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RECENT INTEGRAL REPRESENTATIONS FOR THERMOELASTIC GREEN'S FUNCTIONS AND MANY EXAMPLES OF THEIR EXACT ANALYTICAL EXPRESSIONS

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This article is devoted to derivation of new integral representations for the main thermoelastic Green's functions (MTGFs), based on the presentation of solutions of respective Lamé elliptic differential equations via Green's functions for the Poisson equation (GFPEs). The newly derived integral representations in Cartesian coordinates permitted the proof of a theorem about constructive formulas for MTGFs expressed in terms of respective GFPEs. The thermoelastic displacements are generated by a unitary heat source, applied in an arbitrary inner point of a generalized boundary values problem (BVP) of thermoelasticity for an octant at different homogeneous mechanical and thermal boundary conditions, prescribed on its marginal quadrants. According to the constructive formulas obtained, the derivation of MTGFs for about 20 BVPs for a plane, a half-plane, a quadrant, a space, a quarter-space, and an octant may be obtained by changing the respective well-known GFPEs. All results obtained are in terms of elementary functions with many examples of their validation. Two new MTGFs for quarter-space and octant, together with some of their graphical computer evaluations, are also included. The main advantages of the proposed approach in comparison with the GΘ convolution method for MTGFs constructing are: First, it is not necessary to derive the functions of influence of a unit concentrated force onto elastic volume dilatation - $\Theta^{(i)}$. Second, it is not necessary to calculate an integral of the product of the volume dilatation and Green's function in heat conduction. By using the proposed approach it is possible to extend obtained results for Cartesian domains onto areas of any orthogonal system of coordinates.

***Keywords*:** Elasticity; Green's functions; Heat conduction; Main thermoelastic Green's functions; Thermoelasticity; Volume dilatation

INTRODUCTION

Green's functions (GFs) play a leading role in finding solutions in integrals for boundary value problems (BVPs) for different fields of mathematical physics. Several monographs [1–4] present methods of deriving GFs for ordinary and partial differential equations; and for Laplace's, Poisson's, Helmholtz's, and

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other elliptic, parabolic, hyperbolic scalar equations of mathematical physics. The constructing and applications of GFs and matrices for two-dimensional (2D) BVPs for elliptic system of Lamé equations in the theory of elasticity are presented in the monographs [5–7].

A large, systematic list of GFs for 2D BVPs to Poisson equation, derived for canonical Cartesian and polar domains is given in the encyclopedia [8]. Respectively, in a handbook on Green's functions [9] is found a large, systematic table of GFs and Green's matrices for two- and three-dimensional (3D) BVPs in the theory of elasticity, constructed for canonical Cartesian domains.

Until now, most GFs were derived for BVPs of heat conduction and for BVPs of the theory of elasticity for Cartesian canonical domains. In the theory of thermoelasticity, that is a synthesis of the theory of heat conduction and of the theory of elasticity, the situation is not similar. Presently, a number of theories of thermoelasticity are available in the literature [10–16]. But many new developments of thermoelasticity and many references are included in the book by Hetnarski and Eslami [17].

The best developed theory, which is widely used in practical calculations, is the theory of thermal stresses, i.e., the theory of uncoupled thermoelasticity, when the temperature field does not depend on the field of elastic displacements. In that theory, a few observations are worth mentioning. In the theory of uncoupled heat conduction, that is, a constitutional part of the theory of thermal stresses, to solve BVPs, a Green's integral formula provides the temperature field that results from a given thermal exposure. The analogous Green's integral formula determines the field of elastic displacements produced by the known mechanical actions.

But in the integral Maysel's formula [12, 14, 15], the desired solution (the thermoelastic displacements) is not represented directly in terms of the given data, but in terms of a temperature field, which is found in most cases. This fact introduces certain inconveniences in applying Maysel's formula, except for the case when the temperature field is known a priori. To obtain the integral Maysel's formula in uncoupled thermoelasticity, a two-stage procedure must be applied. First, we find a temperature field, and in the second stage we construct a function of influence corresponding to a unit point body force and representing a volume dilatation. To avoid the inconvenience of Maysel's formula, the author proposes, for the first time, the following generalization of Maysel's and Green's integral formulas in thermoelasticity [9, 18–22]:

$$u_i(\xi) = a^{-1}\int_V F(x)U_i(x,\xi)dV(x) - \int_{\Gamma_D} T(y)\frac{\partial U_i(y,\xi)}{\partial n_y}d\Gamma_D(y) + \int_{\Gamma_N}\frac{\partial T(y)}{\partial n_y}U_i(y,\xi)d\Gamma_N(y)$$
$$+ a\int_{\Gamma_M}\left[\alpha T(y) + a\frac{\partial T(y)}{\partial n_y}\right]U_i(y,\xi)d\Gamma_M(y); \quad i = 1, 2, 3 \tag{1}$$

where Γ_D, Γ_N and Γ_M denote the surfaces on which the boundary conditions of Dirichlet's, Neumann's and mixed type are applied, respectively; temperature $T(y)$; heat flux $a(\partial T(y)/\partial n_y)$ or a heat exchange between exterior medium and surface of the body represented by $\alpha T(y) + a[\partial T(y)/\partial n_y]$ are prescribed, where F is the heat source; a is thermal conductivity; α is the coefficient of convective heat conductivity; $\gamma = \alpha_t(2\mu + 3\lambda)$ is the thermoelastic constant; λ, μ are Lamé constants of elasticity; and, α_t is the coefficient of the linear thermal expansion.

The main advantage of Eq. (1) is that the searched thermoelastic displacements u_i are determined in the form of integrals directly via the prescribed inner heat source and other thermal data, shown on the boundary. The considered main thermoelastic Green's functions (MTGFs) $U_i(x, \xi)$ have the physical sense as displacements at an inner point of observation $x \equiv (x_1, x_2, x_3)$, generated by a unit heat source, applied at an inner point $\xi \equiv (\xi_1, \xi_2, \xi_3)$ and described by a δ-Dirac function. The functions $U_i = U_i(x, \xi)$ are determined by the following integral formula [9, 18–22]:

$$U_i(x, \xi) = \gamma \int_V G_T(x, z)\, \Theta^{(i)}(z, \xi) dV(z);\; x, z, \xi \in V \tag{2}$$

where G is the GF for a heat conduction BVP corresponding to a unit internal point heat source, and $\Theta^{(i)}$ are functions of influence of unit concentrated body forces on elastic volume dilatation. Finally, the MTGFs $U_i(x, \xi)$ are functions of double influence [9, 18–22], which take into consideration two physical phenomena (heat conduction and elasticity) in a solid body:

1. over the coordinates of the point of observation $x \equiv (x_1, x_2, x_3)$ for thermoelastic displacements, they satisfy the equations of the BVPs for determining GFs in the theory of heat conduction, in which the unit heat source is replaced by $\gamma\Theta^{(k)}(x, \xi)$:

$$\nabla_x^2 U_i(x, \xi) = -\gamma\Theta^{(i)}(x, \xi) \tag{3}$$

   and suitable boundary conditions are imposed for $U_i = U_i(x, \xi)$.
2. over the coordinates of the point of application $\xi \equiv (\xi_1, \xi_2, \xi_3)$ of the unit point heat source they satisfy the equations of BVP for determining components of the Green's matrix, in which the unit concentrated body forces are replaced with the derivatives of GFs for the heat conduction problems:

$$\mu\nabla_\xi^2 U_i(x, \xi) + (\lambda + \mu)\Theta_{,\xi_i}(x, \xi) - \gamma G_{T,\xi_i}(x, \xi) = 0 \tag{4}$$

   with the respective homogeneous mechanical boundary conditions.

The other functions in (1), $U_i(y, \xi), y \in \Gamma_N$; $\partial U_i(y, \xi)/\partial n_y, y \in \Gamma_N$ and $U_i(y, \xi), y \in \Gamma_M$, represent functions of influence of a unit point heat flux $a(\partial T(y)\partial n_y) = \delta(x - y)$, of a unit point temperature $T(y) = -\delta(x - y)$ and of a unit point heat exchange of the body with exterior medium $\alpha T(y) + a[\partial T(y)/\partial n_y] = \delta(x - y)$ on the surfaces Γ_N, Γ_D and Γ_M, respectively. They are easily determined, if MTGFs $U_k(x, \xi)$ are known.

The proposed integral formula in Eq. (1) can also be treated as a generalization of Mayzel's formula [12, 14, 15] for those cases when the temperature field satisfies the BVPs of heat conduction. The advantage of the proposed integral formula in Eq. (1) is that it allows us to unite the two-stage process of solving the BVPs in the theory of thermoelasticity (the first stage comprises finding temperature fields and the second stage comprises finding thermoelastic displacements) in one single stage.

Also, the advantage of the integral formula in Eq. (1) in comparison with the well-known Maysel's integral formula is that the thermoelastic displacements are determined directly via given heat actions. Besides, for any concrete type of

BVP we can obtain all possible solutions for different laws describing the above-mentioned heat actions. Using formulas in Eqs. (1) and (2), the author derived, in elementary functions, some new very useful thermoelastic GFs and Green-type integral formulas for a quadrant [23], a half-space [24, 25], a quarter-space [26], a wedge [27] and a half-wedge [28]. For these BVPs the difficulties associated with construction of the additional influence functions for elastic volume dilatation and with the computing the volume integral (2) have been successfully overcome.

Furthermore, the author has observed that for more complicated BVPs of thermoelasticity these difficulties are substantial. This is the reason to search for new methods to derive MTGFs. However, the preliminary investigations made by this author have shown that the classical methods [10–17] such as: method of body-force analogy [10, 15], Goodier's method [10], method of thermoelastic potentials [10, 14, 15] and many other methods [11–17] leads to the need to solve additional BVPs of elasticity and to calculate the volume integral (2).

The main objective of this article is to prove a theorem on derivation of constructive formulas for MTGFs to a general BVP for an octant $V(0 \leq x_1, x_2, x_3 < +\infty)$ (or quadrant $V(0 \leq x_1, x_2 < +\infty)$), which is bounded by the quarter-planes $\Gamma_{10}(y_1 = 0, 0 \leq y_2, y_3 < +\infty)$, $\Gamma_{20}(0 \leq y_1, y_3 < +\infty, y_2 = 0)$ and $\Gamma_{30}(0 \leq y_1, y_2 < +\infty, y_3 = 0)$ (or half-planes $\Gamma_{10}(y_1 = 0, 0 \leq y_2 < +\infty)$, $\Gamma_{20}(0 \leq y_1 < +\infty, y_2 = 0)$) and with different types of homogeneous mechanical and thermal boundary conditions. The main advantage of the constructive formulas obtained is that by changes of GFPE, it is possible to easily write MTGFs for about 20 BVPs of thermoelasticity. To reach this objective it is first necessary to establish the general integral representations for MTGFs in the Cartesian system of coordinates.

GENERAL INTEGRAL REPRESENTATIONS FOR MTGFs

As was pointed in the introduction, note that all results obtained earlier by this author in constructing MTGFs were based on Eq. (2), which follows from Eq. (3). A crucial moment of the author's next investigation was when he discovered that Eq. (4) permits us to present a form of three independent equations of Poisson type with respect to functions

$$V_i(x, \xi) = U_i(x, \xi) + \frac{\xi_i}{2\mu}[(\lambda + \mu)\Theta(x, \xi) - \gamma G_T(x, \xi)] \tag{5}$$

as follows:

$$\nabla_\xi^2 V_i(x, \xi) = \gamma[2(\lambda + 2\mu)]^{-1}\xi_i\delta(x - \xi) \tag{6}$$

where

$$(\lambda + 2\mu)\nabla_\xi^2\Theta(x, \xi) - \gamma\nabla_\xi^2 G_T(x, \xi) = 0; \quad \Rightarrow \nabla_\xi^2 G_T(x, \xi) = -\delta(x - \xi);$$
$$\nabla_\xi^2\Theta(x, \xi) = -\frac{\gamma}{\lambda + 2\mu}\delta(x - \xi) \tag{7}$$

was used.

So, representing the solutions of Poisson, Eq. (6) in terms of respective fundamental solutions (with the accuracy up to some regular functions) $G_T(x, \xi)$, $G_i(x, \xi)$ and $G_\Theta(x, \xi)$ that are linked to temperature T and thermoelastic volume dilatation $\Theta(x, \xi)$, we obtain the following integral representations for MTGFs:

$$\begin{aligned} &U_i(x, \xi) + \frac{\lambda+\mu}{2\mu}\xi_i\Theta(x, \xi) \\ &= -\frac{\gamma}{2(\lambda+2\mu)}x_iG_i(x, \xi) + \frac{\gamma\xi_i}{2\mu}G_T(x, \xi) \\ &- \int_\Gamma \left\{ \left[U_i(x, y) + y_i\left(\frac{\lambda+\mu}{2\mu}\Theta(x, y) - \frac{\gamma}{2\mu}G_T(x, y)\right)\right] \frac{\partial G_i(y, \xi)}{\partial n_\Gamma} \right. \\ &\left. - \frac{\partial}{\partial n_\Gamma}\left[U_i(x, y) + y_i\left(\frac{\lambda+\mu}{2\mu}\Theta(x, y) - \frac{\gamma}{2\mu}G_T(x, y)\right)\right] G_i(y, \xi) \right\} d\Gamma(y) \end{aligned} \quad (8)$$

Next, if we introduce in Eq. (8) the following integral representation for $\Theta(x, \xi)$ of the last Poisson Eq. (7) via respective fundamental solution (with exactitude up to regular functions) $G_\Theta(x, \xi)$:

$$\Theta(x, \xi) = \frac{\gamma}{\lambda+2\mu}G_\Theta(x, \xi) + \int_\Gamma \left[\frac{\partial\Theta(x, y)}{\partial n_\Gamma} - \Theta(x, y)\frac{\partial}{\partial n_\Gamma}\right] G_\Theta(y, \xi)d\Gamma(y), \quad (9)$$

then we obtain the following general integral representations for MTGFs:

$$\begin{aligned} U_i(x, \xi) = &-\frac{\gamma}{2(\lambda+2\mu)}\left[\frac{(\lambda+\mu)\xi_i}{\mu}G_\Theta(x, \xi) + x_iG_i(x, \xi) - \frac{(\lambda+2\mu)\xi_i}{\mu}G_T(x, \xi)\right] \\ &- \int_\Gamma \left\{ [U_i(x, y) + (2\mu)^{-1}((\lambda+\mu)(y_i - \xi_i)\Theta(x, y) - \gamma y_iG_T(x, y))]\frac{\partial G_i(y, \xi)}{\partial n_\Gamma} \right. \\ &- \frac{\partial}{\partial n_\Gamma}[U_i(x, y) + (2\mu)^{-1}((\lambda+\mu)(y_i - \xi_i)\Theta(x, y) \\ &\left. - \gamma y_iG_T(x, y))]G_i(y, \xi) \right\} d\Gamma(y) \end{aligned} \quad (10)$$

Note that integral representations (8)–(10) at i = 1, 2 are also applicable to 2D BVPs of thermoelasticity.

Thus, the result is a new idea for deriving MTGFs, using Eq. (4), which in comparison with the state-of-the-art methods (classical methods), is more efficient and unified. This new idea permits us to derive MTGFs directly from Eq. (4), using only the GFPE. Also, in this new concept (in comparison with some classical existing methods) it is not necessary to compute very complicated volume integrals (2). Furthermore, Şeremet's preliminary investigations have shown that at the base of the discovered new idea it becomes possible to develop a new very efficient unified method of constructing new MTGFs (this article). In turn, using this method, it becomes possible to create a large database of MTGFs that is very useful in applications. This is explained by the fact that, unlike existing classical methods, where each problem may be solved in isolation, the expected new method will solve immediately the whole class of thermoelastic problems.

CONSTRUCTIVE FORMULAS FOR MTGFs IN TERMS OF GFPE

Let us consider some canonical semi-infinite domains, whose surfaces represent planes (straight lines) of the Cartesian system of coordinates. Also, these domains do not have parallel planes (parallel straight lines). For domains considered, if homogeneous locally mixed boundary conditions (zero normal stresses and tangential displacements or zero normal stresses and tangential displacements are given in any combinations) are given on the surfaces (lines), then we can prove the following theorem.

Theorem. *Let the field of displacements $U_i(x, \xi)$ and temperature $G_T(x, \xi)$ at inner points $\xi \equiv (\xi_1, \xi_2, \xi_3)$ of the thermoelastic octant $V(0 \le x_1, x_2, x_3 < \infty)$ be determined by non-homogeneous Lamé equations* (4) *and the Poisson equation $\nabla_x^2 G_T(x, \xi) = -\delta(x - \xi)$ and in the points $y \equiv (0, y_2, y_3)$, $y \equiv (y_1, 0, y_3)$ and $y \equiv (y_1 y_2, 0)$ of boundary quadrants $\Gamma_{10}(y_1 = 0, 0 \le y_2, y_3 < +\infty)$, $\Gamma_{20}(0 \le y_1 < \infty, y_2 = 0, 0 \le y_3 < +\infty)$ and $\Gamma_{30}(0 \le y_1, y_2 < +\infty, y_3 = 0)$ the following homogeneous mechanical and thermal conditions are given*:

$$U_1(x; 0, \xi_2, \xi_3) = \sigma_{12}(x; 0, \xi_2, \xi_3) = \sigma_{13}(x; 0, \xi_2, \xi_3) = 0; \quad \partial G_T(x; 0, \xi_2, \xi_3)/\partial n_{\xi_1} = 0$$
$$\sigma_{11}(x; 0, \xi_2, \xi_3) = U_2(x; 0, \xi_2, \xi_3) = U_3(x; 0, \xi_2, \xi_3) = 0; \quad G_T(x; 0, \xi_2, \xi_3) = 0 \quad (11)$$

– *locally mixed boundary conditions on the boundary quadrant $\Gamma_{10}(y_1 = 0, 0 \le y_2, y_3 < +\infty)$*

$$\sigma_{21}(x; \xi_1, 0, \xi_3) = U_2(x; \xi_1, 0, \xi_3) = \sigma_{23}(x; \xi_1, 0, \xi_3) = 0; \quad \partial G_T(x; \xi_1, 0, \xi_3)/\partial n_{\xi_2} = 0$$
$$U_1(x; \xi_1, 0, \xi_3) = \sigma_{22}(x; \xi_1, 0, \xi_3) = U_3(x; \xi_1, 0, \xi_3) = 0; \quad G_T(x; \xi_1, 0, \xi_3) = 0 \quad (12)$$

– *locally mixed boundary conditions on the boundary quadrant $\Gamma_{20}(0 \le y_1 < +\infty, y_2 = 0, 0 \le y_3 < +\infty)$ and*

$$\sigma_{31}(x; \xi_1, \xi_2, 0) = \sigma_{32}(x; \xi_1, \xi_2, 0) = U_3(x; \xi_1, \xi_2, 0) = 0; \quad \partial G_T(x; \xi_1, \xi_2, 0)/\partial n_{\xi_3} = 0$$
$$U_1(x; \xi_1, \xi_2, 0) = U_2(x; \xi_1, \xi_2, 0) = \sigma_{33}(x; \xi_1, \xi_2, 0) = 0; \quad G_T(x; \xi_1, \xi_2, 0) = 0 \quad (13)$$

– *locally mixed boundary conditions on the boundary quadrant $\Gamma_{30}(0 \le y_1, y_2 < +\infty, y_3 = 0)$, where σ_{33} and σ_{21} σ_{31} σ_{23} are the normal and the tangential stresses, which are determined by the well-known Duhamel–Neumann law*

$$\sigma_{ij} = \mu(U_{i,j} + U_{j,i}) + \delta_{ij}(\lambda U_{k,k} - \gamma G_T); \quad i, j = 1, 2, 3 \quad (14)$$

Then representations in Eq. (6) *lead to the following constructive formulae for MTGFs*:

$$U_i(x, \xi) = \gamma[2(\lambda + 2\mu)]^{-1}[\xi_i G_T(x, \xi) - x_i G_i(x, \xi)]; \quad \Theta(x, \xi) = \frac{\gamma}{\lambda + 2\mu} G_T(x, \xi) \quad (15)$$

where $G_T(x,\xi)$ and $G_i(x,\xi)$ are GFPE for respective domains, on the marginal planes (straight lines) are given homogeneous conditions that are similar to boundary conditions for temperature and MTGFs, respectively. So, if $U_i = 0$, then $G_i = 0$ and if $U_{i,n} = 0$, then $G_{i,n} = 0$.

Proof. First, for this purpose we use the general representations (8)–(10) such that in the case of the octant, $V \equiv (0 \le x_1, x_2, x_3 \le \infty)$ can be rewritten in the following form:

$$\begin{aligned}\Theta(x,\xi) = {}& \frac{\gamma}{\lambda+2\mu} G_\Theta(x,\xi) + \int_{\Gamma_{10}} \left[\frac{\partial\Theta(y,x)}{\partial n_{y_1}} - \Theta(y,x)\frac{\partial}{\partial n_{y_1}}\right] G_\Theta(y,\xi)\,d\Gamma_{10}(y) \\ &+ \int_{\Gamma_{20}} \left[\frac{\partial\Theta(y,x)}{\partial n_{y_2}} - \Theta(y,x)\frac{\partial}{\partial n_{y_2}}\right] G_\Theta(y,\xi)\,d\Gamma_{20}(y) \\ &+ \int_{\Gamma_{30}} \left[\frac{\partial\Theta(y,x)}{\partial n_{y_3}} - \Theta(y,x)\frac{\partial}{\partial n_{y_3}}\right] G_\Theta(y,\xi)\,d\Gamma_{30}(y) \end{aligned} \tag{16}$$

– for thermoelastic volume dilatation, and

$$\begin{aligned} U_i(x,\xi) = {}& -\frac{\lambda+\mu}{2\mu}\xi_i\Theta(x,\xi) - \frac{\gamma}{2(\lambda+2\mu)} x_i G_i(x,\xi) + \frac{\gamma\xi_i}{2\mu} G_T(x,\xi) \\ &- \int_{\Gamma_{10}} \left\{ [U_i(x,y) + (2\mu)^{-1} y_i((\lambda+\mu)\Theta(x,y) - \gamma G_T(x,y))] \frac{\partial G_i(y,\xi)}{\partial n_{y_1}} \right. \\ &\left. - \frac{\partial}{\partial n_{y_1}} [U_i(x,y) + (2\mu)^{-1} y_i((\lambda+\mu)\Theta(x,y) - \gamma G_T(x,y))] G_i(y,\xi) \right\} d\Gamma_{10}(y) \\ &- \int_{\Gamma_{20}} \left\{ [U_i(x,y) + (2\mu)^{-1} y_i((\lambda+\mu)\Theta(x,y) - \gamma G_T(x,y))] \frac{\partial G_i(y,\xi)}{\partial n_{y_2}} \right. \\ &\left. - \frac{\partial}{\partial n_{y_2}} [U_i(x,y) + (2\mu)^{-1} y_i((\lambda+\mu)\Theta(x,y) - \gamma G_T(x,y))] G_i(y,\xi) \right\} d\Gamma_{20}(y) \\ &- \int_{\Gamma_{30}} \left\{ [U_i(x,y) + (2\mu)^{-1} y_i((\lambda+\mu)\Theta(x,y) - \gamma G_T(x,y))] \frac{\partial G_i(y,\xi)}{\partial n_{y_3}} \right. \\ &\left. - \frac{\partial}{\partial n_{y_3}} [U_i(x,y) + (2\mu)^{-1} y_i((\lambda+\mu)\Theta(x,y) - \gamma G_T(x,y))] G_i(y,\xi) \right\} d\Gamma_{30}(y) \end{aligned} \tag{17}$$

– for *MTGFs*.

First, let us prove the following hypotheses:

a) Let the surfaces of some domains represent planes or their parts (straight lines or their parts) of Cartesian system of coordinates. If on the marginal planes or their parts (straight lines or their parts) are given zero normal stresses, zero tangential displacements and zero Green's function for temperature, then volume dilatation is equal to zero, $\Theta = 0$. Let on the marginal quadrant $\Gamma_{10}(y_1 = 0, 0 \le y_2, y_3 < \infty)$, the following homogeneous locally mixed boundary conditions are given:

$$\sigma_{11} = U_2 = U_3 = 0; \quad G_T = 0 \tag{18}$$

Then from the well-known Duhamel–Neumann law (14) rewritten for thermal stresses

$$\sigma_{11} = 2\mu U_{1,1} + (\lambda\Theta - \gamma G_T) = (\lambda + 2\mu)U_{1,1} + \lambda(U_{2,2} + U_{3,3}) - \gamma G_T; \quad \Theta = U_{k,k} \tag{19}$$

and (18) follows:

$$\left.\begin{array}{r} U_2 = 0 \Rightarrow U_{2,2} = 0; \quad U_3 = 0 \Rightarrow U_{3,3.} = 0 \\ \sigma_{11} = (\lambda + 2\mu)U_{1,1} + \lambda(U_{2,2} + U_{3,3}) - \gamma G_T = 0; \quad G_T = 0; \quad \Rightarrow U_{1,1} = 0 \end{array}\right\} \Rightarrow \tag{20}$$
$$U_{1,1} = 0, \quad U_{2,2} = 0; \quad U_{3,3.} = 0; \quad \Theta = U_{k,k} \Rightarrow \Theta = 0$$

b) Respectively, if on the marginal planes or their parts (straight lines or their parts) are given zero normal displacements, zero tangential stresses and zero normal derivative of Green's function for temperature, then the normal derivative of volume dilatation is equal to zero, $\Theta_{,n} = 0$.

Let on the marginal quadrant $\Gamma_{20}(0 \le y_1 < +\infty, y_2 = 0, 0 \le y_3 < +\infty)$ the following homogeneous locally mixed boundary conditions are given

$$U_2 = \sigma_{21} = \sigma_{23} = 0; \quad G_{T,2} = 0 \tag{21}$$

Then from the boundary conditions with respect to tangential stresses in Eq. (21) and the equilibrium equation

$$\sigma_{2j,j} = 0; \quad j = 1, 2, 3 \tag{22}$$

follow the relations:

$$\sigma_{21,1} = \sigma_{23,3} = 0 \Rightarrow \sigma_{22,2} = 0 \tag{23}$$

Next from the Duhamel–Neumann law (14) rewritten for thermal stresses σ_{22}

$$\sigma_{22} = 2\mu U_{2,2} + (\lambda\Theta - \gamma G_T); \quad \Theta = U_{i,i} \tag{24}$$

from Eq. (24) for Θ and from Eq. (23) for $\sigma_{22,2}$ it follows:

$$\sigma_{22,2} = 2\mu U_{2,22} + \lambda\Theta_{,2} = (\lambda + 2\mu)\Theta_{,2} - 2\mu(U_{1,12} + U_{3,32}) = 0 \tag{25}$$

From the first boundary conditions in Eq. (21) and Duhamel–Neumann law (14) for the tangential stresses $\sigma_{21},\ \sigma_{23}$ in Eq. (21), it follows:

$$\left.\begin{array}{l} U_2 = 0 \to U_{2,11} = 0; \quad U_{2,33} = 0 \\ \sigma_{21,1} = 0 \to \mu(U_{2,11} + U_{1,21}) = 0 \\ \sigma_{23} = 0 \to \mu\left(U_{2,33} + U_{3,32}\right) = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} U_{1,12} = 0 \\ U_{3,32} = 0 \end{array}\right. \tag{26}$$

Finally, from Eq. (25) and the last equality in Eq. (26) it follows that the boundary conditions in Eq. (21) lead to zero normal derivative from volume dilatation on the boundary quadrant Γ_{20}

$$\Theta_{,2} = 0 \to [\partial\Theta(y, \xi)/\partial n_{y2}] = 0 \tag{27}$$

Next, let in the representations (17) the functions G_i, G_Θ and G_T are the GFPE, those homogeneous boundary conditions are similar to the homogeneous boundary conditions for U_i, Θ and G_T, respectively. So, it means that if $U_i = 0$, $\Theta = 0$ and $G_T = 0$, then $G_i = 0$ and $G_\Theta = G_T = 0$; and if $U_{i,n} = 0$, $\Theta_{,n} = 0$ and $G_{T,n} = 0$, then $G_{i,n} = 0$, $G_{\Theta,n} = G_{T,n} = 0$.

In these cases we can prove that from the boundary conditions (11)–(13) follow equivalent conditions:

$$\begin{aligned} U_1 &= \sigma_{12} = \sigma_{13} = 0; \quad \partial G_T/\partial n_{\xi 1} = 0; \quad \Rightarrow U_1 = 0; \quad U_{1,2} = U_{1,3} = 0; \quad U_{2,1} = U_{3,1} = 0 \\ &\Rightarrow \Theta,_1 = 0; \quad G_1 = G_{2,1} = G_{3,1} = G_{\Theta,1} = G_{T,1} = 0 \\ \sigma_{11} &= U_2 = U_3 = 0; \quad G_T = 0; \quad \Rightarrow U_{1,1} = U_2 = U_{2,2} = U_{2,3} = U_3 = U_{3,2} = U_{3,3.} = 0 \\ &\Rightarrow \Theta = 0; \quad G_{1,1} = G_2 = G_3 = G_\Theta = G_T = 0 \end{aligned} \tag{28}$$

– for locally mixed boundary conditions on the marginal quadrant $\Gamma_{10}(y_1 = 0, 0 \le y_2, y_3 < +\infty)$

$$\begin{aligned} \sigma_{21} &= U_2 = \sigma_{23} = 0; \quad \partial G_T/\partial n_{\xi 2} = 0 \Rightarrow U_{1,2} = 0; \quad U_2 = 0; \quad U_{2,1} = U_{2,3} = U_{3,2} = 0 \\ &\Rightarrow \Theta_{,2} = 0; \quad G_{1,2} = 0; \quad G_2 = 0; \quad G_{3,2} = 0; \quad G_{\Theta,2} = 0; \quad G_{T,2} = 0 \\ U_1 &= \sigma_{22} = U_3 = 0; \quad G_T = 0 \Rightarrow U_1 = U_{1,1} = U_{1,3} = U_3 = U_{3,1} = U_{3,3} = U_{2,2} = 0 \\ &\Rightarrow \Theta = 0; \quad G_1 = G_{2,2} = G_3 = G_\Theta = G_T = 0 \end{aligned} \tag{29}$$

– for locally mixed boundary conditions on the marginal quadrant $\Gamma_{20}(0 \le y_1 < +\infty, y_2 = 0, 0 \le y_3 < +\infty)$ and

$$\begin{aligned} \sigma_{31} &= \sigma_{32} = U_3 = 0; \quad \partial G_T/\partial n_{\xi 3} = 0; \quad \Rightarrow U_{1,3} = U_{2,3} = U_3 = U_{3,1} = U_{3,2} \\ &\Rightarrow \Theta_{,3} = 0; \quad G_{1,3} = 0; \quad G_3 = 0; \quad G_{2,3} = 0; \quad G_{\Theta,3} = 0; \quad G_{T,3} = 0 \\ U_1 &= U_2 = \sigma_{33} = 0; \quad G_T = 0 \Rightarrow U_1 = U_{1,1} = U_{1,2} = U_2 = U_{2,2} = U_{2,1} = U_{3,3} \\ &\Rightarrow \Theta = G_1 = G_2 = G_{3,3} = G_\Theta = G_T = 0 \end{aligned} \tag{30}$$

– for locally mixed boundary conditions on the marginal quadrant $\Gamma_{30}(0 \leq y_1, y_2 < +\infty, y_3 = 0)$.
Indeed, from each boundary condition (11)–(13) follow identical conditions:

$$\begin{aligned} G_{T,1} &= 0;\ \{U_1 = 0 \Rightarrow U_{1,2} = U_{1,3} = 0;\ G_1 = 0;\ \sigma_{12} = 0 \Rightarrow U_{2,1} = 0;\ G_{2,1} = 0; \\ &\qquad \sigma_{13} = 0 \Rightarrow U_{3,1} = 0;\ G_{3,1} = 0\} \\ &\Rightarrow \Theta_{,1} = 0;\ G_{\Theta,1} = 0 \\ G_T &= 0;\ \{U_2 = 0 \Rightarrow U_{2,2} = U_{2,3} = 0;\ U_3 = 0 \Rightarrow U_{3,2} = U_{3,3} = 0;\ \sigma_{11} = 0\} \\ &\Rightarrow U_{1,1} = 0;\ G_{1,1} = 0 \Rightarrow \Theta = 0;\ G_\Theta = 0 \end{aligned} \tag{31}$$

– from locally mixed boundary conditions (11) on the marginal quadrant Γ_{10} $(y_1 = 0, 0 \leq y_2, y_3 < +\infty)$;

$$\begin{aligned} G_{T,1} &= 0;\ \{U_2 = 0 \Rightarrow U_{2,1} = U_{2,3} = 0;\ G_2 = 0;\ \sigma_{21} = 0 \Rightarrow U_{1,2} = 0;\ G_{1,2} = 0; \\ &\qquad \sigma_{23} = 0 \Rightarrow U_{3,2} = 0;\ G_{3,2} = 0\} \\ &\Rightarrow \Theta_{,2} = 0;\ G_{\Theta,2} = 0 \\ G_T &= 0;\ \{U_1 = 0 \Rightarrow U_{1,1} = U_{1,3} = G_1 = 0;\ U_3 = 0 \Rightarrow U_{3,1} = U_{3,3} = 0; \\ &\qquad G_3 = 0;\ \sigma_{11} = 0\} \Rightarrow U_{2,2} = 0;\ G_{2,2} = 0 \\ &\Rightarrow \Theta = 0;\ G_\Theta = 0 \end{aligned} \tag{32}$$

– from locally mixed boundary conditions (12) on the marginal quadrant Γ_{20} $(0 \leq y_1 < +\infty, y_2 = 0, 0 \leq y_3 < +\infty)$; and

$$\begin{aligned} G_{T,3} &= 0;\ \{U_3 = 0 \Rightarrow U_{3,2} = U_{3,1} = 0;\ G_3 = 0;\ \sigma_{31} = 0 \Rightarrow U_{1,3} = 0;\ G_{1,3} = 0; \\ &\qquad \sigma_{32} = 0 \Rightarrow U_{2,3} = 0;\ G_{2,3} = 0;\ G_{T,3} = 0\} \\ &\Rightarrow \Theta_{,3} = 0;\ G_{\Theta,3} = 0 \\ G_T &= 0;\ \{U_1 = 0 \Rightarrow U_{1,1} = U_{1,2} = G_1 = U_2 = U_{2,1} = U_{2,2} = G_2 = 0;\ \sigma_{33} = 0\} \\ &\Rightarrow U_{3,3} = 0;\ G_{3,3} = 0 \\ &\Rightarrow \Theta = 0;\ G_\Theta = 0 \end{aligned} \tag{33}$$

– from locally mixed boundary conditions (13) on the marginal quadrant Γ_{30} $(0 \leq y_1, y_2 < +\infty, y_3 = 0)$.

Substituting the values of the volume dilatation Θ on each boundary quadrant from Eqs. (28)–(30) into representation (16) we can see that all surfaces integrals are zero, and

$$\Theta(x, \xi) = \frac{\gamma}{\lambda + 2\mu} G_\Theta(x, \xi) = \frac{\gamma}{\lambda + 2\mu} G_T(x, \xi) \tag{34}$$

Note, that in Eq. (24) for Green's function for temperature to coincide with Green's function for volume dilatation, it means $G_T(x, \xi) = G_\Theta(x, \xi)$. Next, if we use boundary conditions (18)–(20) and expression (34) in representations (17) we obtain the constructive formulas for MTGFs (15) in terms of GFPE. Note that constructive formulas for MTGFs (15) at $\mathrm{I} = 1, 2$ are applicable also for 2D BVPs of thermoelasticity.

Thus the proposed method will help create a large database of MTGFs in thermoelasticity. Indeed, on the base of constructive formula (15) we can very easily (by changing the respective well-known analytical expressions for GFs $G_T(x, \xi)$ and $G_i(x, \xi)$) to write MTGFs $U_i(x, \xi)$ in elementary functions for about 14 3D and for about 6 2D BVPs of thermoelasticity.

VALIDATION OF THE OBTAINED RESULTS

Here, we show the validation of the obtained constructive formula (15) for MTGFs $U_i(x, \xi)$ in three particular cases of BVPs of thermoelasticity: for half-space-2 BVPs and for half-plane-1 BVP. Note that the MTGFs for these BVPs were obtained earlier [24, 26, 29], using convolution method based on the general formula (2).

Example 1 (MTGFs within half-space). Let the field of displacements $U_i(x, \xi)$ and temperature $G_T(x, \xi)$ at inner points $\xi \equiv (\xi_1, \xi_2, \xi_3)$ of the thermoelastic half-space $V \equiv (0 \le x_1 < +\infty, -\infty < x_2, x_3 < +\infty)$ be determined by non-homogeneous Lamé equations (4) and Poisson equation $\nabla_x^2 G_T(x, \xi) = -\delta(x - \xi)$ and in the points $y \equiv (0, y_2, y_3)$ of the boundary plane $\Gamma_{10}(y_1 = 0, -\infty < y_2, y_3 < +\infty)$, the following homogeneous mechanical and thermal conditions are given

$$\sigma_{11}(x; 0, \xi_2, \xi_3) = U_2(x; 0, \xi_2, \xi_3) = U_3(x; 0, \xi_2, \xi_3) = 0; \quad G_T(x; 0, \xi_2, \xi_3) = 0 \quad (35)$$

Then, substituting the respective GFPE to the general constructive formulas (15) we obtain the final expressions for MTGFs $U_i(x, \xi)$ for the half-space in the form

$$\begin{aligned} U_1(x, \xi) &= \gamma[8\pi(\lambda + 2\mu)]^{-1}[\xi_1(R^{-1} - R_1^{-1}) - x_1(R^{-1} + R_1^{-1})]; \\ U_k(x, \xi) &= \gamma[8\pi(\lambda + 2\mu)]^{-1}(\xi_k - x_k)(R^{-1} - R_1^{-1}); \quad k = 2, 3; \\ \Theta(x, \xi) &= [4\pi(\lambda + 2\mu)]^{-1}(R^{-1} - R_1^{-1}) \end{aligned} \quad (36)$$

Note that derived expressions (36) for MTGFs coincide with the expressions for GFs presented in [24] in the form:

$$U_i(x, \xi) = \gamma[8\pi(\lambda + 2\mu)]^{-1} \frac{\partial}{\partial \xi_i}(R - R_1) \quad (37)$$

Indeed, taking the derivatives in (37) it is observed that expressions (36) and (37) coincide.

Example 2 (MTGFs within half-space). Let the field of displacements $U_i(x, \xi)$ and temperature $G_T(x, \xi)$ at inner points $\xi \equiv (\xi_1, \xi_2, \xi_3)$ of the thermoelastic half-space $V \equiv (0 \le x_1 < +\infty, -\infty < x_2, x_3 < +\infty)$ be determined by non-homogeneous Lamé equations (4) and Poisson equation $\nabla_x^2 G_T(x, \xi) = -\delta(x - \xi)$ and in the points

$y \equiv (0, y_2, y_3)$ of the boundary plane $\Gamma_{10}(y_1 = 0, -\infty < y_2, y_3 < +\infty)$, the following homogeneous mechanical and thermal conditions are given

$$U_1(x; 0, \xi_2, \xi_3) = \sigma_{12}(x; 0, \xi_2, \xi_3) = \sigma_{13}(x; 0, \xi_2, \xi_3) = 0; \quad \partial G_T(x; 0, \xi_2, \xi_3)/\partial n_{\xi 1} = 0 \tag{38}$$

Then, substituting the respective GFPE to the general constructive formulas (15) we obtain the final expressions for MTGFs $U_i(x, \xi)$ for the half-space in the form:

$$\begin{aligned} U_1(x, \xi) &= \gamma[8\pi(\lambda + 2\mu)]^{-1}[\xi_1(R^{-1} + R_1^{-1}) - x_1(R^{-1} - R_1^{-1})]; \\ U_k(x, \xi) &= \gamma[8\pi(\lambda + 2\mu)]^{-1}(\xi_k - x_k)(R^{-1} + R_1^{-1}); \quad k = 2, 3 \\ \Theta(x, \xi) &= \gamma[4\pi(\lambda + 2\mu)]^{-1}(R^{-1} + R_1^{-1}). \end{aligned} \tag{39}$$

Note, that expressions (39) for the MTGFs coincide with the expressions $U_i(x, \xi) = \gamma[8\pi(\lambda + 2\mu)]^{-1}\frac{\partial}{\partial \xi_i}(R + R_1 + R_2 + R_{12})$ for GFs presented in [26], if functions R_1 and R_{12} are omitted and function R_2 is changed by R_1, so that final expression for GFs have the form

$$U_i(x, \xi) = \gamma[8\pi(\lambda + 2\mu)]^{-1}\frac{\partial}{\partial \xi_i}(R + R_1) \tag{40}$$

Indeed, in taking the derivatives in (40), it is observed that the expressions (39) and (40) coincide.

Example 3 (MTGFs within half-plane). Let the field of displacements $U_i(x, \xi)$ and temperature $G_T(x, \xi)$ at inner points $x \equiv (x_1, x_2)$ and $\xi \equiv (\xi_1, \xi_2)$ of the thermoelastic half-plane $V \equiv (0 \le x_1 < \infty, -\infty < x_2 < +\infty)$ be determined by non-homogeneous Lamé equations (4) and Poisson equation $\nabla_x^2 G_T(x, \xi) = -\delta(x - \xi)$ and in the points $y \equiv (0, y_2)$ of the boundary straight line $\Gamma_{10}(y_1 = 0, -\infty < y_2 < +\infty)$, the following homogeneous mechanical and thermal conditions are given

$$U_1(x; 0, \xi_2) = \sigma_{12}(x; 0, \xi_2) = \partial G_T(x; 0, \xi_2)/\partial n_{\xi 1} = 0 \tag{41}$$

Then, substituting the respective GFPE to the general constructive formulas (15) we obtain the final expressions for MTGFs $U_i(x, \xi)$ for the half-space in the form

$$\begin{aligned} U_1(x, \xi) &= -\gamma[4\pi(\lambda + 2\mu)]^{-1}[\xi_1(\ln r + \ln r_1) - x_1(\ln r - \ln r_1)] \\ U_2(x, \xi) &= -\gamma[4\pi(\lambda + 2\mu)]^{-1}(\xi_2 - x_2)(\ln r + \ln r_1) \\ \Theta(x, \xi) &= -\gamma[2\pi(\lambda + 2\mu)]^{-1}(\ln r + \ln r_1) \end{aligned} \tag{42}$$

Note, that expressions (42) for the MTGFs coincide with the expressions for the term $U_k(x, \xi) = \frac{\gamma}{8\pi(\lambda+2\mu)}\left\{\frac{\partial}{\partial \xi_k}[r_1^2(\ln r_1 - 1) - r_2^2(\ln r_2 - 1) - r^2(\ln r - 1) + r_{12}^2(\ln r_{12} - 1)]\right.$ of GFs presented in [29], if functions r_1, $\ln r_1$, r_{12}, $\ln r_{12}$ are omitted, the functions r_2, $\ln r_2$ are changed by r_1, $\ln r_1$ and symbol $k = 1, 2$ is changed by symbol $i = 1, 2$ so that final expressions for MTGFs have the form:

$$U_i(x, \xi) = -\gamma[8\pi(\lambda + 2\mu)]^{-1} \frac{\partial}{\partial \xi_i} \left[\mathrm{r}^2(\ln r - 1) + r_1^2(\ln r_1 - 1) \right]; \quad i = 1, 2 \tag{43}$$

Indeed, taking the derivatives in (43) it is observed that the expressions (42) and (43) coincide.

NEW EXPLICIT 3D MTGFs FOR THERMOELASTIC QUARTER-SPACE AND OCTANT

1. New MTGFs within quarter-space. Let the field of displacements $U_i(x, \xi)$ and temperature $G_T(x, \xi)$ at inner points $x \equiv (x_1, x_2, x_3)$ and $\xi \equiv (\xi_1, \xi_2, \xi_3)$ of the thermoelastic quarter-space $V(0 \le x_1, x_2, -\infty < x_3 < +\infty)$ be determined by non-homogeneous Lamé equations (4) and Poisson equation $\nabla_x^2 G_T(x, \xi) = -\delta(x - \xi)$ and in the points $y \equiv (0, y_2, y_3)$ and $y \equiv (y_1, 0, y_3)$ of boundary half-planes $\Gamma_{10}(y_1 = 0, 0 \le y_2 < +\infty, -\infty < y_3 < +\infty)$ and $\Gamma_{20}(0 \le y_1 < +\infty, y_2 = 0, -\infty < y_3 < +\infty)$ the following homogeneous mechanical and thermal conditions are given:

$$\sigma_{11} = U_2 = U_3 = 0; \quad G_T = 0 \tag{44}$$

– locally mixed boundary conditions on the boundary half-plane $\Gamma_{10}(y_1 = 0, 0 \le y_2 < +\infty, -\infty < y_3 < +\infty)$

$$\sigma_{21} = U_2 = \sigma_{23} = 0; \quad \partial G_T / \partial n_{\xi 2} = 0 \tag{45}$$

– locally mixed boundary conditions on the boundary half-plane $\Gamma_{20}(0 \le y_1 < +\infty, y_2 = 0, -\infty < y_3 < +\infty)$.

Then, according to the second boundary conditions (28) and the first boundary conditions (29), the respective boundary conditions for GFs $G_i(x, \xi)$ for Poisson's equation are:

$$G_{1,1} = G_2 = G_3 = G_T = 0 \tag{46}$$

on the boundary half-plane Γ_{10}, and

$$G_{1,2} = G_2 = G_{3,2} = G_{T,2} = 0 \tag{47}$$

on the boundary half-plane Γ_{20}.

So, substituting well-known expressions of GFs for quarter-space

$$G_1(x, \xi) = (4\pi)^{-1}\left(R^{-1} + R_1^{-1} + R_2^{-1} + R_{12}^{-1}\right)$$
$$G_2(x, \xi) = (4\pi)^{-1}\left(R^{-1} - R_1^{-1} - R_2^{-1} + R_{12}^{-1}\right)$$
$$G_3(x, \xi) = G_T(x, \xi) = (4\pi)^{-1}\left(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1}\right)$$

$$R_2 = \sqrt{(x_1 - \xi_1)^2 + (x_2 + \xi_2)^2 + (x_3 - \xi_3)^2};$$
$$R_{12} = \sqrt{(x_1 + \xi_1)^2 + (x_2 + \xi_2)^2 + (x_3 - \xi_3)^2} \quad (48)$$

in constructive formula (15), we obtain final explicit expressions for MTGFs $U_i(x, \xi)$ in the form:

$$\begin{aligned} U_1(x,\xi) &= \gamma[8\pi(\lambda+2\mu)]^{-1}\big[\xi_1(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1}) \\ &\qquad - x_1(R^{-1} + R_1^{-1} + R_2^{-1} + R_{12}^{-1})\big] \\ U_2(x,\xi) &= \gamma[8\pi(\lambda+2\mu)]^{-1}\big[\xi_2(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1}) \\ &\qquad - x_2(R^{-1} - R_1^{-1} - R_2^{-1} + R_{12}^{-1})\big] \\ U_3(x,\xi) &= \gamma[8\pi(\lambda+2\mu)]^{-1}(\xi_3 - x_3)\big(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1}\big) \\ \Theta(x,\xi) &= \gamma[4\pi(\lambda+2\mu)]^{-1}\big(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1}\big) \end{aligned} \quad (49)$$

or in the more compact form:

$$U_i(x,\xi) = \gamma[8\pi(\lambda+2\mu)]^{-1}\frac{\partial}{\partial \xi_i}(R - R_1 + R_2 - R_{12}) \quad (50)$$

Indeed, taking the derivatives in (50) it is observed that the expressions (49) and (50) coincide.

Note, that from formulas (49) and (50) for quarter-space in the particular case of half-space $V(0 \le x_1 \le \infty, -\infty < x_2, x_3 < \infty)$ we obtain the respective formulas (36) and (37).

Finally, calculating on the basis of the functions (50) the other influence functions $\partial U_k(0, y_2, y_3; \xi)/\partial n_{10}$; on marginal half-plane Γ_{10} and $U_k(y_1, 0, y_3; \xi)$ on marginal half-plane Γ_{20}) and substituting these functions in Eq. (1) we obtain the following solution in integrals of the above-mentioned BVP for the thermoelastic quarter-space in the author's form:

$$\begin{aligned} U_i(\xi) &= \gamma[8\pi(\lambda+2\mu)]^{-1}\frac{\partial}{\partial \xi_i}\Bigg\{a^{-1}\int_0^{+\infty}\int_0^{+\infty}\int_{-\infty}^{+\infty} F(z)[R(z,\xi) - R_1(z,\xi) + R_2(z,\xi) \\ &\qquad - R_{12}(z,\xi)]dz_1 dz_2 dz_3 \\ &\quad - 2\int_0^{+\infty}\int_{-\infty}^{+\infty} T(0, y_2, y_3)[\xi_1(R(0, y_2, y_3; \xi) + R_2(0, y_2, y_3; \xi))]dy_2 dy_3 \\ &\quad + 2\int_0^{+\infty}\int_{-\infty}^{+\infty} \frac{\partial T(y_1, 0, y_3)}{\partial n_{20}}[R(y_1, 0, y_3; \xi) - R_1(y_1, 0, y_3; \xi)]dy_1 dy_3; \\ &\quad z \equiv (z_1, z_2, z_3); \quad \xi \equiv (\xi_1, \xi_2, \xi_3) \end{aligned} \quad (51)$$

where n_{20} is the exterior normal on the boundary half-plane, $\Gamma_{20}(y_2 = 0, 0 \le y_1, y_3 \le +\infty)$.

2. New MTGFs within octant. Let the field of displacements $U_i(x, \xi)$ and temperature $G_T(x, \xi)$ at inner points $\xi \equiv (\xi_1, \xi_2, \xi_3)$ of the thermoelastic octant

$V(0 \leq x_1, x_2, x_3 < +\infty)$ be determined by non-homogeneous Lamé equations (4) and the Poisson equation $\nabla_x^2 G_T(x,\xi) = -\delta(x-\xi)$ and in the points $y \equiv (0, y_2, y_3)$, $y \equiv (y_1, 0, y_3)$ and $y \equiv (y_1 y_2, 0)$ of boundary quadrants $\Gamma_{10}(y_1 = 0, 0 \leq y_2, y_3 < +\infty)$, $\Gamma_{20}(0 \leq y_1 < +\infty, y_2 = 0, 0 \leq y_3 < +\infty)$ and $\Gamma_{30}(0 \leq y_1, y_2 < +\infty, y_3 = 0)$, the following homogeneous mechanical and thermal conditions are given:

$$\sigma_{11} = U_2 = U_3 = 0; \quad G_T = 0 \tag{52}$$

– locally mixed boundary conditions on the marginal quadrant $\Gamma_{10}(y_1 = 0, 0 \leq y_2, y_3 < +\infty)$

$$\sigma_{21} = U_2 = \sigma_{23} = 0; \quad \partial G_T/\partial n_{\xi 2} = 0 \tag{53}$$

– locally mixed boundary conditions on the marginal quadrant $\Gamma_{20}(0 \leq y_1 < \infty, y_2 = 0, 0 \leq y_3 < +\infty)$
and

$$\sigma_{31} = \sigma_{32} = U_3 = 0; \quad \partial G_T/\partial n_{\xi 3} = 0 \tag{54}$$

– locally mixed boundary conditions on the marginal quadrant $\Gamma_{30}(0 \leq y_1, y_2 < +\infty, y_3 = 0)$.
Then, according to the second boundary conditions (28), the first boundary conditions (29) and, the first boundary conditions (30), the respective boundary conditions for GFs $G_i(x,\xi)$ for the Poisson equation are the following:

$$G_{1,1} = G_2 = G_3 = G_T = 0 \tag{55}$$

on the marginal quadrant Γ_{10},

$$G_{1,2} = G_2 = G_{3,2} = G_{T,2} = 0 \tag{56}$$

on the marginal quadrant Γ_{20} and

$$G_{1,3} = G_{2,3} = G_3 = G_{T,3} = 0 \tag{57}$$

on the marginal quadrant Γ_{30}.
So, substituting well-known expressions of GFs for octant V

$$\begin{aligned}
G_1(x,\xi) &= (4\pi)^{-1}\left(R^{-1} + R_1^{-1} + R_2^{-1} + R_{12}^{-1} + R_3^{-1} + R_{13}^{-1} + R_{23}^{-1} + R_{123}^{-1}\right)\\
G_2(x,\xi) &= (4\pi)^{-1}\left(R^{-1} - R_1^{-1} - R_2^{-1} + R_{12}^{-1} + R_3^{-1} - R_{13}^{-1} - R_{23}^{-1} + R_{123}^{-1}\right)\\
G_3(x,\xi) &= (4\pi)^{-1}\left(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} - R_3^{-1} + R_{13}^{-1} - R_{23}^{-1} + R_{123}^{-1}\right)\\
G_T(x,\xi) &= (4\pi)^{-1}\left(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} + R_3^{-1} - R_{13}^{-1} + R_{23}^{-1} - R_{123}^{-1}\right)\\
R_2 &= \sqrt{(x_1-\xi_1)^2 + (x_2+\xi_2)^2 + (x_3-\xi_3)^2};\\
R_{12} &= \sqrt{(x_1+\xi_1)^2 + (x_2+\xi_2)^2 + (x_3-\xi_3)^2}
\end{aligned}$$

$$R_3 = \sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 + \xi_3)^2};$$
$$R_{13} = \sqrt{(x_1 + \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 + \xi_3)^2}$$
$$R_{23} = \sqrt{(x_1 - \xi_1)^2 + (x_2 + \xi_2)^2 + (x_3 + \xi_3)^2};$$
$$R_{123} = \sqrt{(x_1 + \xi_1)^2 + (x_2 + \xi_2)^2 + (x_3 + \xi_3)^2} \quad (58)$$

in the constructive formula (15) we obtain the final explicit expressions for MTGFs $U_i(x, \xi)$ in the form:

$$\begin{aligned} U_1(x, \xi) &= \gamma[8\pi(\lambda + 2\mu)]^{-1} \\ &\quad \times [\xi_1(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} + R_3^{-1} - R_{13}^{-1} + R_{23}^{-1} - R_{123}^{-1}) \\ &\quad - x_1(R^{-1} + R_1^{-1} + R_2^{-1} + R_{12}^{-1} + R_3^{-1} + R_{13}^{-1} + R_{23}^{-1} + R_{123}^{-1})] \\ U_2(x, \xi) &= \gamma[8\pi(\lambda + 2\mu)]^{-1} \\ &\quad \times [\xi_2(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} + R_3^{-1} - R_{13}^{-1} + R_{23}^{-1} - R_{123}^{-1}) \\ &\quad - x_2(R^{-1} - R_1^{-1} - R_2^{-1} + R_{12}^{-1} + R_3^{-1} - R_{13}^{-1} - R_{23}^{-1} + R_{123}^{-1})] \\ U_3(x, \xi) &= \gamma[8\pi(\lambda + 2\mu)]^{-1} \\ &\quad \times [\xi_3(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} + R_3^{-1} - R_{13}^{-1} + R_{23}^{-1} - R_{123}^{-1}) \\ &\quad - x_3(R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} - R_3^{-1} + R_{13}^{-1} - R_{23}^{-1} + R_{123}^{-1})] \\ \Theta(x, \xi) &= \frac{\gamma}{4\pi(\lambda + 2\mu)}[R^{-1} - R_1^{-1} + R_2^{-1} - R_{12}^{-1} + R_3^{-1} - R_{13}^{-1} + R_{23}^{-1} - R_{123}^{-1}] \end{aligned} \quad (59)$$

or in the more compact form:

$$U_i(x, \xi) = \gamma[8\pi(\lambda + 2\mu)]^{-1} \frac{\partial}{\partial \xi_i} \left(R - R_1 + R_2 - R_{12} + R_3^{-1} - R_{13}^{-1} + R_{23}^{-1} - R_{123}^{-1}\right) \quad (60)$$

Indeed, taking the derivatives in (60), it is observed that expressions (59) and (60) coincide.

Note that from formulas (59) and (60) for the thermoelastic octant, in the particular case of quarter-space $V(0 \le x_1, x_2, -\infty < x_3 < \infty)$, we obtain the respective formulas (49) and (50). The Appendix presents graphics of behavior of MTGFs $U_2(x, \xi)$ in Eq. (59) in dependence of the variables $\xi \equiv (\xi_1, \xi_2, \xi_3)$ at some values of the variables $x \equiv (x_1, x_2, x_3)$ obtained by using Maple 15 software.

Finally, calculating on the basis of functions in Eq. (60) the other influence functions

$$\begin{aligned} \partial U_i(y, \xi)/\partial n_{10} = -\partial U_i(0, y_2, y_3, \xi)/\partial y_1 &= 2\gamma[8\pi(\lambda + 2\mu)]^{-1} \\ &\quad \times [\xi_1(R_3^{-1}(0, y_2, y_3; \xi) + R_{23}^{-1}(0, y_2, y_3; \xi) \\ &\quad - R^{-1}(0, y_2, y_3; \xi) - R_2^{-1}(0, y_2, y_3; \xi))] \end{aligned} \quad (61)$$

$$U_i(y,\xi) = U_i(y_1,0,y_3;\xi) = 2\gamma[8\pi(\lambda+2\mu)]^{-1}[R(y_1,0,y_3;\xi) - R_1(y_1,0,y_3;\xi) + R_3(y_1,0,y_3;\xi) - R_{13}(y_1,0,y_3;\xi)] \quad (62)$$

$$U_i(y,\xi) = U_i(y_1,y_2,0;\xi) = 2\gamma[8\pi(\lambda+2\mu)]^{-1}[R(y_1,y_2,0;\xi) - R_1(y_1,y_2,0;\xi) + R_2(y_1,y_2,0;\xi) - R_{12}(y_1,y_2,0;\xi)] \quad (63)$$

and substituting these functions in formula (1), we obtain the following solution in integrals of the above-mentioned BVP for the thermoelastic octant in the author's form:

$$\begin{aligned} U_i(\xi) = \gamma[8\pi(\lambda+2\mu)]^{-1}\frac{\partial}{\partial\xi_i}\Bigg\{ & a^{-1}\int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} F(z)[R(z,\xi) - R_1(z,\xi) + R_2(z,\xi) \\ & - R_{12}(z,\xi) + R_3(z,\xi) - R_{13}(z,\xi) \\ & + R_{23}(z,\xi) - R_{123}(z,\xi)]dz_1dz_2dz_3 \\ & - 2\int_0^{+\infty}\int_0^{+\infty} T(0,y_2,y_3)\left[\xi_1\left(R_3^{-1}(0,y_2,y_3,\xi) + R_{23}^{-1}(0,y_2,y_3,\xi)\right.\right. \\ & \left.\left. - R^{-1}(0,y_2,y_3,\xi) - R_2^{-1}(0,y_2,y_3,\xi)\right)\right]dy_2dy_3 \\ & + 2\int_0^{+\infty}\int_0^{+\infty} \frac{\partial T((y_1,0,y_3))}{\partial n_{20}}[R(y_1,0,y_3;\xi) - R_1(y_1,0,y_3;\xi) \\ & + R_3(y_1,0,y_3;\xi) - R_{13}(y_1,0,y_3;\xi)]dy_1dy_3 \\ & + 2\int_0^{+\infty}\int_0^{+\infty} \frac{\partial T(y_1,y_2,0)}{\partial n_{30}}[R(y_1,y_2,0;\xi) - R_1(y_1,y_2,0;\xi) \\ & + R_2(y_1,y_2,0;\xi) - R_{12}(y_1,y_2,0;\xi)]dy_1dy_2\Bigg\} \end{aligned}$$

$$z \equiv (z_1,z_2,z_3); \quad \xi \equiv (\xi_1,\xi_2,\xi_3) \quad (64)$$

where n_{20} and n_{30}- are the exterior normal on the boundary quarter-plane $\Gamma_{20}(y_2 = 0, 0 \le y_1, y_3 \le \infty)$ and $\Gamma_{30}(y_3 = 0, 0 \le y_1, y_2 \le +\infty)$, respectively.

CONCLUSIONS

A new approach for derivation of MTGFs $U_i(x,\xi)$ directly from the respective Lamé equations (4) is proposed for the first time. This approach is based on new general integral representations for functions $U_i(x,\xi)$ that are presented in Eqs. (8)–(10). A theorem on constructive formulas (15) for MTGFs $U_i(x,\xi)$ in terms of GFPE is proved. According to the constructive formulas obtained, the derivation of MTGFs for about 20 BVPs for a plane, a half-plane, a quadrant, a space, a quarter-space and an octant may be obtained by changing the respective well-known GFPE.

All results are obtained in terms of elementary functions with three examples of their validation. Two new MTGFs for thermoelastic quarter-space and octant together with graphical computer evaluation are also included. The main advantages of the proposed approach, in comparison with the $G\Theta$ convolution method for constructing MTGFs, are: (a) It is not necessary to derive the functions of influence of a unit concentrated force onto elastic volume dilatation - $\Theta^{(i)}$, and (b) it is not necessary to calculate a complicated volume integral of the product of the function $\Theta^{(i)}$and Green's function in heat conduction. The proposed approach may be extended to canonical domains of any orthogonal system of coordinates.

APPENDIX

GRAPHICS OF MTGFs U_2 WITHIN THERMOELASTIC OCTANT $V(0 \leq X_1, X_2, X_3 < +\infty)$

Here, we present Figures 1–3, showing the MTGFs U_2 determined by Eq. (68) for 3D BVP of thermoelasticity (4), (61)–(63) within the octant $V(0 \leq x_1, x_2, x_3 < +\infty)$, constructed using computer program Maple 15 [30].

The MTGFs U_2 are created by the unitary inner point heat source $F = \delta(x - \xi)$. All six graphics for the MTGFs U_2 were constructed at the following values of the constants: Poisson ratio, $v = 0.3$; elasticity modulus $E = 2, 1 \times 10^5\, MPa$ and coefficient of linear thermal expansion $\alpha = 18 \times 10^{-6}(K)^{-1}$, $\mu = E/[2(1 + v)]$; $\mu = 80769.23077\, MPa$; $\lambda = 2v\mu/(1 - 2v)$; $\lambda = 1.211538462 \times 10^5\, MPa$; and $\gamma = \alpha(2\mu + 3\lambda)$; $\gamma = 9.450000002\, MPa$.

Graphics of the MTGFs U_2 within the octant $V(0 \leq x_1, x_2, x_3 < +\infty)$, constructed independently of $0 \leq \xi_1 \leq 10\, m$; $0 \leq \xi_2 \leq 10\, m$ at $x_1 = x_2 = x_3 = 5\, m$ *and: a.* $\xi_3 = 0.2\, m$; b. $\xi_3 = 5\, m$ are shown in Figure 1.

On Figure 1 it can be seen that:

1. At $\xi_1 = 0\, m$ *and* $\xi_2 = 0\, m$ the MTGFs U_2 are zero. This means that boundary condition $U_2(x, \xi_1 = 0,\ \xi_2, \xi_3) = 0$ in Eq. (61) on the marginal quadrant Γ_{10} and boundary condition $U_2(x, \xi_1,\ \xi_2 = 0, \xi_3) = 0$ (62) on the marginal quadrant Γ_{20} are satisfied (see Figures 1a and b);
2. At $\xi_3 = 0.2\, m$ when $\xi_1 \to 1\, m$ and $\xi_2 \to 2\, m$ the MTGFs U_2 are maximal, $U_2 = U_{2\max} = 5 \times 10^{-7}\, m$ and tike positive magnitudes only (see Figure 1a);
3. At $\xi_3 = 3\, m$, when $\xi_1 \to 1\, m$ and $\xi_2 \to 2 + 0\, m$ the MTGFs U_2 tend to maximal magnitude, $\lim U_2 \to 5 \cdot 10^{-5}\, m$. When $\xi_1 \to 1\, m$ and $\xi_2 \to 2 - 0\, m$ at $\xi_3 = 3\, m$ displacements U_2 tend to minimal magnitude, $\lim U_2 \to -1 \cdot 10^{-6}\, m$ (see Figure 1b). So in the point $\xi_i = x_i (i = 1, 2, 3)$ the function U_2 has the following singularity: $\lim\limits_{\xi_2 \mapsto x_2 + 0} U_2 \to +\infty$ and $\lim\limits_{\xi_2 \mapsto x_2 - 0} U_2 \to -\infty$;
4. At $\xi_3 = 3\, m$, when $\xi_1 > 1\, m$ and $\xi_2 > 2\, m$ the MTGFs U_2-diminishes (see Figure 1b).

Graphics of the MTGFs U_2 within the octant $V(0 \leq x_1, x_2, x_3 < +\infty)$, constructed in dependence of $0 \leq \xi_1 \leq 10\, m$; $0 \leq \xi_2 \leq 10\, m$ at $x_1 = x_2 = x_3 = 5\, m$ *and: a.* $\xi_3 = 0.2\, m$; b. $\xi_3 = 5\, m$ are shown on Figure 2.

On Figure 2 it can be observed:

1. At $\xi_1 = 0\, m$ *and* $\xi_2 = 0\, m$ MTGFs U_2 are zero. This means that boundary condition $U_2(x, \xi_1 = 0,\ \xi_2, \xi_3) = 0$ in Eq. (61) on the marginal quadrant Γ_{10} and boundary condition $U_2(x, \xi_1,\ \xi_2 = 0, \xi_3) = 0$ (62) on the marginal quadrant Γ_{20} are satisfied (see Figures 2a and b);

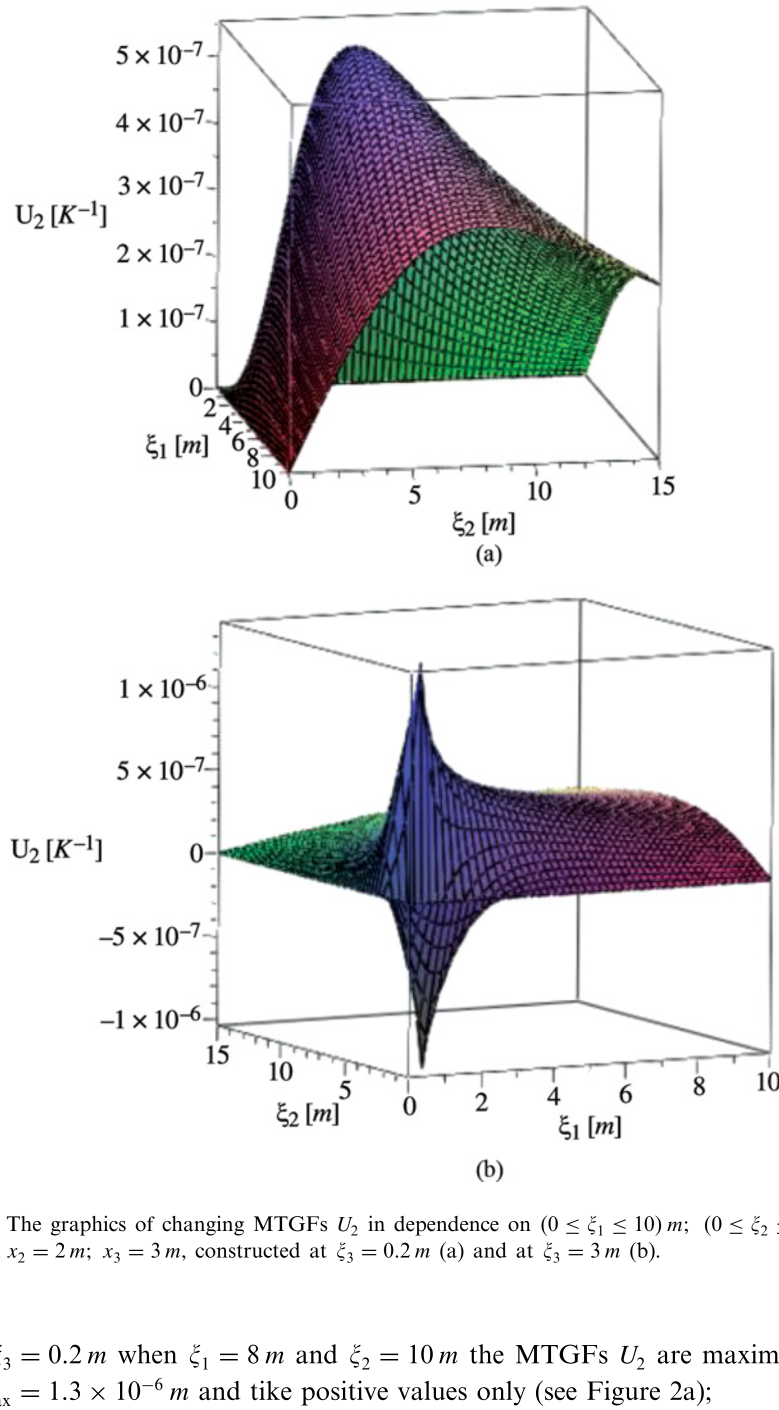


Figure 1 The graphics of changing MTGFs U_2 in dependence on $(0 \leq \xi_1 \leq 10)\,m$; $(0 \leq \xi_2 \leq 15)\,m$ at $x_1 = 1\,m$; $x_2 = 2\,m$; $x_3 = 3\,m$, constructed at $\xi_3 = 0.2\,m$ (a) and at $\xi_3 = 3\,m$ (b).

2. At $\xi_3 = 0.2\,m$ when $\xi_1 = 8\,m$ and $\xi_2 = 10\,m$ the MTGFs U_2 are maximal, $U_2 = U_{2\max} = 1.3 \times 10^{-6}\,m$ and tike positive values only (see Figure 2a);
3. At $\xi_3 = 5\,m$, when $\xi_1 \to 5\,m$ and $\xi_2 \to 5 + 0\,m$ the MTGFs U_2 tend to maximal value, $\lim U_2 \to 1.6 \cdot 10^{-6}\,m$. When $\xi_1 \to 5\,m$ and $\xi_2 \to 5 - 0\,m$ at $\xi_3 = 5\,m$ displacements U_2 tend to minimal value, $\lim U_2 \to -5 \cdot 10^{-7}\,m$ (see Figure 2b).

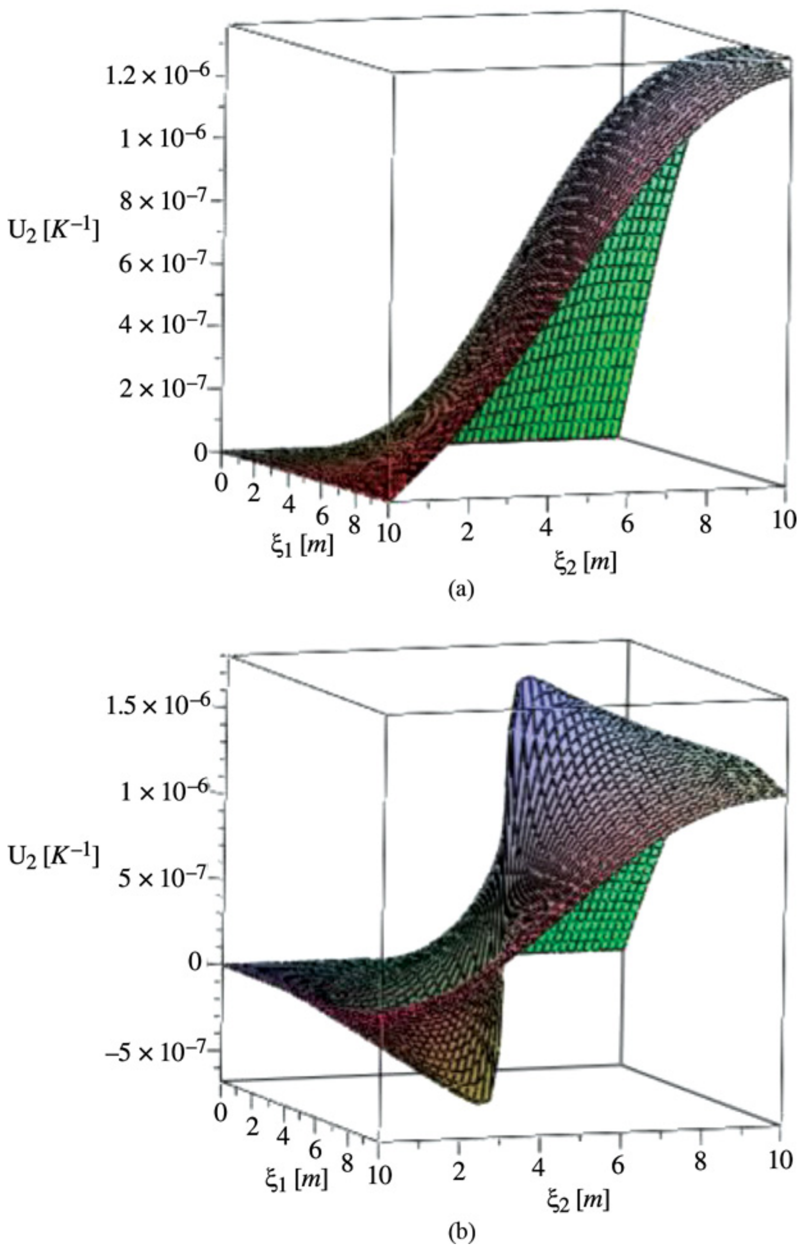
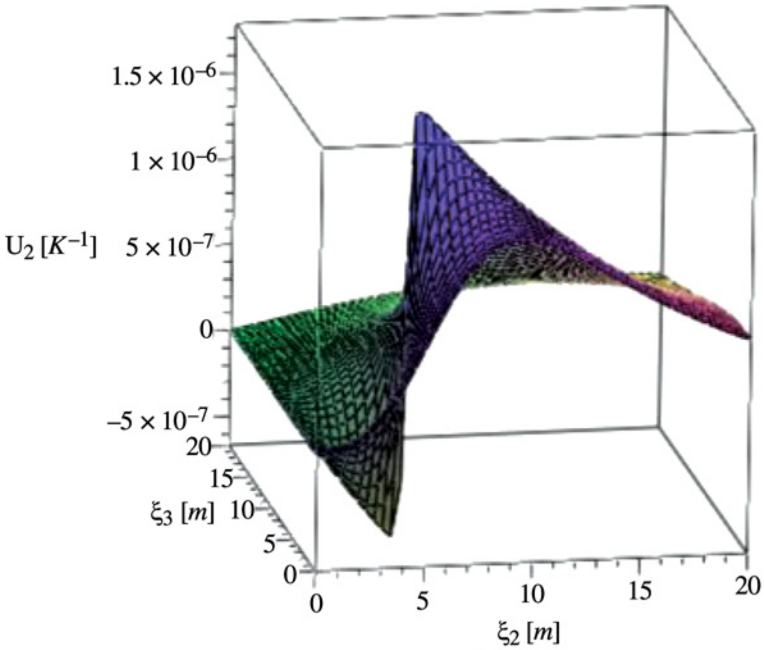


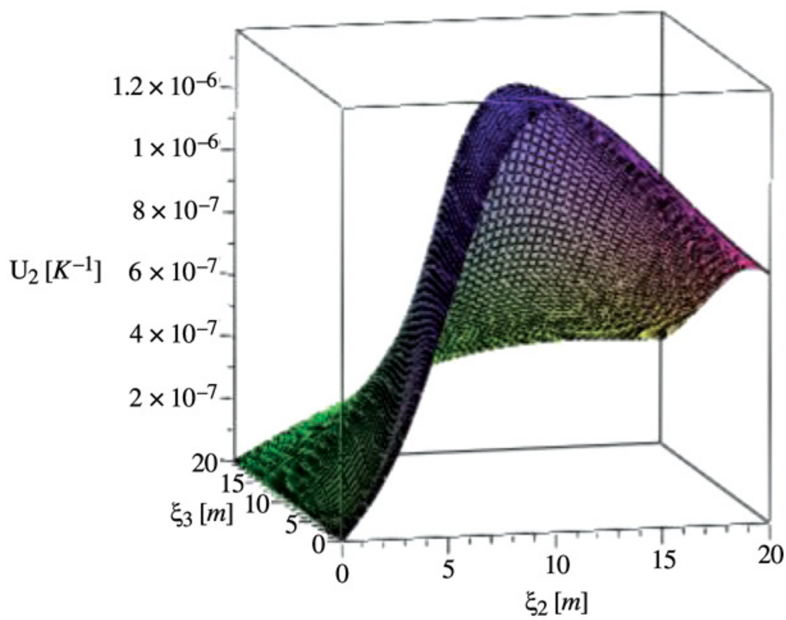
Figure 2 The graphics of changing MTGFs U_2 in dependence on $0 \le \xi_1 \le 10\,m$; $0 \le \xi_2 \le 10\,m$ at $x_1 = x_2 = x_3 = 5\,m$, constructed at $\xi_3 = 0.2\,m$ (a) and at $\xi_3 = 5\,m$ (b).

So, in the point $\xi_i = x_i$ $(i = 1, 2, 3)$ the function has the following singularity: $\lim\limits_{\xi_2 \mapsto x_2+0} U_2 \to +\infty$ and $\lim\limits_{\xi_2 \mapsto x_2-0} U_2 \to -\infty$.

4. At $\xi_3 = 5m$, when $\xi_1 > 5\,m$ and $\xi_2 > 5\,m$ the module of the MTGFs $|U_2|$ diminishes (see Figure 2b).



(a)



(b)

Figure 3 The graphics of changing MTGFs U_2 in dependence on $0 \le \xi_1 \le 10\,m$; $0 \le \xi_2 \le 10\,m$ at $x_1 = x_2 = x_3 = 5\,m$, constructed at $\xi_1 = 5\,m$ (a) and at $\xi_1 = 8\,m$ (b).

Graphics of the MTGFs U_2 within the octant $V(0 \leq x_1, x_2, x_3 < +\infty)$, constructed in dependence of $0 \leq \xi_2 \leq 20\,m$; $0 \leq \xi_3 \leq 20\,m$ at $x_1 = x_2 = x_3 = 5\,m$ *and: a.* $\xi_1 = 5\,m$; b. $\xi_1 = 8\,m$ are shown on Figure 3.

On Figure 3 it can be observed:

1. At $\xi_2 = 0\,m$ the MTGFs U_2 are zero. This means that boundary condition $U_2(x, \xi_1,\ \xi_2 = 0, \xi_3) = 0$ in Eq. (62) on the marginal quadrant Γ_{20} are satisfied (see Figures 3a and b);
2. The character of the changing MTGFs U_2 in the interval $(1 \cdot 10^{-7} \leq \xi_1 < 8)m$, is the same as that shown in Figure 3a at $\xi_1 = 5m$;
3. The character of the changing MTGFs U_2 in the interval $(8 \leq \xi_1 < +\infty)m$, is the same as that shown in Figure 3b at $\xi_1 = 8m$;
4. At $\xi_1 = 5m$ MTGFs U_2 are negative when $0 \leq \xi_2 < 5 - 0\,m$ and positive when $5 + 0 \leq \xi_2 < 20\,m$. At $\xi_2 \geq 20\,m$, $U_2 = 0$. At $\xi_2 \to 5 - 0\,m$ the function U_2 has minimal value $U_2 = U_{2\min} = -5 \times 10^{-7}\,m$. At $\xi_2 \to 5 + 0\,m$ the function U_2 has maximal value $U_2 = U_{2\max} = 1.5 \times 10^{-6}\,m$(see Figure 3a).

NOMENCLATURE

BVP	boundary values problem
GF	Green's function
GFPEs	Green's functions for the Poisson equation
MTGFs	main thermoelastic Green's functions
2D	two-dimensional
3D	three-dimensional
MPa	mega Pascal
K	grades Kelvin

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