}^{-1}\right)$$
$$- x_3\left(R^{-1} + R_1^{-1} + R_2^{-1} + R_{12}^{-1} + R_3^{-1} + R_{13}^{-1} + R_{23}^{-1} + R_{123}^{-1}\right)$$
$$+ 2\Big[-x_3\left(R_1^{-1} + R_{12}^{-1} + R_{13}^{-1} + R_{123}^{-1}\right) - x_1\frac{\partial}{\partial x_3}\ln\Big(|x_1+\xi_1+R_1(x,\xi)|$$
$$\cdot|x_1+\xi_1+R_{12}(x,\xi)|\cdot|x_1+\xi_1+R_{13}(x,\xi)|\cdot|x_1+\xi_1+R_{123}(x,\xi)|\Big)\Big]\Big\} \tag{26c}$$

and for TVD

$$\Theta(x,\xi) = \frac{\gamma}{4\pi(\lambda+2\mu)}\left(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} - R_3^{-1} + R_{13}^{-1} - R_{23}^{-1} + R_{123}^{-1}\right) \tag{27}$$

Also, the final explicit expressions for MTGFs $U_i(x,\xi)$ may be presented in the compact form:

$$U_i(x,\xi) = \gamma\,[8\pi(\lambda+2\mu)]^{-1}\frac{\partial}{\partial\xi_i}\Big[(R - R_1 + R_2 - R_{12} - R_3 + R_{13} - R_{23} + R_{123})$$
$$- 2x_1\ln\left(\frac{|x_1+\xi_1+R_{13}|\cdot|x_1+\xi_1+R_{123}|}{|x_1+\xi_1+R_1|\cdot|x_1+\xi_1+R_{12}|}\right)\Big] \tag{28}$$

Indeed, taking derivatives in (28) we see that they coincide with expressions (26a)–(26c). As example of validation of the obtained MTGFs (26a)–(26c) or (28) in the Appendix we present their graphics, constructed using Maple 15 computer software.

Finally, calculating on the basis of the functions (28) the other influence functions ($\partial U_i(y,\xi)/\partial n_{10} = -\,\partial U_i\,(0,y_2,y_3,\xi)\,/\partial y_1$ on boundary quadrant Γ_{10}, $U_i(y,\xi) = U_i\,(y_1,0,y_3;\xi)$ on boundary quadrant Γ_{20} and $\partial U_i(y,\xi)/\partial n_{30} = -\partial U_i\,(y_1,y_2,0;\xi)\,/\partial y_3$ on boundary quadrant Γ_{30}) and substituting these functions in the general integral formula [19–21], we obtain the following solution in the form of integrals of the above-mentioned BVP for the thermoelastic octant in the author's form:

$$u_i(\xi) = a^{-1}\int_0^\infty\int_0^\infty\int_0^\infty F\,(z)\,U_i\,(z,\xi)\,dz_1dz_2dz_3$$
$$+\int_0^{+\infty}\int_0^{+\infty} T\,(0,y_2,y_3)\,\frac{\partial U_i\,(0,y_2,y_3;\xi)}{\partial y_1}dy_2dy_3$$

$$- \int_0^{+\infty} \int_0^{+\infty} \frac{\partial T(y_1, 0, y_3)}{\partial y_2} U_i(y_1, 0, y_3; \xi)\, dy_1 dy_3$$
$$+ \int_0^{\infty} \int_0^{\infty} T(y_1, y_2, 0) \frac{\partial U_i(y_1, y_2, 0; \xi)}{\partial y_3} dy_1 dy_2;$$
$$z \equiv (z_1, z_2, z_3); \quad \xi \equiv (\xi_1, \xi_2, \xi_3), \tag{29}$$

where $U_i(z, \xi)$ are determined by Eq. (28); the other kernels are:

$$\frac{\partial U_i(0, y_2, y_3; \xi)}{\partial y_1} = -\frac{\partial U_i(0, y_2, y_3; \xi)}{\partial n_{10}} = \frac{2\gamma}{8\pi(\lambda + 2\mu)} \frac{\partial}{\partial \xi_i} \left[\xi_1 \left(-R^{-1} - R_2^{-1} + R_3^{-1} + R_{23}^{-1} \right) - \ln \left(\frac{|\xi_1 + R_3| \cdot |\xi_1 + R_{23}|}{|\xi_1 + R_1| \cdot |\xi_1 + R_2|} \right) \right] \tag{30a}$$

$$U_i(y_1, 0, y_3; \xi) = 2\gamma [8\pi(\lambda + 2\mu)]^{-1} \frac{\partial}{\partial \xi_i} \left[(R - R_1 - R_3 + R_{13}) - 2y_1 \ln \left(\frac{|y_1 + \xi_1 + R_{13}|}{|y_1 + \xi_1 + R_1|} \right) \right] \tag{30b}$$

and

$$\frac{\partial U_i(y_1, y_2, 0; \xi)}{\partial y_3} = -\frac{\partial U_i(y_1, y_2, 0; \xi)}{\partial n_{30}} = 2\gamma [8\pi(\lambda + 2\mu)]^{-1} \times \frac{\partial}{\partial \xi_i} \left[\xi_3 \left(-R^{-1} + R_1^{-1} - R_2^{-1} + R_{12}^{-1} \right) - 2y_1 \xi_3 \left(R_1^{-1} (y_1 + \xi_1 + R_1)^{-1} + R_{12}^{-1} (y_1 + \xi_1 + R_{12})^{-1} \right) \right] \tag{30c}$$

Note, that from formulas (26a)–(26c) or (28) for the thermoelastic octant, we can obtain respective new MTGFs for quarter-space $V(0 \le x_1, x_2 < +\infty, \ -\infty < x_3 < \infty)$. To achieve these results is sufficient to omit in the formulas (26a)–(26c) or (28) for the thermoelastic octant the terms that contain inferior index 3.

Validation of the New MTGFs Obtained for Octant

Validation of MTGFs $U_i(x, \xi)$ obtained before for thermoelastic octant is confirmed by respective already known MTGFs for particular case of half-space $V(0 \le x_1 < +\infty, \ -\infty < x_2, x_3 < +\infty)$ derived before [23]. To confirm MTGFs for half-space $V(0 \le x_1 < +\infty, \ -\infty < x_2, x_3 < +\infty)$ [23] is enough to omit in the formulas (26a)–(26c) or (28) for thermoelastic octant $V(0 \le x_1, x_2, x_3 \le +\infty)$ the terms that contain inferior indexes 2 and 3. Also the correctitude of obtained MTGFs $U_i(x, \xi)$ for octant is checked by using given below Eqs. (31) and (32). Finally in the Appendix is presented MTGFs $U_i(x, \xi)$ and GFPE $G_T(x, \xi)$ for octant, which were evaluated numerically and graphically using Maple 15 computer software.

We checked to confirm that MTGFs (26a)–(26c) or (28) for the thermoelastic octant derived previously with respect of point $x \equiv (x_1, x_2, x_3)$ satisfy Eqs. [19–21]

$$\nabla_x^2 U_i(x, \xi) = -\gamma \Theta^{(i)}(x, \xi) \tag{31}$$

where according to handbook [10] (see answer to problem 16.L.3) the volume dilatation is determined by the expression:

$$\Theta^{(i)}(x,\xi) = -\,[4\pi(\lambda+2\mu)]^{-1}\frac{\partial}{\partial\xi_i}\left(R^{-1}+R_1^{-1}+R_2^{-1}+R_{12}^{-1}-R_3^{-1}-R_{13}^{-1}-R_{23}^{-1}-R_{123}^{-1}\right) \tag{32}$$

Also, MTGFs (26a)–(26c) or (28) satisfy the homogeneous boundary conditions for $U_i = U_i(x,\xi)$ that are identical to whose of G_T:

$$U_i\,(0, y_2, y_3; \xi) = 0, \quad y \equiv (0, y_2, y_3) \in \Gamma_{10} \tag{33a}$$

$$\partial U_i\,(y_1, 0, y_3; \xi)\,/\partial n_{20} = 0, \quad y \equiv (y_1, 0, y_3; \xi) \in \Gamma_{20} \tag{33b}$$

$$U_i\,(y_1, y_2, 0; \xi) = 0, \quad y \equiv (0, y_2, y_3) \in \Gamma_{30} \tag{33c}$$

Also, we checked and confirm that with respect to point $\xi \equiv (\xi_1, \xi_2, \xi_3)$ the derived MTGFs (26a)–(26c) or (28) satisfy Lamé's equations in thermoelasticity

$$\mu\nabla_\xi^2 U_i(x,\xi) + (\lambda+\mu)\Theta_{,\,\xi_i}(x,\xi) - \gamma G_{T,\,\xi_i}(x,\xi) = 0; \quad i = 1,2,3 \tag{34}$$

and the homogeneous mechanical boundary conditions (17)–(19).

CONCLUSIONS

A new approach for derivation of MTGFs $U_i(x,\xi)$ directly from respective Lamé's equations (1) is proposed. To achieve this aim some special general integral representations for functions $U_i(x,\xi)$ presented in Eqs. (8)–(9), are used. A new theorem on structural formulas (7) for functions $U_i(x,\xi)$ in terms of GFPE is proved. According to the structural formulas obtained, the derivation of MTGFs for about 12 BVPs for a plane, a half-plane, a quadrant, a space, a quarter-space, and an octant may be obtained by changing the respective well-known GFPEs and its regular parts. New MTGFs for octant and quarter-space are derived.

All results are obtained in terms of elementary functions with an example of their validation and checking. The derived MTGFs $U_i(x,\xi)$ and GFPE $G_T(x,\xi)$ for an octant are evaluated numerically and graphically using Maple 15 software. Using the proposed technique, it is possible to extend all results obtained here to many BVPs for canonical Cartesian domains that do not have parallel straight lines or their parts (planes or their parts) to the respective coordinates axes (coordinates planes).

APPENDIX

Graphics of MTGFS U_i and GFPE G_T within an Octant

Here we present Figures 1–6, showing the MTGFs $U_i(x,\xi)$, determined by Eqs. (26a)–(26c) or (28) for 3D BVP of thermoelasticity (1) and (17)–(19) for the octant $V(0 \le x_1, x_2, x_3 < \infty)$. Also we present Figure 7 showing the GFPE $G_T(x,\xi)$

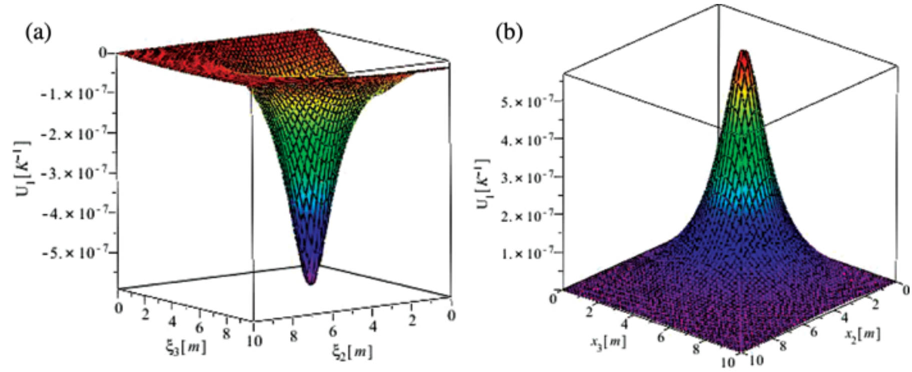

Figure 1 Graphics of changing MTGFs U_1 in dependence of ξ_2, ξ_3 constructed at $\xi_1 = 1m$; $x_1 = 2m$; $x_2 = 3m$; $x_3 = 4m$–Figure 1a; and in dependence of x_2, x_3 constructed at $x_1 = 1m$; $\xi_1 = 2m$; $\xi_2 = 3m$; $\xi_3 = 4m$–Figure 1b.

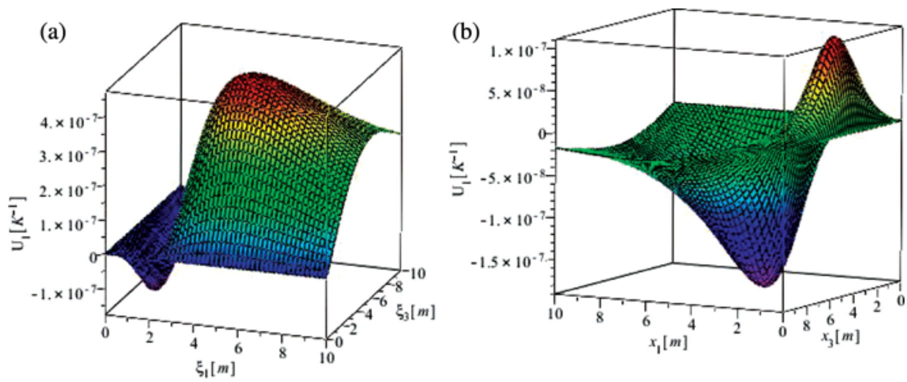

Figure 2 Graphics of changing MTGFs U_1 in dependence of ξ_1, ξ_3, constructed at $\xi_2 = 1m$; $x_1 = 2m$; $x_2 = 3m$; $x_3 = 4m$–Fig. 2a; and in dependence of x_1, x_3, constructed at $x_2 = 1m$; $\xi_1 = 2m$; $\xi_2 = 3m$; $\xi_3 = 4m$–Figure 2b.

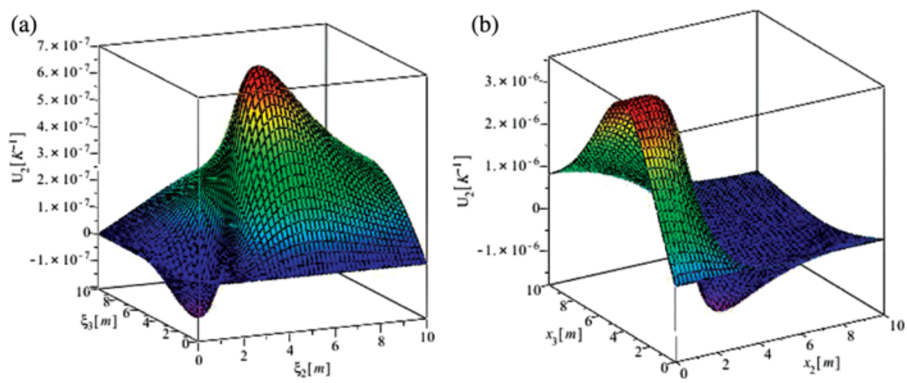

Figure 3 Graphics of changing MTGFs U_2 in dependence of ξ_2, ξ_3 constructed at $\xi_1 = 1m$; $x_1 = 2m$; $x_2 = 3m$; $x_3 = 4m$–Figure 3a; and in dependence of x_2, x_3 constructed at $x_1 = 1m$; $\xi_1 = 2m$; $\xi_2 = 3m$; $\xi_3 = 4m$–Figure 3b.

for 3D BVP that consist from Poisson equation (2) and boundary conditions (20)–(22) of heat conduction theory within the octant V, plotted using Maple 15 software. The MTGFs $U_i(x, \xi)$ are generated by the unitary inner point heat source $F = \delta(x - \xi)$. All 12 graphics for the MTGFs $U_i(x, \xi)$ were constructed at the following values of the constants: Poisson's ratio $\nu = 0.3$; elasticity modulus $E = 2, 1 \cdot 10^5\, MPa$ and coefficient of linear thermal expansion $\alpha_{\mathrm{i}} = 1.2 \times 10^{-5}$. Graphics of changing MTGFs $U_1(x, \xi)$ are shown in Figures 1 and 2.

Graphics of changing MTGFs $U_2(x, \xi)$ are shown in Figures 3 and 4.

Graphics of changing MTGFs are shown in Figures 5 and 6.

The main conclusions that follow from Figures 1–6 are: (1) the derived new MTGFs $U_i(x, \xi)$ satisfy all boundary conditions with respect to both points: $x \equiv (x_1, x_2, x_3)$ and $\xi \equiv (\xi_1, \xi_2, \xi_3)$; and, (2) In the point $x = \xi$ the most MTGFs $U_i(x, \xi)$ have jumps.

Finally, the graphics of changing GFPE G_T are shown in Figure 7.

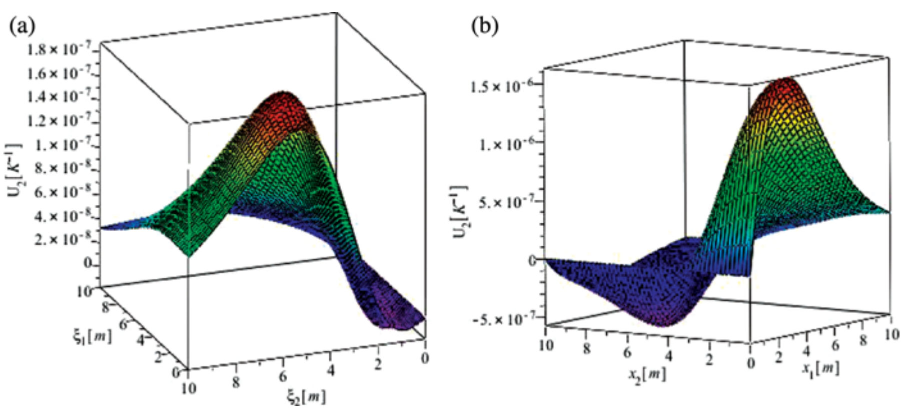

Figure 4 Graphics of changing MTGFs U_2 in dependence of ξ_1, ξ_2 constructed at $\xi_3 = 1m$; $x_1 = 2m$; $x_2 = 3m$; $x_3 = 4m$–Fig. 4a; and in dependence of x_1, x_2, constructed at $x_3 = 1m$; $\xi_1 = 2m$; $\xi_2 = 3m$; $\xi_3 = 4m$–Figure 4b.

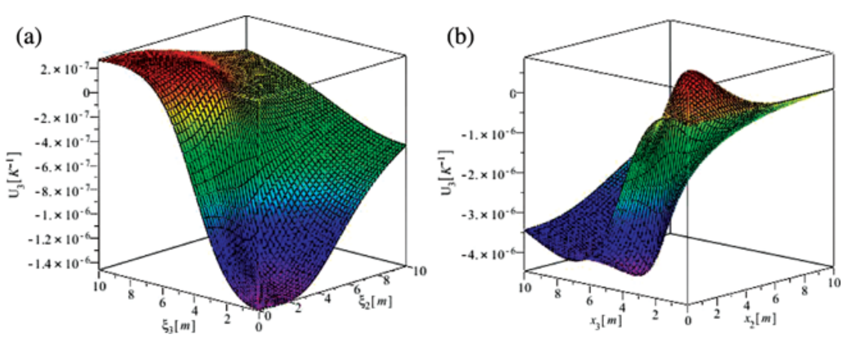

Figure 5 Graphics of changing MTGFs U_3 in dependence of ξ_2, ξ_3 constructed at $\xi_1 = 1m$; $x_1 = 2m$; $x_2 = 3m$; $x_3 = 4m$–Figure 5a; and in dependence ofx_2, x_3, constructed at $x_1 = 1m$; $\xi_1 = 2m$; $\xi_2 = 3m$; $\xi_3 = 4m$–Figure 5b.

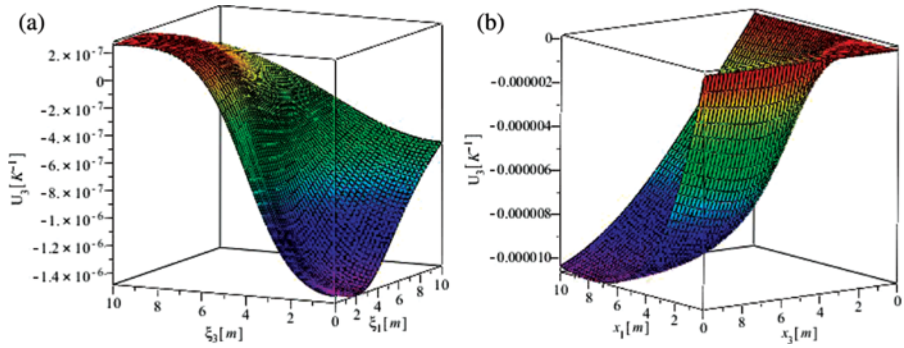

Figure 6 Graphics of changing MTGFs U_3 in dependence of ξ_1, ξ_3 constructed at $\xi_2 = 1m$; $x_1 = 2m$; $x_2 = 3m$; $x_3 = 4m$–Figure 6a; and in dependence of x_1, x_3, constructed at $x_2 = 1m$; $\xi_1 = 2m$; $\xi_2 = 3m$; $\xi_3 = 4m$–Figure 6b.

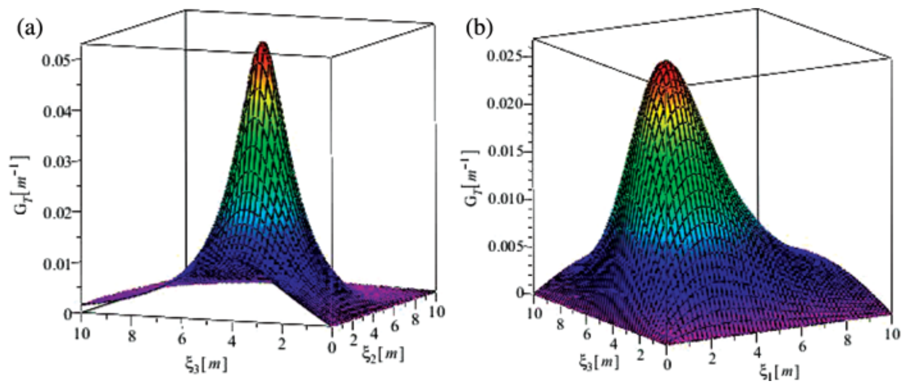

Figure 7 Graphics of changing GFPE G_T in dependence of ξ_2, ξ_3, constructed at $\xi_1 = 1m$; $x_1 = 2m$; $x_2 = 3m$; $x_3 = 4m$–Figure 7a; and in dependence of ξ_1, ξ_3, constructed at $\xi_2 = 1m$; $x_1 = 2m$; $x_2 = 3m$; $x_3 = 4m$–Figure 7b.

The main conclusions that follows from Figure 7 are: (1) The GFPE G_T satisfy all boundary conditions; and, (2) In the point $x = \xi$ the function G_T has a singularity of the R^{-1} type.

NOMENCLATURE

GFs – Green's functions
GFM – Green's function method
MTGFs – main thermoelastic Green's functions
GFPEs – Green's functions for Poisson equation
BVP – boundary values problem
BVPs – boundary values problems
TVD – thermoelastic volume dilatation
2D – two-dimensional

3D – three-dimensional
MPa – mega Pascal
K– degrees Kelvin

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